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Research Article

Volume of geometric solids on the Desmos platform – A didactic experience in Cape Verde

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ABSTRACT

This work intends to disseminate a didactic experience in mathematics, in times of pandemic, in an emergency remote teaching situation, at the Portuguese School of Cape Verde, using the Desmos digital platform. The topic addressed was the study of the volume of geometric solids. The main objective was to contribute to the learning of mathematical concepts, using digital tools that promote students' autonomy while respecting their learning pace, thus improving teaching practices through more assertive methodologies and more innovative resources. In order to understand how the Desmos platform contributes to improve the learning of geometry an explanatory case study was conducted. The participants in the study are 9th grade students from a Cape Verde school. Preliminary analysis of the data collected through several techniques, using diverse data collecting mechanisms, indicates that the contribution is very positive with clear advantages in the construction of knowledge on the part of the students, centered in a logic of skill development.

Keywords: emergency remote teaching, Cape Verde, volume of geometric solids, Desmos

INTRODUCTION

The pandemic associated with COVID-19 forced the closure of schools, with strict confinements and, in a unique context, students' homes became the classroom. This urgency was a decisive factor for the necessary changes to be carried out in schools. With no time for great discussions, debates and studies, which normally postpone the implementation of reforms, the world's educational systems were forced to change the paradigm of their practices. What takes years to implement had to be assimilated in a short period of time, namely the application of new methodologies in the teaching and learning process. We are at a stage where we enter the unknown, where it is necessary to adapt actions as challenges arise, as happened in the age of maritime discoveries (Osório, 2020).

The response to this Emergency Distance Learning (Carrillo & Flores, 2020; Hodges et al., 2020; Iglesias-Pradas et al., 2021; Shim & Lee, 2020) gave rise to a proliferation of digital tools that brought both positive and negative aspects. On the positive side, working in virtual learning environments promotes student autonomy, adjusts the learning pace, is more motivating, enhances collaboration between peers and develops higher-level skills (Barker & Gossman, 2013; Beluce & de Oliveira, 2015; Chen & Jang, 2010). The use of these environments has considerable gains in the field of evaluation, since there is a record of the work carried out with the possibility of giving feedback, that is, there is an almost permanent monitoring of the activity developed (Lima, 2019). However, they create more barriers to the emotional relationship between the teacher and the student, and can trigger greater social inequalities, to the detriment of the most disadvantaged, given the need to use computer equipment, which often cannot be provided by schools (Gautam, 2020). In addition, they pose problems of security and privacy of the data of its users, since almost all platforms require registration. Also, many of them require fee payments for use of their general potential.

This technological boom can be used to carry out trans-disciplinary projects, which do not place the curriculum and programs at the center of the educational action but instead promote more active teaching methodologies (Osório, 2020). It is up to the teachers to give pedagogical intentionality to the digital resources available to them (Barbosa et al., 2014).

If, at present, the conditions for the implementation of information and communication technologies (ICT) are clearly better, this does not mean that learning has improved. In many cases, there was only an attempt to replicate face-to-face teaching in online classes, maintaining a very teacher-centered education, where the role of students continues to be too passive. In fact, the epidemiological situation has been very unstable, at times allowing the opening of schools and the return to face-to-face teaching and, at other times, forcing them to close and return to online classes. This alternation has not allowed us to advance towards truly active learning in which the student has a central role, regardless of whether it is an online or face-to-face class.

Remote learning is not synonymous with active learning, nor does it automatically develop students' autonomy (Fonseca & Mattar, 2017). There will always have to be a paradigm shift in which the teacher changes from being a transmitter of knowledge to a facilitator of student learning (Barbosa et al., 2014).

Today's children and young people are digital natives (Prensky, 2001) who assimilate information and knowledge differently. Despite their natural dexterity for handling computer equipment, their digital skills are still at the most basic level. Teachers do not need to be ICT professionals, but rather integrators in their teaching practices, turning digital technologies into educational technologies. This integration will allow students to develop more advanced skills through the understanding of the contents and the use of digital tools. Digital tools have been continuously optimized, making it possible to diversify teaching and learning methodologies, as well as their forms of assessment, namely formative assessment (Decreto-Lei n.o 55/2018 Do Ministério da Educação, 2018).

In the field of mathematics teaching, the use of technology is already quite widespread, namely in geometry. However, it has only recently been possible to carry out broader learning tasks that enhance the students' autonomy, through self-correcting activities that respect each student's learning pace.

From an early age, the basic geometric concepts are worked on, but it is not uncommon for students to confuse the notions of perimeter and area. Associated with these notions, students also have difficulties in distinguishing objects in different dimensions, identifying, for example, a cube as a square (Leung, 2010). The importance of this branch of mathematics in the structuring of the world around us is highlighted and its perception depends on the geometric knowledge that we have (Gonçalves, 2019). In certain geometric problems, there is a great connection with other areas of mathematics, which require a phased and, consequently, more challenging resolution. It is, therefore, important to take a step-by-step approach, which contextualizes and justifies certain results, using digital resources to support learning. Task solving must give meaning to the geometric concepts to be addressed, thus ensuring student involvement (Gonçalves, 2019).

This work intends to disseminate a didactic experience in mathematics, in times of pandemic, in an emergency remote teaching situation, at the Portuguese School of Cape Verde, using the Desmos digital platform. The topic addressed was the study of the volume of geometric solids. The main objective was to contribute to the learning of mathematical concepts, using digital tools that promote students' autonomy while respecting their learning pace, thus improving teaching practices through more assertive methodologies and more innovative resources.

Emergency Remote Teaching in Cape Verde

In Cape Verde, the health emergency context was extremely challenging. Schools had to adapt to a new reality, but with more limited resources. Classes were divided into two shifts, one of them had face-to-face classes on Mondays, Wednesdays and Fridays and the other on Tuesdays and Thursdays. During those days when students stayed at home, all activities were asynchronous.

In 2019, only 25.4% of Cape Verde's population used the computer at least once in three months. If we consider the age group that attends basic education, the percentage drops to 12.6% (INE, 2020). The Internet access is more widespread, with 73.6% of households, in an urban context, connected to the web. However, the main means of access is the mobile phone (92.9%) (INE, 2020). As the reality of Cape Verde, at a socio-economic level, is quite different from that of European countries, namely in relation to wages, many families have difficulty in bearing the costs of internet connection, especially with regard to faster and stable connections (INE, 2020).

The Portuguese School of Cape Verde is a Portuguese school abroad, thus with different characteristics from other schools. In order to respond to this problem, it was equipped with videoconference systems, through the Teams platform, to alleviate the difficulties created by the various periods of quarantine. But, the biggest difficulty, as mentioned above, was the lack of equipment and stable internet access. Despite the school population being from a higher social stratum, few students had the opportunity to use a computer exclusively. The equipment had to be shared by family members in telecommuting and siblings, creating several constraints to the normal functioning of school activities. In its distance learning plan, the school chose to use Microsoft's Teams platform (educational version). This platform is a hub that brings together multiple features for a school, namely, classes, meetings, tasks, files, among others.

Digital Platforms in Mathematics Teaching

Regarding the subject of mathematics, in accordance with the essential learnings document (Ministério da Educação [Ministry of Education], 2018), learning practices must include digital technology and the calculator, both in problem solving and in other learning tasks. In geometry, in particular, students should develop the ability to visualize and understand the properties of geometric figures (Ministério da Educação [Ministry of Education], 2018).

The most used digital platforms, because they are associated with textbooks used in schools, are the "Escola Virtual" (Virtual School), by the Porto Editora group and the "Aula Digital" (Digital Class), by the Leya group. In both, students have to register to have access to materials that their teachers provide and have a wide range of resources, which can be assigned to students through assignments. It is also possible to build learning sequences, with links to the adopted textbook.

Also, in the field of gamification, the platforms are very diverse. Kahoot! and Quizziz are two examples commonly used. Both are based on very motivating multidisciplinary activities and a very attractive graphic design. However, the free version does not give you access to all features.

One of the best-known platforms in mathematics teaching is GeoGebra, a dynamic math software. Recently, the GeoGebra Classroom virtual platform was developed, which allows assigning interactive tasks, monitoring students' progress and creating discussion moments for all or just a part of the students. Access is open and the activity bank is enriched by an extensive community of users from all over the world. The experience that was developed in this research used the Desmos platform.

The Desmos Platform

Desmos is a digital platform accessible through the internet (www.desmos.com/?lang=pt-BR, Portuguese version of Brazil) with free access, which includes several mathematical tools that can be used either on computers, or on mobile devices. In addition, it has a set of activities, developed by the entire Desmos community, which aim to enrich math classes with interactive content. These free digital activities are designed according to a pedagogical philosophy, obeying the following characteristics:

1. Students have the opportunity to be right or wrong in different and interesting ways, as they can create creative representations of their thinking that do not fit the simple label of right and wrong.

2. Students can share their creations.
3. Feedback gives meaning to the student's thinking, without making judgments.
4. The activities create a need, that is, they make students feel that they need to add new tools to existing ones.
5. The activities promote the use of a variety of resources, from digital, paper and pencil, to manipulative materials, creating opportunities to share each student's work with the rest of the class.
6. The philosophy aims to interrupt prejudices, that is, a lot of time is invested in discussing, sharing and questioning our beliefs (Moynihan et al., 2021).

In 2011, its creator, Eli Luberoff, presented his creation to the world, at Disrupt NYC 2011, a meeting of startups. Initially, Desmos was a free interactive calculator, capable of graphically displaying functions. The idea was behind a principle of equity, which argued that students should not have to pay to access a graphing calculator. Its ease of use, at no cost, and appealing interface were decisive for its exponential growth, the result of large investments by large companies, namely Google. The idea evolved and, today, through a multifaceted team, thousands of free digital activities are available, which are already used by around 40 million teachers and students around the world. In 2020, the K-12 math program was launched, which combines the accessible US K-12 curriculum from illustrative mathematics and open up resources with the powerful technology, humanized pedagogy and intuitive design of Desmos (Antunes & Cambrainha, 2020).

The use of Desmos in the classroom can be done in two ways: by students and by teachers. Teachers must create an account so they can store their activities. After logging in, the teacher has access to a very wide range of activities, organized by theme and in different languages. After exploring and choosing the activity, it is possible, in most cases, to copy and edit it so that it can be changed. In addition to using already built activities, users can build their own activities from scratch. The activity edit menu allows them to define the details of the activity. Next, the activity creation layout is available, with multiple items/components: note, text input, math input, multiple choice, checkboxes, ordered list, graph, sketch, media (image or video), geometry, table, action button, graphing calculator, marble slides, and card sort. Each activity page/screen can contain one or a combination of several components, as intended by the user.

The "media" and "note" components are the most used to build activity texts. They can include text with or without math language, images, videos (maximum size 100 MB), and GIF animations. Another very useful component is the "table", which can be used alone or in combination with others. They can be filled in by students and allow for diversified graphic explorations, using the "graphic" component. The "sketch" component is the one that best expresses the students' informal thinking, because they have at their disposal a pen for free drawing, a tool for inserting text, with or without mathematical formulas, and another for representing line segments. The components "text input" and "math input" serve to collect the students' answers and, normally, appear combined with other components. These answers can be given by writing with the virtual keyboard or by uploading images that contain the student's work in another format (for example, a photo of the notebook). These answers can be shared with classmates and there is also the option, in the case of short answers, to allow students to explain their answer. Two of the most interesting components are the "multiple choice" and "checkboxes", because they allow you to diversify the response options. Students can select one or more options, and these options can include text with or without math formulas, images and graphics.

For more enriching activities, there is also the possibility of programming each of these items, through the computation layer (CL), using, for this, a specific programming language. It is a scripting language that allows you to customize and add more interactivity to activities, allowing the connection between different components and screens. To work in this programming environment, one can start with the following steps:

1. identify how you intend to make the activity more interactive,
2. search for existing activities, which already contain pages that serve one's purpose, copy and edit them so that they can be reformulated or adjusted as intended,
3. explore activities that contain simple CL resources and work from them,
4. consult the CL documentation available at https://teacher.desmos.com/computation-layer/documentation?lang=pt-BR#, and

5. access the different sharing forums, namely the CL forum (https://cl.desmos.com/), sending a message to support@desmos.com or simply using Facebook to post a question in the Desmos Educators Group.

The various components can be programmed, according to the intended objectives, and this work is essential to enrich the activities, namely in the characteristics of the students' responses, which can be self-correcting, personalized and interacting with other items.

After the preparation of the activity, it can be assigned to a class, through prior student registration or through unique session codes. Registering students on the platform is more advantageous than assigning them through single-session codes, because in this way students can consult the history of activities carried out. This registration can be done through the Desmos student page at https://student.desmos.com/?lang=pt-BR. Once logged in, the student can access his activities and carry out the work proposed by the teacher. For better organization, the teacher can, in advance, create his own classes, generate an access code and the students will automatically be integrated into the class.

Once the activity is assigned, the teacher has at his disposal an activity panel that serves to monitor, in real time, the students' work. This panel has several very useful features for the teaching and learning process in virtual environments: Once the activity is assigned, the teacher has at his disposal an activity panel that serves to monitor, in real time, the students' work. This panel has several very useful features for the teaching and learning process in virtual environments:

1. summary of the activity, where the teacher can check, which students have already solved each task,
2. teacher's view, where you can view students' responses,
3. student's view, which displays tasks in student mode, that is, as each student sees them, when he accesses them,
4. snapshots, which allows you to take prints of students' work, sequence them and present them to the class for debating the presented resolutions,
5. anonymize, which replaces the students' names with fake names when presenting to the class,
6. pacing, to select the pages that are visible to students, allowing the teacher to control the pace of the class, and
7. pause, to interrupt the visualization of the activity, in case the teacher feels the need to have the students' attention for whatever reason.

Volume of Geometric Solids

Records show that ancient civilizations already made simple area calculations, namely the Babylonians, Egyptians, and Chinese. Regarding geometric solids, there was also a vast knowledge, namely problems related to the volume of the cylinder, with walls and dikes and the number of workers needed to build them (Katz, 2010). The Babylonians also considered the volume of solids related to pyramids, through problems involving a heap of cereals shaped like a rectangular pyramid with an elongated top. All these approaches were practical in nature, susceptible to inaccuracies, but with little effect on final responses (Katz, 2010).

This work focuses on the study of concepts and properties associated with our object of study, the volume of geometric solids, namely prisms, pyramids, cylinders, cones and spheres. The principle that served as the basis for the deductions of the formulas for calculating the volume of the studied solids was the Cavalieri's Principle established in the 17th century by Bonaventura Cavalieri (Katz, 2010):

> **Cavalieri's principle:** Suppose two solids are included between two parallel planes. If every plane parallel to these planes intersects both solids in cross-sections of equal area, then the two solids have equal volume (equivalent solids).

METHOD

Methodological Options

The research question underlying this work is: Does the Desmos platform contribute to promote an active learning of Geometry through student's autonomy?

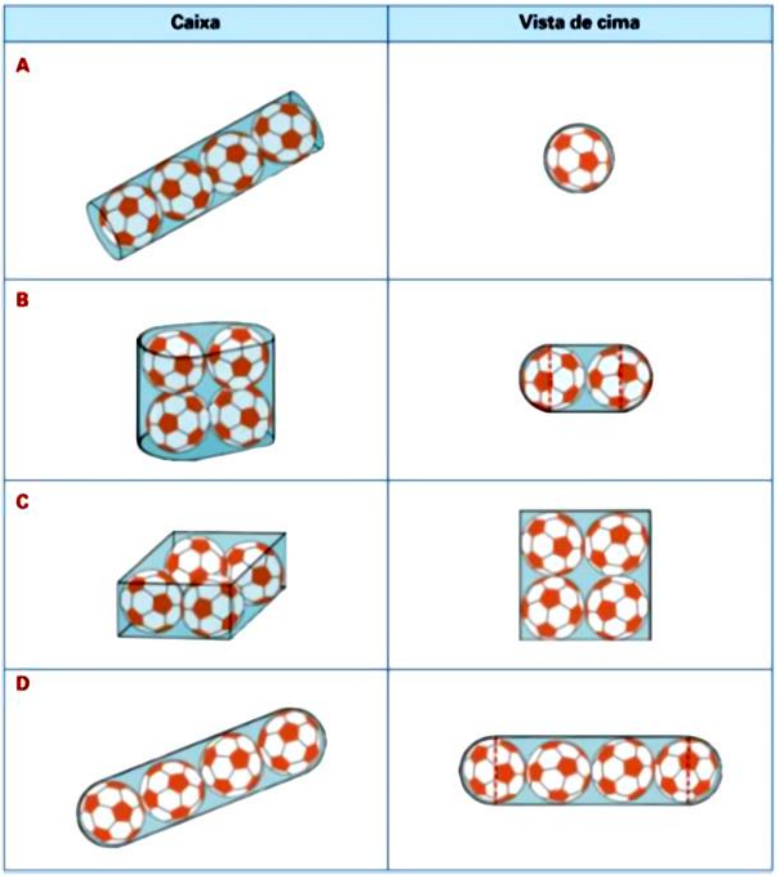

Figure 1. Soccer ball boxes in perspective and top view (Neves & Silva, 2020)

Concerning the methodological options, an explanatory case study (Selltiz et al., 1967) was elected and direct observation and documental analysis techniques were used, supported by the following instruments: assessment tests, a battery of tasks of diverse nature, computerized records of the students' performance on Desmos platform, field notes and interviews.

Research Design

Context description

A didactic sequence on the volume of geometric solids was planned and developed on the Desmos platform. This sequence was intended for the ninth school year and was applied to a class of 29 students from the Portuguese School of Cape Verde. This sequence arose from an initial problem, with some degree of complexity, taken from the textbook "Matemática 9" of Porto Editora (Neves & Silva, 2020).

Initial problem: *A company that produces soccer balls offered a child support institution four boxes of soccer balls, in different packages* (**Figure 1**).

The children were very happy but intrigued by the different shapes of the packaging. So, they asked the mathematics teacher, which packaging took up less space. The teacher challenges the children to investigate, which box takes up less space, that is, which box holds the four balls and has less volume.

The pandemic situation in Cape Verde forced the school to implement a blended learning system and measures for the recovery of topics not addressed in the previous year (due to the first outbreak of the pandemic). The class was divided into two shifts (maximum of 15 students each) so that the number of face-to-face students could comply with the guidelines issued by the health authorities. One of the shifts had face-to-face classes on Mondays, Wednesdays, and Fridays and the other on Tuesdays and Thursdays, changing the following week and so on. Coincidentally, in the students' schedule, mathematics classes were scheduled on Mondays, Wednesdays, and Fridays, which implied that each of the shifts had face-to-face classes on alternate weeks.

To overcome the constraints inherent to blended learning coupled with the need to recover the topics not addressed in the previous school year, the school board decided to allocate six weekly teaching times of 50 minutes to the subject of mathematics, distributed, as follows: two classes of 50 minutes every Monday, Wednesday, and Friday. In addition, the board decided to schedule online classes for the shift that was at home, that is, after each face-to-face class, scheduled an e-learning class with the other shift, in two modalities (a 50-minute class with synchronous activities and a 50-minute class with asynchronous activities). It was under these conditions that this work was carried out. The experience lasted during a whole week, according to the schedule established for the two class shifts.

Description of the didactic experience

In accordance with the Portuguese objectives for the ninth grade students, the didactic sequence was intended to

(i) develop the ability to visualize and understand geometric shapes, namely geometric solids,

(ii) develop the ability to solve problems in more complex situations,

(iii) develop the ability to communicate in mathematics, by writing and using appropriate symbology,

(iv) promote autonomous and collaborative work practices,

(v) use geometric models through digital technology, and

(vi) adapt the mathematical content to the needs and potential of each student.

Given the uncertainty of the pandemic situation, the actions developed were planned in two moments: face-to-face experience and online experience. The different nature of the two moments allowed us to assess different skills:

(i) mastery of digital platforms,

(ii) mastery of mathematical concepts and procedures related to the volume of geometric solids, and

(iii) transversal skills such as communication, reasoning and mathematical problem solving.

As assessment tools, we used:

(i) self-correcting exercises, eminently formative, with the possibility of repetition, on the Desmos platform,

(ii) the teacher's panel on the Desmos platform,

(iii) summative assessment questionnaire, carried out on the Desmos platform, and

(iv) the interventions of the students, in the face-to-face component.

The resources used, with the limitations described above, were

(i) the students' personal computers, with internet access,

(ii) a multimedia projector,

(iii) the Desmos platform, and

(iv) the Teams platform.

Using the potential of the Desmos platform, an activity with 57 screens (pages) was created, which includes several items: videos, images, animations, graphics, tables, questionnaires, etc. All these items were combined to promote autonomy, interactivity and self-correction, enhancing active learning. In the first part of the activity, the concepts of area and volume and respective calculation formulas were worked on in a gradual approach. The second part was dedicated to solving the initial problem. The activity structure is summarized in **Table 1**.

The face-to-face experience, with one of the group's shifts, took place along three 100-minute classes. Due to the lack of computer equipment for the students in the classroom, students were challenged to follow the didactic sequence, projected by the teacher, through the exposition and justification of their ideas and reasoning. The first class began with the presentation of the Desmos platform, in terms of its features and potential. Regarding the mathematical contents, namely the volume of geometric solids, a discussion took place, using the knowledge acquired in previous years. This allowed the teacher to understand the starting

Table 1. Didactic sequence table of contents

point and justify the option for the inclusion of some contents already covered in previous years. In the second class, and having a real-life problem as a starting point, the pages of the activity were worked on and used as a trigger for discussion and exchange of points of view, addressing the difficulties highlighted by the students. In the third class, students had the opportunity to complete the proposed tasks and then proceed to the final conclusions and systematization of the learning developed.

The online experience, with the other group shift (14 students in total), took place along three classes of 100 minutes (50 minutes synchronously and 50 minutes asynchronously), in remote teaching, using the Teams platform. In the first class, students had the opportunity to register on the Desmos platform. Then, the structure of the didactic sequence was explained, and the initial problem presented. In this phase, the teacher took advantage of the rhythm function to control the pages that were available for viewing by the students. In the second class, the teacher used the pause function to explain the methodology to be adopted, namely the most correct ways to explore the various resources used. This was followed by the resolution of the tasks and respective self-correction, previously programmed. In this class, a review of the concepts of area of plane figures and volume of geometric solids was carried out. Finally, the initial problem was resumed. In the third class, students had the opportunity to complete the proposed tasks, in a step-by-step, duly guided resolution, and then proceeded to the final conclusions and systematization of the learning developed. This was followed by the completion of a summative assessment questionnaire addressing the explored contents.

PRESENTATION AND DISCUSSION OF THE RESULTS

The didactic sequence underlying this study is intitled "Sequência de aprendizagem–Volumes de sólidos geométricos" [Learning sequence–Volumes of geometric solids] and may be found in https://teacher.desmos.com/activitybuilder/custom/60a55b9ed3380407e2036c0f?lang=pt-BR.

The didactic experience involved two stages, as follows:

(i) the construction of the didactic sequence on the Desmos platform and

(ii) its implementation.

Construction of the Didactic Sequence

This section contains a brief description of some of the contents, as well as some challenges felt in the construction of the didactic sequence and the way found to overcome them. It is fundamentally intended to give some clues/help to potential Desmos users who are less experienced in creating tasks.

Despite the wide range of existing resources, we tried, whenever possible, to create our own resources:

- Explanatory videos, using GeoGebra,
- Videos to support the understanding of the problem stated, and
- Videos for deducing volume calculation formulas, using transparent solids and water (**Figure 2**).

Figure 2. Moments from the video on screen 22 of the didactic sequence (Source: Authors' own elaboration using the Desmos platform)

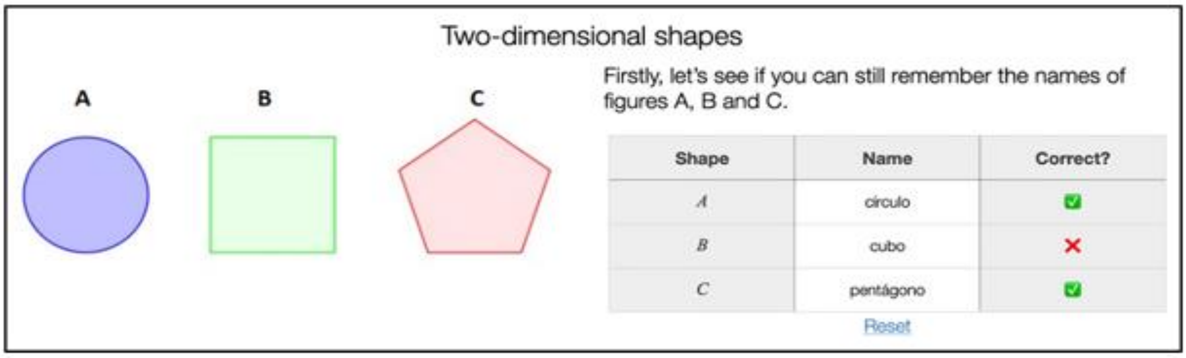

Figure 3. Example of one answer and respective feedback (partly translated into English) (Source: Authors' own elaboration using the Desmos platform)

```
submitButtonText: "verificar"

isCorrect1=tabela3.cellContent(1,2)="círculo"
isCorrect1a=tabela3.cellContent(1,2)="Círculo"
isCorrect1b=tabela3.cellContent(1,2)="CÍRCULO"
isCorrect1c=tabela3.cellContent(1,2)="circulo"
isCorrect1d=tabela3.cellContent(1,2)="Circulo"
isCorrect1e=tabela3.cellContent(1,2)="CIRCULO"
isCorrect2=tabela3.cellContent(2,2)="quadrado"
isCorrect2a=tabela3.cellContent(2,2)="Quadrado"
isCorrect2b=tabela3.cellContent(2,2)="QUADRADO"
isCorrect3=tabela3.cellContent(3,2)="pentágono"
isCorrect3a=tabela3.cellContent(3,2)="Pentágono"
isCorrect3b=tabela3.cellContent(3,2)="pentagono"
isCorrect3c=tabela3.cellContent(3,2)="PENTÁGONO"
isCorrect3d=tabela3.cellContent(3,2)="PENTAGONO"
isCorrect3e=tabela3.cellContent(3,2)="Pentagono"

cellContent(1, 3):
  when tabela3.submitted and (isCorrect1 or isCorrect1a or isCorrect1b or isCorrect1c or isCorrect1d or isCorrect1e) "✅"
  when not(tabela3.submitted) ""
  otherwise "❌"

cellContent(2, 3):

  when tabela3.submitted and (isCorrect2 or isCorrect2a or isCorrect2b) "✅"
  when not(tabela3.submitted) ""
  otherwise "❌"

  cellContent(3,3):

  when tabela3.submitted and (isCorrect3 or isCorrect3a or isCorrect3b or isCorrect3c or isCorrect3d or isCorrect3e) "✅"
  when not(tabela3.submitted) ""
  otherwise "❌"
```

Figure 4. Example of the computation Layer environment of the "table" component (Source: Authors' own elaboration using the Desmos platform)

The video production was elaborated using non-professional and free software and a smartphone. There was a concern not to create very long videos, in order to capture the students' attention, focusing on the essentials. This led to a more time-consuming editing work.

Another concern was to diversify the items and respective answers, using the programming of the different Desmos components, in CL, so that the activities could promote the students' autonomy, either through short feedbacks and suggestions, or with the possibility of self-correction of the answers. Next are some examples:

- "Table" component, with cell filling and immediate feedback (**Figure 3** and **Figure 4**),

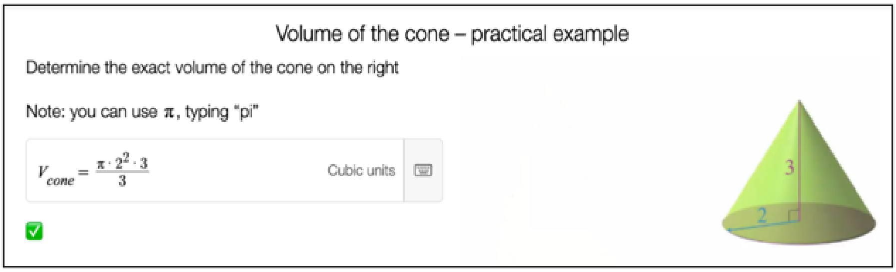

Figure 5. Example of an answer with feedback (Source: Authors' own elaboration using Desmos platform)

```
i = resposta22
n = nota21
v = numericValue("{V}_{cone}=4\pi")

correct = simpleFunction(i.latex).evaluateAt(2)=v

correct: correct

defined = isDefined(simpleFunction(i.latex).evaluateAt(2))

disableEvaluation: true

suffix: when resposta22.numericValue=1 "unidade cúbica" otherwise "unidades cúbicas"

initialLatex: "{V}_{cone}="

showSubmitButton: false
```

Figure 6. Example of the computation layer environment of the "equation" component (Source: Authors' own elaboration using the Desmos platform)

```
content:"${feedback}"
feedback =

when resposta22.script.correct "✅" otherwise "❌"

i = resposta22
n = nota21
v = numericValue("4\pi")

correct = simpleFunction(i.latex).evaluateAt(2)=v

defined = isDefined(simpleFunction(i.latex).evaluateAt(2))
```

Figure 7. Second example of the computation layer environment of the "equation" component with feedback given to the student (Source: Authors' own elaboration using the Desmos platform)

- "Note" component with small feedbacks, depending on the option chosen in the "multiple choice" component,
- "Equation" component that collects and evaluates the students' response, in mathematical language, using a virtual keyboard, in the exercises in which they had to calculate the volume of a certain geometric solid (**Figure 5** (translated into English), **Figure 6**, and **Figure 7**),
- Automatic filling of the area of flat figures and the volume of solids, using unit squares and cubes, for a better understanding of the intuitive notions of area and volume,
- Gradual deduction of formulas for calculating the volume of some solids, with a description of the reasoning step by step, as the student pressed the "button" tool,
- Combination of the "chart" and "tables" components in which filling in the table was accompanied by graphical representation, so students had the opportunity to see whether their answer was correct or not.

These were some of the programming challenges that aimed to provide students with more interactive activities, promoting their autonomy and respecting the different learning paces, while providing immediate feedback, capable of identifying errors and correcting them, promoting truly active learning.

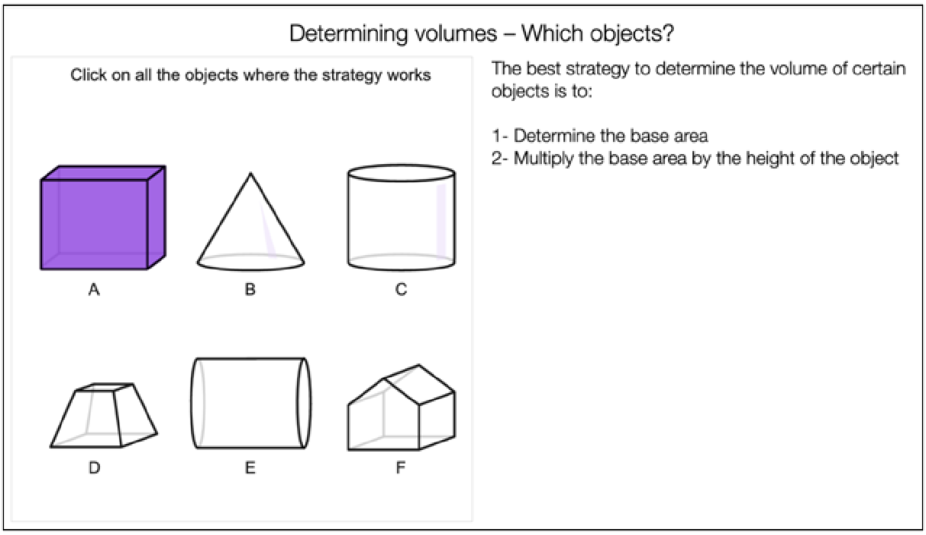

Figure 8. Screen 11: Identifying solids (Source: Authors' own elaboration using the Desmos platform)

Implementation of the Didactic Sequence

One of the fundamental components of this work was its implementation in practice. Despite the uncertainty caused by the pandemic situation, managed to work with the students, however, not in the desirable way, but in the possible way.

The face-to-face experience, with one of the shifts, was conditioned by the lack of computer equipment in the classroom. However, it was particularly interesting to notice the students' enthusiasm, when they saw that the projected materials, for the most part, had been built by their teacher. In a way, they managed to understand that that didactic sequence was built for them, considering their needs. In general, the students had no difficulties in understanding what was asked of them. However, there were some imponderables, which helped them to understand the challenges underlying the construction of an activity on Desmos, and at the same time allowed the improvement of the sequence considering their feedback. For instance, the volume of the pyramid was worked from screen 14 to screen 18. At this stage, the students had some difficulty in understanding the generalization of the volume formula for any pyramid, since the starting point was the cube and its decomposition into congruent triangular pyramids. Screen 16 has been added to the sequence, after the situation described, and the text on screen 17 has also been improved.

Regarding the online experience, with the other shift, it was undoubtedly more enriching, because all the potentialities of Desmos were explored. In general, the students had no difficulties in handling the platform, after registering on it. Students had the opportunity to review the concepts of area and volume, through interactive exercises with automatic feedback to the answers, using appropriate mathematical symbology. The biggest difficulties reported were on screen 11, when students were asked to indicate the objects in which the volume is calculated through the product of the base area by the height (**Figure 8**, translated into English). Object A, already pre-selected, and object C were selected by all students, but objects E and F were not indicated by some, simply because they are in an unfamiliar position (their base planes are not horizontal).

In the remaining didactic sequence, the students reported some problems in validating the answers, because they were sure that the answer was correct. This fact resulted from some incomplete programming of the correct answers (the answers are case sensitive, for instance, and all possible answers have to be listed) or in the limitations of the platform itself, namely, for example, the use of the "." instead of ",". Also in this context, on screens 43, 50, and 56, students had to indicate approximate values of results involving π and reported that their answers were not being validated correctly. In these pages, it was necessary to make additional clarifications because the initial text was not sufficiently clear.

The feedback given by the students was fundamental for the improvement of the didactic sequence. The difficulties they felt allowed its reformulation: adding new screens, clarifying the text here and there, and improving the programming code for the response feedback. The students considered that the sequence was

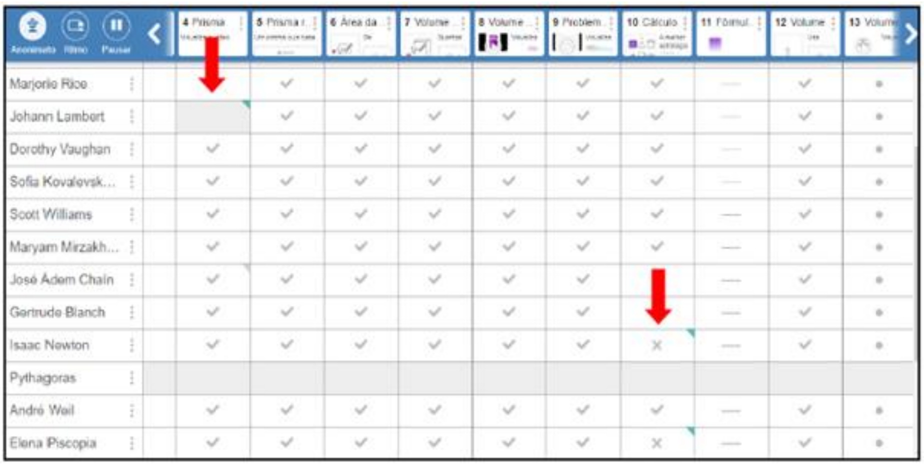

Figure 9. Teacher's panel (Source: Authors' own elaboration using the Desmos platform)

well constructed and presented because it guided their work. Furthermore, they highlighted, as a differentiating factor, the elaboration, by the teacher, of the explanatory videos, using material that can be found in their homes (cups, water, etc.). The programming component of the responses was also enriching because the errors found were discussed in the classroom, and the debate was extended to other programming languages referenced by the students.

As a final remark we would like to point out that, in spite of the overall enthusiasm of all students with the Desmos didactic sequence, the students who worked online on Desmos, managed to solve all the proposed tasks in an autonomous way with very little intervention of the teacher. On the other hand, the group who worked the sequence in the face-to-face classes, through the projection on the screen of the Desmos slides, required much more assistance from the teacher, and needed a continuous guidance to reach the answers to the proposed tasks.

Assessment

The work carried out allowed us to diversify the assessment tools and thus address different skills. Mastery of digital platforms was assessed using the teacher's panel. Through this, it was possible to verify which students registered and access the respective platform. This panel allows teachers to monitor their students' pace, in real time, thanks to one of the best features of the panel, which allows you to check which students are working on the platform, what screen they are on and what kind of response they are giving. This feature was also used as a formative assessment tool, with immediate feedback on the students' learning. Following the teacher's panel in real time, made it possible to act instantly on incorrect or blank answers (**Figure 9**).

Regarding the domain of mathematical concepts, the panel in **Figure 9** illustrates that, in general, the students understood the proposed exercises and revealed a large percentage of correct answers. Considering the total number of self-correcting questions in the didactic sequence (14), there were 87.8% correct answers (172 out of 196).

The transversal skills of mathematical communication and reasoning were also evaluated whether in mathematical writing through a digital platform, or through error, its detection and ability to correct it. For all students, mathematical writing in digital format was a novelty and, therefore, writing in the form of fractions, power, expressions with parentheses and even the writing of π became challenging. For this, they used the virtual keyboard and calculator, already integrated in the platform. These features were so well received by the students that, in later activities, they tried to solve exercises exclusively using the scientific calculator or virtual keyboard, revealing a relevant development of mathematical communication in a remote teaching situation.

Finally, the option of trying to solve the initial problem in a phased manner proved to be effective. As a final goal, students had to find out which of the boxes had the smallest volume (as a function of the ball radius) and for that they were asked to order, from smallest to largest, the volumes of the four boxes. All students except one (13 out of 14, 92.8%) were able to correctly order the volumes. **Figure 10** shows that the

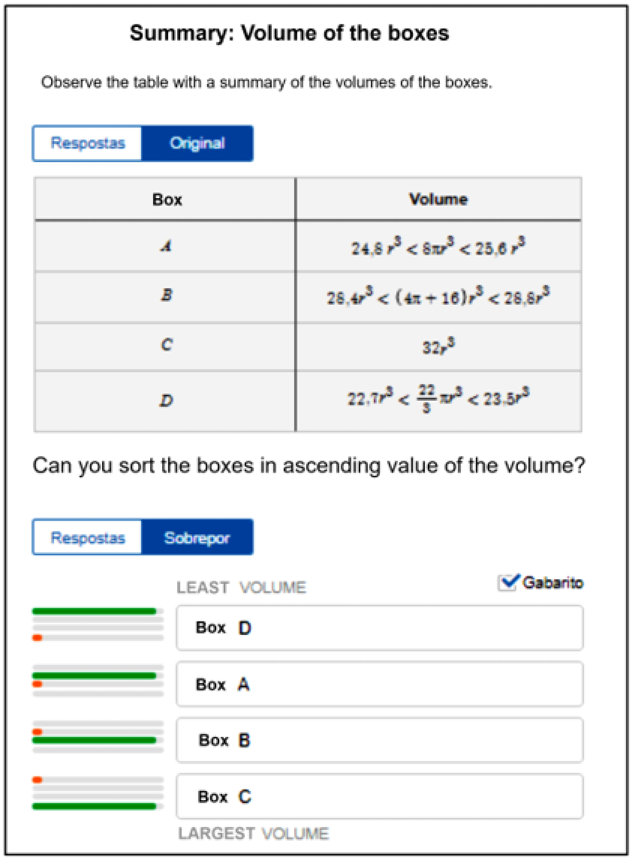

Figure 10. Students' responses to the question raised by the initial problem (partly translated) (Source: Authors' own elaboration using the Desmos platform)

only incorrect answer revealed to be the result of a misunderstanding of the question. The student, who was a good student in mathematics, ordered the volumes from largest to smallest instead of smallest to largest as requested. Considering that the error was in the reading of the question and not on the ordering itself, we can conclude that all students were able to solve the initial problem correctly.

CONCLUSIONS

The current pandemic context brought with it the urgency of implementing remote learning plans, which could replace face-to-face classes. At the same time, the existing digital tools were being improved to respond to the needs of the educational community. There were moments of difficulty, but also of opportunity to diversify methodologies and, with this, provide students with a more active role in their learning.

The current Portuguese government guidelines for the area of education, set out in Decree-Law No. 54/2018, of July 6 (Decreto-Lei n.o 54/2018 Do Ministério da Educação, 2018), aim to eliminate possible barriers to learning that constitute obstacles to the normal learning process of students. The use of digital platforms that promote autonomy, respect learning rhythms and provide immediate feedback are essential for the fulfillment of this aim. In the area of mathematics, and in particular in geometry, the construction of didactic sequences that allow, on the one hand, to explore geometric properties and, on the other, to promote the resolution of more complex problems, in a more oriented way, are enormous contributions to the development of more active learning.

The main objective of this work was to contribute to the improvement of learning in geometry, using digital tools that could develop the students' autonomy and respect their learning pace. It is considered that the use of the Desmos platform, for the construction of a didactic sequence on the volume of geometric solids, had the desired effect.

Using digital tools in the classroom is always incomplete if there is no pedagogical intention. It is always necessary to consider the advantages and disadvantages of each platform and make the most of them.

Therefore, this work had its limitations, since there are no perfect platforms, pointing out as the main disadvantages:

- the need to learn a new programming language for the development of immediate feedback,
- difficulty in using the platform in the classroom, due to the lack of computer equipment for the students,
- the need to create a new copy of the activity, after being assigned to the students, to correct any errors, forcing the distribution of the students' answers to more than one teacher's panel, and
- the lack of a data export functionality in Desmos, for statistical purposes.

Feedback from students was also a decisive factor for the improvement of the experience because it enabled the detection of errors in the activity construction. Throughout the implementation phase, the activity was improved until its final version, thanks to the contribution of students. Their role was more active, and the learning achievements allowed to alleviate the constraints imposed by the pandemic situation.

Because Desmos revealed itself as a very flexible and powerful platform, this work has already been extrapolated to other situations by the teacher who is first author of this paper. He carried on creating didactic sequences around other topics for all his students. Furthermore, Desmos turned out to be a powerful tool in the formative or summative remote assessment of students, through forms and questionnaires, in different topics and school grades.

Concerning the research question underlying this work, "*does the Desmos platform contribute to promote an active learning of geometry through student's autonomy?",* this study clearly suggests that the contribution is a positive one.

To complement the experience, a summative assessment was carried out using Desmos, which allowed the consolidation of the learning developed, as well as the handling of the platform. In this assessment activity, each screen corresponded to a component, either of multiple choice or of open answer. The multiple-choice questions could be automatically evaluated by looking at the teacher's panel and in the remaining questions, students had to use the virtual keyboard for writing their answers (mostly in mathematical symbolic language) or send a photo of the resolution on paper. Students revealed a gratifying ease in using either approach showing that they had improved both their mathematical and ICT knowledge.

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Data availability: Data generated or analyzed during this study are available from the authors on request.

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