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Research Article

The role of metaphors in interpreting students' difficulties in operating with percentages: A mixed method study based on large scale assessment

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ABSTRACT

The issue of students' difficulties in processing operations with percentages has been addressed in several international research studies from a qualitative perspective. In this study, we analyze students' difficulties on this topic, focusing on the transition from middle school to high school with a mixed methods research design. We focus on students' responses in a specific task belonging to the Italian large-scale assessment analyzed through the Rasch model, and we deepen the task analysis thanks to interviews, which enlightened image schemas and metaphors underlying students' reasoning. From the qualitative point of view, the Rasch model shows that students' difficulties in dealing with percentages is a macrophenomenon that involves the higher levels of competences. From the qualitative point of view, the metaphoric approach outlines the image schemas that foster the correct conceptualization of percentage and those that hinder their correct learning and can be one of the possible causes of the emerging aforementioned macrophenomenon.

Keywords: percentages, mixed method, large scale assessment, Rasch analysis, semiotics, sensuous cognition, metaphors

INTRODUCTION

Extensive international research shows students' qualitatively specific difficulties in treating operations with percentages (Lestiana, 2021; Rianasari et al., 2012). Our research aims to analyze students' difficulties on this issue, focusing on the first years of upper secondary school (and in particular on grade 10).

Mathematics educators are designing learning models for teaching and learning percentages as didactic tools for enhancing teachers' professional competences in developing students' learning capacities, mastering their computational skills, and sophisticating their mathematical reasoning about percentages (Confrey & Maloney, 2014; Hodnik-Cadez & Kolar, 2017). The learning of percentages stems from students' intuitive (or informal) knowledge about percentages and build on it different types of students' mathematical knowledge that blends embodied and abstract approaches based on several semiotic resources. A viable approach to percentages starts from a concrete-embodied level, which is then subsumed into general and formal principals that percentages fulfill in order to allow students a meaningful learning (Lampert, 1986; Clements & Sarama, 2009; van den Heuvel-Panhuizen, 2003).

There is a variety of instructional models for teaching and learning percentages with their advantages and disadvantages (Bennet & Nelson, 1994; Parker & Leinhardt, 1995; Scaptura et al., 2007; van Galen & Reitsma, 2008).

Parker and Leinhardt (1995) argue that percent is universal and forms a bridge between real-world situations and mathematical concepts of multiplicative structures. They claim we should consider also long history of the percent concept from its early roots in Babylonian, Chinese, and Indian trading practices, and its parallel roots in Greek proportional geometry to its modern multifaceted meanings. Parker and Leinhardt (1995) focus on the different meanings of percent: fraction, ratio, function, and statistics. They highlight the specific language and semiotics involved in the learning of percent; an extremely concise language that has lost its explicit referents to mathematical practice and has misleading additive terminology for multiplicative meanings, and multiple uses for the preposition "of" in its operational meaning.

The variety of instructional models available for teaching and learning percentages motivated many researchers to analyze the advantages of the implementation of different instructional models in building a particular type of students' mathematical knowledge about percentages (Bennet & Nelson, 1994; Parker & Leinhardt, 1995; Scaptura et al., 2007; van Galen & Reitsma, 2008).

Parker and Leinhardt (1995) also pointed out the disadvantages of their implementation in addressing a certain issue within the teaching and learning of percentages. Indeed, there are arguments showing that the implementation of the 10×10 squares grid is an effective instructional model for building students' understanding of percentage as a part of the whole, in which the whole has 100 equal parts (Bennett & Nelson, 1994; Ningsih et al., 2017), but its implementation has a limited success in developing students' skills in solving problems with percentages greater than 100%. Parker and Leinhardt (1995) emphasized that

> "it is not uncommon to see 150% illustrated by shading all of a hundreds board and half of a second. This representation is a good illustration of 1 1/2 hundreds boards (an extensive quantity) but in no way makes it apparent that the representation is meant to illustrate an area that is 150% of one hundreds board" (p. 469).

In Italy, we have the possibility to track students' difficulties over time thanks to INVALSI tests (tests with the purpose of measuring students' levels of competence in relation to the Italian curricular guidelines), which were administered since 2008 in grades 2, 5, 8, 10, and 13 from the National Institute for the Evaluation of the Educational System (from 2009 to 2013, the tests also covered grade 6). Most of the information related to INVALSI tests (e.g., questions texts, percentage of correct answers, and percentage of wrong answers) are collected in the Gestinv database (www.gestinv.it). The Gestinv database includes 101 items that require operating with percentages. 24 of these items were administered at grade 8 (last year of middle school in Italy) and 49 were administered at grade 10 (last year of mandatory school in Italy).

Difficulties in the transition from middle school to high school are recognized in the literature (Gambini et al., 2022). Remarkably, operating with percentages is a difficult topic for students. In fact, the items administered to grade 8 that have a percentage of correct answers greater than 60% are 12, while the items administered at grade 10 that have a percentage of correct answers greater than 60% are seven. These difficulties are evident in both grades, not only considering students with low competence levels, but, in particular at grade 10, students with medium competence levels (in terms of ability measured over the entire mathematics test using the Rasch model), struggle to overcome obstacles related to percentages. This finding emerges from the Rasch model's interpretation, which will be discussed in more detail in the following sections.

Conducting an in-depth survey of the Gestinv database, we found that the types of difficulties related to percentages change from grade 8 to grade 10. More specifically, the main difficulties in grade 8 are related to the calculation of percentages, while the main difficulties in grade 10 are related not only to the calculation, but also to algebraic modelling, to the splitting of percentages and interpretation in terms of probability and extrapolating data from tables. This partly helps delineate the persistence and development of their difficulties in transitioning from middle school to high school. In the article we focus on the following item administered to high school students (grade 10), which scored an astonishing 24% of correct answers:

> In a shop a dress is sold at the discount of 30% on the original price. During the sales season the already discounted price is still lowered by 10%. What is the total percentage discount on the original price of the dress?

In this research we will adopt a mixed method approach (Johnson & Onwuegbuzie, 2004), integrating quantitative and qualitative methods. In particular, Santi et al. (2021) have introduced a new research methodology for mathematics education based on the insertion of large-scale assessment (LSA) theoretical and experimental paradigms in research practice. They have outlined a new self-contained methodological block that encompasses qualitative and quantitative elements, structured along the following scheme (Johnson & Onwuegbuzie, 2004):

QUAL--->QUAN--->QUAL+QUAN.

Two legs sustain this methodological block:

(i) a theoretical framework appropriate for the mathematical issue under study and

(ii) a structured repository of LSA tools (Bolondi et al., 2017; Gestinv, 2018; Santi et al., 2021; Spagnolo et al., 2022).

Our research method is based on a quantitative methodology, driven by semiotic theoretical perspectives, that utilizes the results deriving from LSA. We have expanded the original scheme of Santi et al. (2021a) into the following:

aprioriQUAL--->QUAN--->expQUAL--->QUAL+QUAN.

We remark that the three parts of the methodology are carried out sequentially. The QUAN phase is the most consistent one, given the relevance of the statistic sample and the preparation, administration and analysis of the tests carried out by INVALSI. The expQual phase has an exploratory role to complement the quantitative data. It is possible and advisable to weigh more the role of the expQual but it was not possible in this research due to humans and time constraints.

We cast our analysis in a theoretical framework based on the notion of sensuous cognition (Radford, 2014) and the emergence of image schemas (Johnson, 1987) that allow students to metaphorize the notion of percentage. Image schemas and metaphors open a window on students' cognition and their attitude to semiotic transformation (Duval, 2017). Based on the results of the national large-scale assessment, the quantitative analysis allowed us to observe specific difficulties on percentages on a large sample of students, representative of the Italian population. This preliminary quantitative analysis based on the Rasch model (Rasch, 1980) enables us to make hypotheses on the causes of students' persistent incorrect answers, which will be further analyzed, confirmed, or disconfirmed through semi-structured interviews. In this respect, we will focus on an LSA item administered to high school students that would have been accessible to lower secondary school students. Furthermore, high school students can tap into a wide range of resources to arrive at the correct solution to the problem; for example, arithmetical or algebraic solutions using different semiotic resources (drawings, natural language, symbolic language, etc.). We propose a metaphorical approach to interpret the origin of the macrophenomenon (Ferretti et al., 2022; Spagnolo et al., 2022) that emerges from the LSA.

The following section will include the theoretical framework which enable us to interpret both the results of the quantitative and qualitative phases. Then the two parts of our research will be presented separately (in terms of methodology, results, and discussion) and the conclusions will re-connect the two phases as a whole study.

THEORETICAL FRAMEWORK

In order to understand the macro-phenomenon that emerges from INVALSI LSA, we need theoretical tools that allow us to fathom different facets that make up the unexpected high school students' mathematical behavior. The item we are analyzing would have been accessible to primary school and lower secondary school students. Furthermore, high school students can tap into a wide range of resources to arrive at the correct solution to the problem; for example, arithmetical or algebraic solutions using different semiotic resources (drawings, natural language, symbolic language, etc.).

To understand the origins of the large scale results, we focus on the relationship between the meaning of mathematical concepts and the use of semiotic resources with a two-fold understanding, as mediators of sociocultural activity and signs that stand for mathematical objects and belong to structured systems of signs. We present the tenets of Radford's (2008, 2021) theory of objectification (TO) to frame the socio-cultural approach to signs and Duval's (2017) semio-cognitive approach to frame the structural one. We will also give a glimpse of the metaphoric approach to cognition introduced by Johnson (1987) and Lakoff (1987) to link the two perspectives mentioned above.

The Theory of Objectification

TO (Radford, 2008, 2021) is a socio-cultural semiotic approach rooted in activity theory (Leont'ev, 1978). Activity is at the core of the theory, and mathematical thinking, knowledge and learning are inseparable from the social practices of individuals. The TO developed at the beginning of the century and reached its stable architecture around the year 2010. The TO suggests a different understanding of signs with respect to the traditional uses within the signifier-signified relationship. Indeed, the theory broadens the notion of sign according to Arzarello (2006) outer enlargement of semiotic resources, in that signs are interlocked with social activity, and they broaden the traditional horizon of semiotic systems in mathematics. Semiotic resources, besides symbolic language, natural language, geometrical drawings, diagrams, and cartesian registers, include less formal and non-formal components such as gestures, rhythm, material objects, kinesthetic activity, indexical use of natural language, etc. (Radford, 2000, 2003). In the remainder of this section, we briefly outline how the TO conceives mathematical knowledge and learning.

The development of mathematical knowledge and learning stems from the dialectical movement between individuals and a cultural-historical dimension that occurs in the activity groups of individuals engage in. Signs and artefacts are bearers of cultural-historical knowledge and consubstantial to activity. In its interaction with individuals (their objects, actions, division of labor, etc.) via the use of signs and artefacts, the cultural dimension gives rise, on the one hand, to *forms or modes of activities*, and, on the other hand, to specific modes of knowing or *epistemes* (Radford, 2008). The modes of knowing are distinguishable but inseparable from the modes of activity they are intertwined with.

Mathematical objects are "fixed patterns of reflexive human activity encrusted in the ever-changing world of social practice mediated by artefacts" (Radford, 2008, p. 222). Mathematical knowledge is a system of ideal archetypes of reflexive activity reified in the cultural-historical dimension (Radford, 2021).

The issue of *learning* is rooted in the dialectics between the individual and their culture. Learning is a movement pushed by the intrinsic differential between individual and cultural knowledge. In fact, in attending to knowledge the student has to cope with something that in the beginning is different from him, an alterity that challenges, resists and opposes him. Learning is the process that erases such a difference to make sense of knowledge and transform it into something familiar, allowing new forms of action, thinking, imagination and feeling. In order to reduce the distance between the individual and cultural knowledge, acts as a specific human endeavor is required on the part of the student. Radford (2008) conceives learning as

> the social processes of progressively becoming aware of cultural-historical systems of thinking and doing–something we gradually notice and at the same time endow with meaning [...] those acts of meaningfully noticing something that reveals to our consciousness through our bodily, sensory, and artefactual semiotic activity (p. 78).

The movement towards knowledge on the part of the student to notice and become aware of mathematical meaning is termed by the theory process of objectification. A process allows the student to transform the mathematical object into an object of consciousness.

We remark that, according to TO, signs and artefacts are constitutive of processes of objectification referred to as semiotic means of objectification:

> These objects, tools, linguistic devices, and signs that individuals intentionally use in social meaning-making processes to achieve a stable form of awareness, to make apparent their intentions, and to carry out their actions to attain the goal of their activities, I call semiotic means of objectification (Radford, 2003, p. 41).

As mentioned above, the outcome of learning as a process of objectification is the encounter with mathematical cultural objects and their transformation into objects of consciousness. From this standpoint, learning has a strong phenomenological nature where noticing occurs in an enlarged notion of mind and consciousness, termed by TO *sensuous cognition*, that includes not only ideal and mental features but also embodied ones such as perception feelings and kinesthetic activity. Within this enlarged notion of mind and consciousness, the encounter with knowledge and its transformation into an object of consciousness occurs within the features of *sensuous cognition*. In light of the dialectic-materialist approach underpinning the TO, the basic tenet behind sensuous cognition is that the body, the senses, and the objects of sensation are not a priori entities but are mutually transformed by cultural-historical activity *entangled with the use of signs and artefacts*.

From the standpoint of sensuous cognition, human perception is, in the words of Wartofsky (1984, p. 865), "a cultural artefact shaped by our own historically changing practices." In this regard, perception deploys cultural forms of seeing, touching, hearing, etc. that characterize our relationship with the world. How do students change their perception from "spontaneous" forms of attending to objects to a mathematical and theoretical ones? To answer this question, we must consider learning as a "domestication of the eye" (Radford, 2010), a long process that allows students–in cultural-historical activity intertwined with the use of signs and artefacts - *to transform the eye (and other senses)* into *sophisticated theoretical organs* able to notice and make sense of certain things in mathematical manners–for example, recognizing numerosity, algebraic structures, geometric invariants, etc.

We underline the multimodal nature of the "domestication of the eye" in the various sensorial channels and the richness of signs and artefacts interwoven with cultural-historical activity involved in the transformation of perception (Radford, 2021):

> "Mathematics is visual, tactile, olfactory, aural, material, artefactual, gestural, and kinesthetic and, being all of that, becomes an object of consciousness and thought. School mathematics, in this materialist and phenomenological line of thought, is what is made sensible through the teachers' and students' activity" (Radford, 2008).

The Semio-Cognitive Approach

TO accounts for the emergence of mathematical objects as fixed patterns of cultural-historical activity and learning as a process of objectification. When mathematical objects assume an interpersonal reality as archetypes of the cultural-historical dimension, we need to linguistically and semiotically refer to such emerging forms of activity in the signifier-signified relation. Wittgenstein (1953) in the *Philosophical investigations* observes that the denotative character of language is one of its possible "uses"–in the terminology of TO, one of the *possible forms of activity*–which is suitably outlined by the semio-cognitive approach.

At the beginning of the '90s, Duval (1993, 1995) proposed semiotics as a new theoretical lens to investigate and characterize mathematical thinking and learning. His forefront research (Duval, 1993, 1995, 2017) introduced a structural and functional approach to semiotics, outlining the features and potentials of semiotic systems and the transformative functions that inform mathematical thinking and knowledge.

According to Duval (2017), every mathematical concept refers to *objects* that do not belong to our perceptive experience. In mathematics, *ostensive referrals are impossible* since we cannot display "objects" that are directly accessible. Therefore, every mathematical concept intrinsically requires working with semiotic representations (semio-cognitive) not the objects themselves.

The lack of ostensive referrals led Duval (2017) to assign a constitutive role in mathematical thinking to using representations belonging to specific semiotic systems. From this point of view, Duval 2017 (p. 23) claims that there is no "*noesis without semiosis,*" i.e., no conceptual comprehension without sign use.

The peculiar nature of mathematical objects (algebraic in particular) allows us to outline a specific cognitive functioning that characterizes the evolution and the learning of mathematics. We can say that conceptualization in mathematics, can be identified with the complex coordination of several semiotic systems.

A semiotic system (or *register*) is defined (Duval, 1995; Ernest, 2006) as a *set of basic signs*, a set of *organizing rules* for the production of signs and for the transformation of signs; an *underlying meaning* resulting from the combination of the *basic* signs in structured semiotic representations.

D'Amore (2003) identifies conceptualization with the following *semiotic functions*, which are specific to mathematics*: choice of the distinctive traits* of a mathematical object; *treatment*, i.e., the transformation in the *same* semiotic system; *conversion*, i.e., the transformation from one semiotic system to *another* semiotic system. The combination of these three semiotic functions can be considered the "construction of knowledge in mathematics" both in its historical development and the teaching learning process. But it is not spontaneous nor easily managed and represents the cause for many difficulties in the learning of mathematics, because the student has to overcome the *cognitive paradox*:

> On the one hand the learning of mathematical objects cannot be but a conceptual learning, on the other an activity on mathematical objects is possible only through semiotic representations [...] How can learners master mathematical treatments, necessarily bound to semiotic representations, if they do not already possess a conceptual learning of the represented objects? (Duval, 1993, p. 38).

In other words, we know mathematical objects through symbols representing them, but effective use of those symbols requires an understanding of the objects they represent. We seem to be caught in a trap.

Mathematical objects are recognized as invariant entities that bind different semiotic representations when treatment and conversion transformations are performed, and as such, they cannot be referred to directly.

We need to establish a coherent connection between using signs as mediators of activity and representations in the signifier-signified relationship. The notion of image schema introduced by cognitive linguistics provides a fruitful bridge between the above perspectives.

Cognitive linguistics suggests that the relationship between elements of language and their referents stems from *human embodied action and experience* (Lakoff, 1987; Lakoff & Johnson, 1980, 1999). A fundamental tool in cognitive linguistics is the notion of image schema. They are foundational elements for forms of embodied imaginative thought as basic perceptual units that develop through our interaction with the environment; among perceptual units forming in our mind, they are of particularly simple (primary) and abstract forms.

> Image schemas are "recurrent, stable patterns of sensorimotor experience ... [that] preserve the topological structure of the perceptual whole ... having internal structures that give rise to constrained inferences" (Johnson, 2007, p. 144).

Corni and Fuchs (2020) and Corni et al. (2018) call image schemas small-scale perceptual gestalts that exhibit limited structure (aspects) with an internal logic that is used in reasoning. Examples of image schemas (**Table 1**) are PATH, CONTAINER, IN-OUT, UP-DOWN (verticality), SUB- STANCE, AGENCY or CAUSATION, SCALE, CYCLE, PROCESS, and many more. **Table 1** presents the most common image schemas used in meaning making processes that Johnson (1987) and Lakoff (1987) identified in their studies of cognitive linguistics.

It is advantageous to bind image schemas with Egan's (1997, 2002) notion of *cognitive tools.* They characterize the phylogenetic and ontogenetic development in the mastery of forms of language use and forms of embodied imaginative thought; for example, polarity (binary opposites), metaphors, analogical thinking, and narratives. For the scope of our study, we focus on conceptual metaphors, that is, figurative structures that are created by projecting an embodied understanding of a domain (source domain) into another domain of experience (target domain). A possible and potent source domain can be an image schema in the encounter with mathematical knowledge. They allow us to produce concrete metaphoric expressions.

Table 1. The most fundamental image schemas outlined by Johnson (1987)

Image schema category	Components
Opposition	*Binary* opposition, *polarity*
Scale	Path, *gradient*
Space	Up-down, *verticality, high-low*, front-back, left-right, near-far, center-periphery, contact
Process	Process, state, cycle, *change*
Container	Containment, in-out, surface, full-empty, content
Force/causation	Balance, counterforce, compulsion, restraint, enablement, blockage, diversion, attraction, manipulation, *tension*
Unity/multiplicity	Merging, collecting, splitting, iteration, pat-whole, mass-count, link
Identity	Matching, superimposition, *difference*
Existence	Removal, bounded space, object, *(fluid)* substance

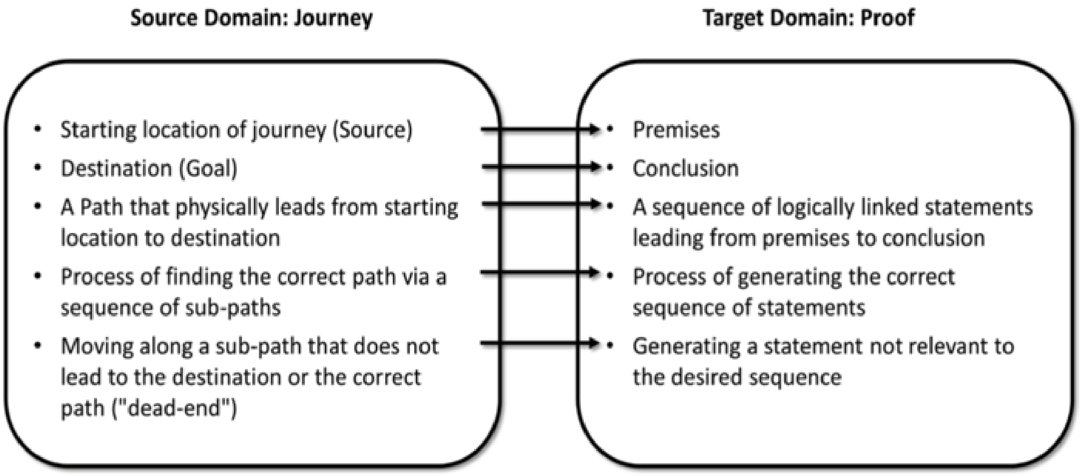

Figure 1. Schema of a metaphoric conceptualization of proof (Edwards, 2010)

For example, Edwards (2010) shows how the concept of mathematical proof can be conceptualized as a metaphor whose source domain is the source-path-goal image schema and the target domain is the explicit structure of a proof (**Figure 1**). Asenova (2022) shows the role of non-classical approaches to argumentation and proof in taking into account their epistemic aspects and thereby grasping the formal aspects of a mathematical proof.

The author describes the important role of gesturing and speech in the student's activity to support the emergence of the source-path-goal image schemas in their sensorimotor activity and the ensuing mapping between the source and target domains.

Image schemas and conceptual metaphors allow us to bridge the use of signs and artefacts in sensuous cognition and the use of signs in the signifier-signified relationship. Mathematical objects emerge as fixed patterns of cultural-historical activity, they are recognized as image schemas that give rise to metaphors that involve different semiotic systems as a target domain to refer in a meaningful manner to the mathematical object. In our example, the target domain was expressed in natural language, but it might as well be converted into the symbolic language used in formal logic.

RESEARCH QUESTIONS

The present study aims to outline a macrophenomenon that characterizes the learning of percentages in the Italian educational system, emerging from LSA. We address the following research questions:

1. **RQ1**. Looking at INVALSI data what type of difficulties do students encounter when facing tasks regarding percentages? What are levels of competencies mainly affected by such difficulties?
2. **RQ2.** What image schemas and metaphors do students presumably use to conceptualize percentages? What forms of sensuous cognition with its related use of semiotic means of objectification give rise to such image schemas?

3. **RQ3.** How do students handle semiotic transformations (treatment and conversion) in conceptualizing percentages? How does the coordination of semiotic systems relate to sensuous cognition and emergent image schemas?

As already explained, our mixed method research is based on a quantitative methodology, driven by semiotic theoretical perspectives, that utilizes the results deriving from LSA:

aprioriQUAL--->QUAN--->expQUAL--->QUAL+QUAN.

Then **RQ1** will be addressed thanks to the QUANtitative phase, while **RQ2** and **RQ3** will be investigated through the explorative QUALitative phase and the connection between QUALitative and QUANtitative phases.

PART 1: AprioriQUAL AND QUAN PHASES

Methods Part 1

In the school year 2007/2008, in Italy, INVALSI tests were introduced for grade 8, i.e., for the third year of secondary school. Currently, the tests have been administered annually in school grades 2, 5, 8, 10, and 13, then from the beginning of primary school to the end of high school. The purpose of these census surveys involving around 500,000 students for each grade is to measure the competencies in mathematics, Italian language and English related to the Italian curricular guidelines (MIUR, 2010, 2012).

The INVALSI tests are large-scale tests consisting of closed-ended or open-ended questions. The construction of the INVALSI tests, like other LSA, requires a process that never takes less than 15/18 months (but can be even longer) and interdisciplinary work involving experts from different sectors (such as mathematics education, linguistics, statistics, etc.). After an initial qualitative analysis, the tests must be pretested on a first sample of students (field trial) to assess whether the test and each question have solid measurement characteristics. The items are analyzed using classical test theory and Rasch analysis. The technical characteristics of the tests and other psycho-metrical aspects are published each year in reports prepared by INVALSI experts. The items are analyzed using classical test theory and Rasch analysis. The articles are then validated, modified, or deleted and, after a qualitative analysis, the final booklets are composed.

The AprioriQUAL Phase

This phase requires pinpointing the research focus and the suitable and effective theoretical lenses. A clear research focus and its theoretical framework allow us to identify the research questions and the didactical variables. They will be of extreme importance in the interpretation of the data and in the selection of the macro-phenomena. The authors of the article were interested in the learning of percentages at the level of the Italian educational system. The focus was on how the items' semiotic representations and type of question influenced the student's reasoning across the different school levels. This phase triggered the *QUAN* phase with Gestinv.

The QUAN Phase

This phase is based on the implementation of Gestinv (Ferretti et al., 2020) exploiting its rich resources in terms of available items of the INVALSI tests indexed according to the national guidelines, the results from the statistical point of view, the mathematical content, the key words, the percentage of correct, wrong, and invalid answers and the other characteristics mentioned above.

Ferretti et al. (2018) provide significant examples of the use of Gestinv in mathematics education research. Gestinv allows us to conduct a quantitative analysis based on the INVALSI tests pertaining to the research focus, questions, and didactical variables. The functions of Gestinv provide items that match the research needs in terms of cognitive processes, mathematical content and learning objectives underpinning the research questions of the investigation. We point out that the selection of the most significant items using Gestinv is strongly driven by the QUAL phase not only with regard to the research questions and the didactical variables but also to other systemic characteristics that include contextual educational and socio-economic information.

In a shop a dress is sold at a discount of 30% on the original price.
During the sales season the already discounted price is still lowered by 10%.
What is the total percentage discount on the original price of the dress?

A. 20%
B. 33%
C. 37%
D. 40%

Figure 2. Item D25–INVALSI maths test administered in 2012 at grade 10 (Source: www.gestinv.it; translation by the authors)

This paper presents an item-level analysis of the INVALSI maths test administered in Italy in 2012 at grade 10. We focus on a specific multiple-choice item which requires students to operate with percentages and analyze it through the lenses of maths education theories presented in the theoretical framework.

The sample, representative of the entire Italian population, consists of 41,812 grade 10 students to whom the INVALSI test was administered under controlled conditions.

The quantitative analysis is based on the outputs of the Rasch model (Barbaranelli & Natali, 2005), which is the simplest among the models of the theory of item response and allows to make a joint estimate of the ability of the students and the difficulty of the items in a test. The difficulty parameter of each item (delta) and the students' ability to respond to the test are estimated on the same scale, with values from -4 to +4 (higher values correspond to higher item difficulties and students with higher abilities). From this information, it is possible to obtain a graph for each item, called *distractor plot*, which describes the trend of the correct response and the other options according to the students' ability over the entire test. In particular, the distractor plot of each item reports the ability of the students (measured over the entire test) on the x axes and the probability of choosing the correct answer on the y axes, then the curve estimated by the model (item characteristic curve-ICC) is represented to describe the probability of answering correctly as function of students' ability. The same plot also reports the trend of the empirical data: the sample is divided in deciles considering students' ability and then for each decile, the percentage of students choosing each possible answer is reported. In this way the distractor plots also allows us to compare the empirical trend of the correct answer and the ICC expected by the model, and to observe the empirical trend of the incorrect/missing answers.

Results Part 1

The research stems from the authors' interest in learning percentages at the Italian educational system level. Based on the theoretical framework described before, the item analyzed in this paper belongs to the INVALSI test administered at grade 10 in 2012 (**Figure 2**). The item is a word problem (Boone, 1959) and students must calculate the percentage relative to the total discount of a dress discounted twice: the first discount is 30% and the second one is the 10% discounted price.

The results of the INVALSI sample are reported in **Table 1**. The item highlights good psychometric features with an optimal fit (weighted=0.98), good discrimination (0.43), and the delta parameter (1.36) shows the great difficulty of the item. Only 24% of students choose the correct answer, a low percentage since it is a multiple-choice question with four possible options, and this confirms students' difficulties also at grade 10 in operating with percentages.

Almost half of the students choose distractor D, which, by observing the distractor plot (**Figure 3**), is very attractive for all ability levels but particularly for students with medium ability levels, then students with an ability parameter measured through the Rasch model with values between -1 and +1 (**Figure 4**).

```
item:38 (M25)
Cases for this item  41812   Discrimination  0.43
Item Threshold(s):      1.37   Weighted MNSQ   0.98
Item Delta(s):      1.36
------------------------------------------------------------------------------
 Label       Score     Count   % of tot  Pt Bis     t  (p)    PV1Avg:1 PV1 SD:1
------------------------------------------------------------------------------
 1 A          0.00      3074      7.35   -0.21   -44.10(.000) -0.72      0.82
 2 B          0.00      6783     16.22    0.10    20.80(.000)  0.21      1.04
 3 C (corr)   1.00     10141     24.25    0.43    96.15(.000)  0.70      1.03
 4 D          0.00     20073     48.01   -0.29   -61.50(.000) -0.28      0.78
 7 Not reached 0.00       85      0.20   -0.03    -5.20(.000) -0.60      1.29
 9 Missing    0.00      1656      3.96   -0.10   -20.70(.000) -0.50      1.04
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Figure 3. Results of item D25 output of the Rasch model implemented using ConQuest software (Source: www.gestinv.it)

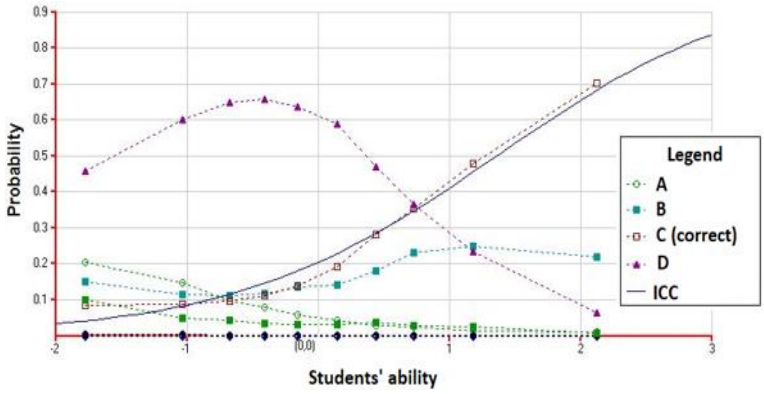

Figure 4. Distractor plot of Item D25 (Source: www.gestinv.it)

Table 2. Percentage of correct answers of item D25 according to type of high school

Type of high school	Percentage of correct answers
Gymnasium	28%
Technical schools	24%
Vocational schools	15%

Previous research has shown that such a trend, named the "humped performance trend", can be explained in terms of excessive adherence of students of medium ability levels to teaching practices (Bolondi et al., 2018; Ferretti et al., 2018; Ferretti & Giberti, 2021). Indeed, this distractor might be interpreted in terms of misconceptions: students choosing option D operate with percentages following the same procedures used with natural numbers and find the total discount by adding the two discounts. This specific behavior in operating with other percentages was already found in a previous study in which several INVALSI tasks on percentages were compared (Giberti, 2018).

In light of the theoretical framework, the students' reasoning is likely not supported by meaningful activities with proper signs and artefacts. A possible interpretation is that students refer to inappropriate metaphors of percentages and discounts that do not consider the fact that the two percentages operate in different quantities in the first and second discounts. They probably cling to the container image schema as the metaphor's source domain, and the discount is seen as a fixed quantity removed at each discount, independent of the remaining quantity at each change of price. Even distractor B has a particular trend, and it is chosen mainly by students with medium-high ability levels: indeed, these are students who do not incur the misconception but reverse the two discounts.

Furthermore, it is interesting to notice the different percentages of correct answers according to the type of school (**Table 2**).

Considering data from gymnasium schools, less than the 30% of students choose the correct answer, and this percentage is reduced to 24% if we consider technical schools and halve considering vocational schools. The remarkable difficulties also encountered by students at gymnasium schools can be explained by considering that, in gymnasium schools, there are better results, but at the same time, it is easier to find errors related to the incorrect use of procedures and algorithms. Learning mathematics too much based on rules and procedures can in fact lead students with difficulty to rely obediently on the procedures followed in class and, therefore, run more easily into misconceptions such as the one examined.

The quantitative analysis of this particular item, based on the Rasch model and in particular on the distractor plot analysis, highlights the difficulties of most of the students in operating with percentages, even in a concrete context. Furthermore, we observed that almost half of the students operate with percentages as if they are natural numbers: they add the two percentages and answer by choosing option D. This kind of mistake is then particularly interesting from a didactical point of view and will be further investigated through qualitative analysis. In particular, it is interesting to underline that students choosing this option are also students with medium ability levels, then this behavior could be connected to a strong relationship with didactical practices (Barbaranelli & Natali, 2005).

PART 2: expQUAL PHASE

Methods Part 2

The expQUAL phase

The expQUAL phase uses qualitative tools in a broad sense to single out to provide experimental qualitative data. The expQUAL phase resorts to several tools such as interviews, discussion groups, group activity, and on-the-ground observations. In our research, we enriched the QUAL phase by collecting qualitative data that consisted of interviews to grade 10 students regarding the item under scrutiny that we will present in the following sections. The data collection involved an Italian high school with a scientific and technical curriculum in December 2018. 16 grade 11 students were involved in the trial. They were exposed to the item D25 (described before) in an interview with two authors of the paper. The students were firstly asked to answer the INVALSI item, and their solution triggered a discussion regarding their reasoning and the strategies they implemented. The interviews were voice-recorded. We have chosen grade 11 students to allow them considering the INVALSI tests they faced the year before from the perspective of more mature individuals.

The interview was structured as following:

1. The researchers introduce the activity explaining the students they would have to solve an INVALSI item concerning percentages and that the interview is audio recorded. The students are informed that the interview is anonymous, and no information is going to be shared with their mathematics teacher.
2. The students solve the INVALSI item and are asked to argue for their solution. In this phase the researcher does not prompt any specific semiotic behavior on the part of the student. The researcher follows an inquiry approach with the student to dig into the nature of the problem, the structure of the solution and the possible forms of reasoning. The aim is to trigger the student's semiotic activity typical of the high school level - mainly transformations involving natural and symbolic language - and, within the inquiry attitude, unveil sensuous cognition and domestication of the eye, image schemas and metaphors grounding the treatments and conversions.
3. If the student provides the correct answer, the researcher asks them to pretend they have to explain the solution to another student at a lower school level in order to go deeper into their metaphoric background and check that the correct answer is not the outcome of meaningless symbolic manipulations.

The analysis of the protocols aims at outlining the relationship between sensuous cognition and the domestication of the eye and the kind of metaphors students tap into to carry out the semiotic transformations. The data collection based on audio recorded interviews does not allow us to fully grasp the sensuous activity and the range of semiotic means of objectification deployed by the students. We get only, sometimes significant, tokens of the objectification processes.

The main image schemas used to build the metaphors for percentages are, according to the students' needs, a blending of the categories scale, space, containment, and unity-multiplicity (**Table 1**). The scale and space image schemas allow students to metaphorize the increase or decrease of the price of the dress. These

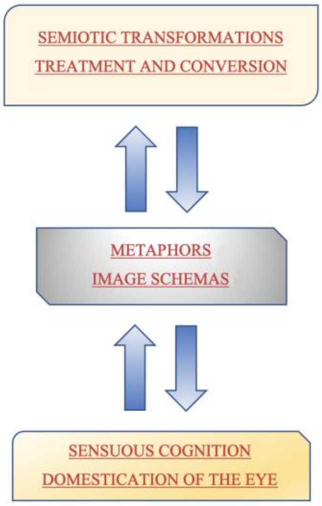

Figure 5. The theoretical structure of the qualitative analysis (Source: Authors)

image schemas are the outcome of the action of going up and down in space connected to the gradient of a quantity that can be experienced along a path. The containment image schemas play the same role but based on the action of filling in or pouring out something of a container. Such image schemas alone can lead to the misconception that you have to add the two discounts disregarding the functional meaning of the percentage. The introduction of the unity/multiplicity image schemas include not only the decrease of the price but also the relationship between the two subsequent discounts in which percentages are conceptualized as operators. They are related to the action of dividing, splitting something and merging or re-collecting the pieces together. Another important action related this set of image schema is perceiving the relationship between the part and the whole of a physical object. At high school level, the sensuous activities related to these image schemas are interwoven with the use of gestures, indexical use of natural language, natural language, and drawings.

The interviews were the only interactions we had with the students. We did not have the possibility to set up any activity with students and the mathematics teachers. The students belonged to several classes of the school and from different course studies: scientific lyceum and technical courses studies.

The only data available consist of the audio recordings and the students' written protocols we collected during the interviews. So, the analysis basically starts from the students' semiotic transformations to infer the image schemas and sensuous cognition that behind their semiotic activity. During the interview it can happen that the student puts forward recognizable sensuous activity with the ensuing image schemas that inform the semiotic transformations. **Figure 5** shows the two paths along which the analysis is conducted: from semiotic transformations to sensuous cognition and vice versa.

Results Part 2

Qualitative analysis (expQUAL phase)

In this section, we analyze four protocols out of the 16 we gathered during the study. On the one hand the analysis is carried out looking at the image schemas and metaphors behind the students' reasoning and, where possible, outlining the semiotic means of objectification and modes of activity that engenders the schemas. Coherently with our quantitative analysis, we have chosen two protocols referring to the incorrect answer D since it was the most interesting for the distractor plot analysis and two for the correct answer C. We have chosen the protocols that provided more information regarding the image schemas and the semiotic resources (treatment and conversions, semiotic means of objectification).

Analysis of protocol 1 (incorrect answer D): The following is the transcript of Miriam's interview:

1. Miriam: Well, it's 40% because I do the sum of the percentages.
2. Researcher: If you were to show me with a drawing, a calculation how would you do it?

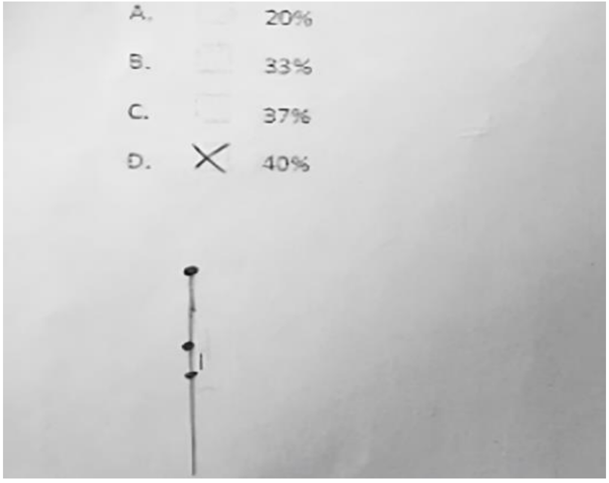

Figure 6. Miriam's working sheet (Source: Scanned working sheet of Miriam)

3. Miriam: Like, I have kind of a scale where at the highest point is the original price, then it tells you it's lowered by 30% so it gets here, and it's lowered again by 10% already the discounted one (scrolling her pen up and down the diagram), so we lower it here and you do the sum of that plus this (**Figure 5**).

4. Researcher: If you were to do a calculation with numbers.

5. Miriam: Like what?

6. Researcher: A process with algebra or with numbers. What you have done is a diagram, if you were to use symbols, numbers and/or letters how would you do it?

7. Miriam: I would not know how to do it.

Miriam's reasoning is based on connecting two image schemas: gradient and up-down. In fact, she uses the terms "scale", "highest point", and "lowered" as the source domain to metaphorize the contribution of the two discounts to the final price (line 3). The drawing (**Figure 6**) and the movement of the pen mediates her action of moving downwards to materialize the decreasing quantity of money needed to buy the dress. Miriam's metaphorization of the percentage does not consider the discount as a percentage that operates on two different quantities, the original price and the first discounted price. Interestingly, Miriam in line 3 refers to the "already discounted one" that she recollects from the item, but it is disregarded from her reflexive activity and the emerging image schemas. We remark the student cannot give an algebraic or arithmetical solution performing treatments and conversions. Given her school grade and the scientific curriculum, she should be able to do the calculation needed to pursue the correct answer. The structure of the task does not allow Miriam to trigger the necessary treatments and conversions, which she would presumably handle in standard contexts although lacking the encompassing mathematical meaning behind the semiotic transformations. Indeed, the image schemas she brings to the fore and the ensuing metaphor of percentages and discounts do not embrace all the components of the concept of percentage as an operator on a given quantity.

Miriam provides a very poor semiotic activity. She sums the two percentage in natural language without any conceptual control due to an inappropriate metaphorization of the two discounts. The source domain based on the scale and up-down image schemas blurs the operational meaning of the two discounts into the decrease of the original price made up of two independent and subsequent descents on the scale. The sensuous activity behind the emergence of this image schema is the scrolling of the pen along the segment in **Figure 5** interwoven with natural language and indexical use of natural language (here, there, high, and low). The domestication of the eye cannot see the two discounts as percentages that operate on different quantities, only quantities that decreases in terms of subtractions.

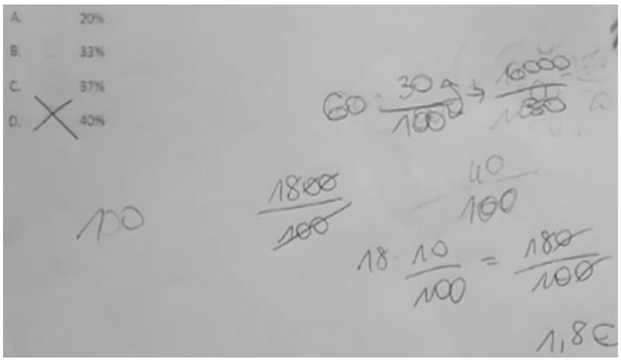

Figure 7. Sara's working sheet (Source: Scanned working sheet of Sara)

Analysis of protocol 2 (incorrect answer D): The following is the transcript of Sara's interview (**Figure 7**):

8. Sara: … Here I the percentages … the statistics … it's a bit of a problem … so, if I have 100 … and I add 10 … it should be like this … (writes 30/100+10/100=40/100).

9. Researcher: So that 100 is what?

10. Sara: Is the percentage … so theoretically a 40% … if I divide 40:100, I get 4 out of 10 … so, ok I get a fraction so nothing … so, ok 40%

11. Researcher: What if I told you that the initial price is 100 euros or 60 euros? How would you have done that?

12. Sara: I would have to multiply by the i.e., I would have to do like 60×30/100 and then so, I find the discounted price and then reapply it on the 10 …

13. Researcher: Come on try! you price the dress … whatever you prefer.

14. Sara: (Rewrites above previous writing by adding a 60 in front of 30/100 calculates 1,800/100 and simplifies arriving at 1.8) … Something does not add up …

15. Researcher: What is that 1.8?

16. Sara: Eh indeed … it is impossible … maybe it will cost more … much more …

17. Researcher: You want to start with an easier price? Try 100 …

18. Sara: Eh but it does not come anyway because I subtract it costs you 30 … (repeats the same calculations) … it would come out €3 … with the method I apply that is surely right because it can't come something like that i.e., it would come out 100 I simplify it means 100 in the denominator so it would come out €30.

19. Researcher: And what is that 30?

20. Sara: Eh theoretically it should be the discounted price however it is not possible because if I then do 30×10 that is I would get 300 so 3 euros …

21. Researcher: And if a dress that costs 100 euros … they tell you it's discounted to 30% how much do you expect it to cost? even roughly …

22. Sara. Boh … 80?

23. Sara: Ah but maybe it's the other way around … (and reverses the fraction 30/100 … but then wraps up again …) … I do that in maths … I often go by trial and error …

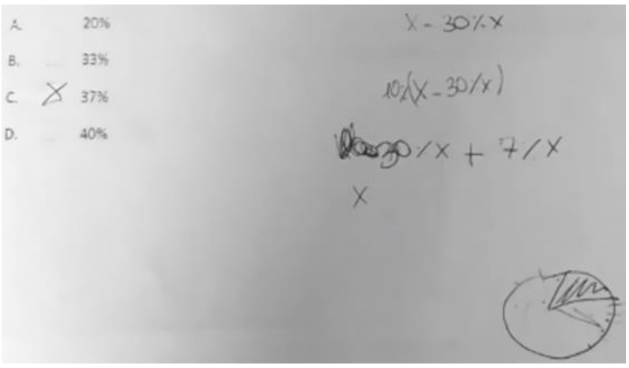

Figure 8. Ava's working sheet (Source: Scanned working sheet of Ava)

Sara faces the task by resorting to the metaphor of the discount as the decreasing cost of the dress, where we infer that the source domain is the coordination of the gradient and up-down image schemas. It is as if the decrease in each discount is independent of the quantity it is being decreased. Like Sara, she is not metaphorizing the fact that the percentages operate on two different quantities from the first to the second discount. The sum of fractions she uses in line 8 is coherent with her metaphorical reasoning, although it does not reflect the correct mathematical concept that leads to the correct solution. The researcher's questions in lines 9 and 11 puzzles Sara, who recalls the meaning of percentage as an operator on a given quantity. She carries out syntactically correct semiotic transformations (treatments) but they do not root in consistent mathematical meaning. In lines 12-16, Sara correctly calculates 30% of 60 that turns out 18 and calculates the further 10% discount on this quantity and not on discounted quantity, i.e., 60-18. In line 17, the researcher invites Sara to work with 100 as the original price but it does not help. Sara comes up with the same reasoning (lines 18-20). In line 21, the researcher uncouples the two discounts directly asking the price she expects with a 30% discount. Sara intuitively comes up with 80 but is completely lost at this point.

In this protocol, Sara carries out her activity to encounter mathematical objects having recourse mainly to natural language and arithmetical symbolic language. There are no instances of sensuous cognition in terms of kinesthetic activity, deployment of material objects, or use of icons that share a link to embodied experience. Sara's semiotic activity, which involves treatments in symbolic language and conversions between natural and symbolic language, allows us to infer what image schemas and metaphors support her reasoning. In the beginning, she uses a metaphor similar to Miriam's. The price of the dress is on a scale, and at each discount, the "price level" decreases on the scale. Prompted by the researcher, Sara interprets the percentage as an operator on the beginning price (lines 14-20). She is in a state of confusion, and her correct treatments have no meaning. She has no image schema to metaphorize her semiotic transformation. Sara is probably bridled in the cognitive paradox without a domesticated eye to encounter percentages as a meaningful cultural object in her sensuous experience and the emerging image schema to set off appropriate metaphors that sustain the calculation.

Sara carries out an intense semiotic activity that tries to incorporate the concept of percentage in its operational meaning. The semiotic transformations appear as a mechanical and meaningless manipulation of signs unrelated to the concept of percentage. In fact, Sara does not answer correctly to the item. We assume that her reasoning grounds in a metaphorization of the concept of percentages and discounts where the source domain the scale and up-down image schemas with the limitations outlined above. We do not get many hints about her sensuous activity in this interview, just terms like "subtract" and "add up" that recall the metaphor behind her intuitive calculation, the sum of the two discounts. Her eye is not domesticated to "see" percentages and discounts in its operational meaning.

Analysis of protocol 3 (correct answer C): The following is the transcript of Ava's interview (**Figure 8**):

24. Ava: [After thinking about it for a few minutes] I would do 30% of *x* and 10% of what's left.

25. Researcher: OK, go ahead.

26. Ava: So, how do I solve it? 70% of *x* minus

27. Researcher: This 70% of *x* comes from where?

28. Ava: From this [*x*-30%*x*].

29. Researcher: What does this represent?

30. Ava: The normal discount on the original price.

31. Researcher: So, this is the price with the discount of?

32. Ava: 30.

33. Researcher: How much should you take away from it?

34. Ava: 10%, however of that discounted … [Puzzled waits]. Another 7%, 7%*x* is the second discount. So, it is *x*, minus this, minus that, (deletes). This one is the normal one. Minus 30, 37.

35. Researcher: This *x*?

36. Ava: That's the price, no it is misspelt.

37. Researcher: This one minus.

38. Ava: Also. These are the two discounts.

39. Researcher: Explain it to me.

40. Ava: I have to make 30% of the *x*, which is the original price, then I have to make 10% of the remainder.

41. Researcher: Where did you get the 7 from?

42. Ava: From the 70% of the original price that I have left I make 10%. The total makes 37.

43. Researcher: Which is?

44. Ava: Which is 37, the total is 37.

45. Researcher: Because your fellow Italians got it wrong by choosing mostly D, 40%.

46. Ava: Because they make 30%+10%

47. Researcher: Where is the error?

48. Ava: Actually 10% is of the price that has already been discounted.

49. Researcher: If I had to explain this reasoning to an elementary or middle school child who doesn't know the variable?

50. Ava: I would do the percentages with a diagram.

51. Researcher: For example?

52. Ava: Oh my gosh, I tell him this 30 percent I have to take away and then out of all this I have to take away another one.

53. Researcher: From the pie you take away this wedge which would be 30%. And then what?

54. Ava: Of that, I divide it into parts and take out 10%.

55. Researcher: And then how would you go through the calculations?

56. Ava: In what sense?

57. Researcher: How do I get to the total discount without using *x*?

58. Ava: First I divide the initial one into wedges of 10 and see that there are 7 (wedges of 10) left. I divide the 70 into 10 and it is 7. So, I see that it is 30+7.

Ava performs the algebraic calculations correctly resorting to treatments in symbolic language and conversions between natural language and symbolic language (lines 24-45). The use of symbolic language is rooted in sensuous cognition that via subsequent objectifications has presumably domesticated her algebraic eye to encounter and give meaning to the notion of percentages. The student clearly explains the image schema that acts as the source domain of the metaphor that informs her reasoning up to the algebraic abstraction. The image schema is slightly different from Miriam's in the first protocol; a coordination of containment, in-out, part-whole, and splitting.

Ava performs effective, concise, and neat semiotic transformations, both treatments and conversions involving natural language and algebraic language. The metaphor behind her reasoning is based on an efficacious blending of image schemas as source domain. In fact, the operational meaning of percentages that operate on different quantities is metaphorized via the part-whole image schema. The initial price is the whole and the part obtained with the splitting pours out of the container. The remaining quantity plays the role of the whole for the second discount carried out in the same fashion. The actions of outlining the whole and the part split in wedges, and the "leakage out" of the container is carried out by Ava using the drawing, gesture, and natural language. The domesticated eye allows her to construct the correct metaphor of percentages that inform the effective use of semiotic transformations.

Analysis of Protocol 4 (correct answer C): The following is the transcript of Mia's interview:

59. Mia: I do it by numbers, always, because I get better. Discount, so it goes to 70. Discount 10% of 70 because it's already discounted. So, it comes, 7, so it comes 63 Euro.

60. Researcher: And that's …

61. Mia: The final price of the dress.

62. Researcher: The discount?

63. Mia: The discount is … oh God, I do not know what I'm doing.

64. Researcher: How is it possible that you don't know what you're doing?

65. Mia: I'm getting lost.

66. Researcher: In a teacup.

67. Mia: for sure …

68. Researcher: Let's start from here, this is the …

69. Mia: That's the price of the final dress.

70. Researcher: What does it ask?

71. Mia: What is the overall percentage discount on the original price. So, then, OK, so on the original price, so 63 stands at 100, which is the money, right, I don't know, no, then, then. This is the original price, and this is the final price. Ah, OK it was 30%, it's the original one, so it's definitely

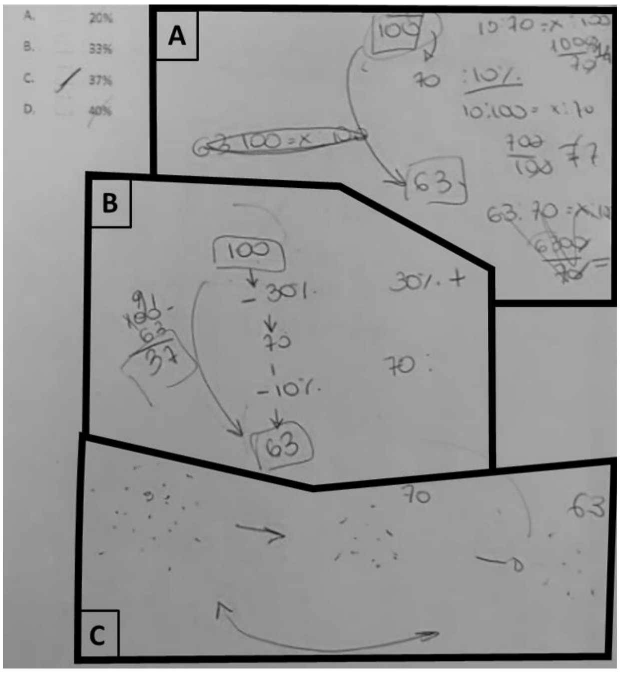

Figure 9. Mia's working sheet (Source: Scanned and edited working sheet of Mia)

not this one, because it's less. I think it's not even the sum. So, it's one of those two, now we have to understand, and that's the passage here but I'm getting lost. So, by a further 10%.

72. Researcher: That's the final price.

73. Mia: Yes, 10% of 70 is equal to what percentage of 100? (Long pause) It's more or less 12. 7,1, remainder of 3, 30, 7, 14, 21, 28, and 4.

74. Researcher: If you need a calculator, use it.

75. Mia: It comes out more or less 14, so then I would do the sum and it would come out ... I don't think so. So, no. So, during the sale season the already discounted price comes ... Let's do the steps again right (drawing the schema in **Figure 9**). So, 10, 10 I take 30% off and it comes to 70 then I take 10% off again and it comes to 63. So, 30% off the original I have it for sure (long pause).

76. Researcher: 63 is the final price, right?

77. Mia: And 100 is the original one.

78. Researcher: So how much is the discount?

79. Mia: 40, I make. So here (**Figure 9**) ... OK so I have to calculate how much he discounted it to me, so I do 100-63, 37. So 37 is what he took it off me, so then you do 37% I would say.

80. Researcher: OK.

81. Mia: Then if I want to, yes because I do what I get taken off is 37 because it's less than 100, that seems fair, that seems fair to me.

82. Researcher: If you had to explain it to a child, with a drawing, a diagram, how would you do it? Without using numbers, to make them understand that you don't have to add up the percentages.

83. Mia: Then you do … Or you do it like with little balls. That is, like little drawings like in primary schools that were in the books that you put in, maybe instead of 100 you decrease the quantities. But then you do everything in proportion, i.e., you put a total of balls, lowered to, i.e., you consider that these balls are the percentage like, we could see. Like if I have 100 balls which is the price of my dress, then I have 70 balls left and then I have 63 balls left. So then afterwards you look, you wonder how many balls were lost, like. And so, you look at how many have gone, I mean you do not do the sum of this and this.

84. Researcher: So, this is the original one.

85. Mia: These are the final balls.

86. Researcher: Here 70 are left because I took out 30, then from that.

87. Mia: You take out 10%

88. Researcher: Of my original amount 63 remain.

Mia wisely decides to perform the calculation using numbers–taking 100 as the initial price–and performing treatments in the arithmetical symbolic language and conversions from natural language to arithmetical symbolic language. In line 59, she swiftly arrives at the conclusion that the final prize after the two discounts is 63. The calculations are supported by a metaphor of the discounts with an image schema as the source domain similar to Miriam's. However, there is an important difference. The decrease of the level of the gradient in each discount considers the new price after the previous discount. She is not using the percentage as a given quantity that "spills out" the original price but as an operator *on the price as a given* quantity that *changes* after each discount.

Mia is bewildered when asked to determine the total discount. It is interesting how her metaphoric reasoning is not working to support the calculation, which is just a simple subtraction, as if her "theoretical eye" becomes "blind" (lines 63-75). In lines 73-75, the student uses a proportion to calculate 10% of 70 (**Figure 9a**) that she already did with a mental calculation! In line 75 she realizes the result is inconsistent and goes through the reasoning using the drawing in **Figure 9a** that led to the final price (63) after the two discounts, according to the image schema described above.

The researcher's question in line 76 ignites an adjustment of her original metaphor that was latent in Mia's sensuous cognition. Mia immediately arrives at the correct solution. Referring to the drawing in **Figure 9a** and **Figure 9b**, she adds an arrow from 100 to 63, scrolling it with the pen incorporating in the drawing the discounts that led from 100 to 63 at each step. This convergence of gesturing and drawing allows Mia's theoretical eye to see the missing subtraction, as shown in her drawing in **Figure 9b**–which was not in her perceptive field - as an outcome of a more encompassing metaphor'. The second drawing prompted by the researcher's input (**Figure 9b**) shows another metaphoric understanding of the item. Now, the original source domain is the image schema completed also considering the part-whole image schema that provides a strong metaphor to sustain the subtraction and calculate the total discount. Mia also interweaves the container, full-empty, splitting and part-whole image schemas as the source domain to provide another metaphoric understanding of percentages and discounts.

Mia shows a dialectics between her semiotic activity and the metaphors she brings into play. In the beginning, she cannot connect via a trivial subtraction the final price with the total discount. Although the calculation is trivial, she needs to ground it into a meaningful metaphor. Indeed, before adjusting the scale and up-down image schemas, she performs nonsense calculations to figure out the total discount. The first metaphor is based on the scale and up-down image schemas used in a subtle way. To subsume the operational meaning of the percentages and discounts, each decrease is dependent on where you are positioned on the scale. Such a position determined how much you go down given the discount. The second metaphor is based on the containment and unity/multiplicity image schemas as described in Ava's protocol. The actions of moving up and down the scale and connecting different levels are carried out with drawings to materialize the scale, and gestures to move up and down along the scale. The actions of outlining the whole and the part split in "balls", and the "leakage out" of the container is carried out by Mia using the drawing, gestures, and natural language. This protocol shows Mia's domestication of her theoretical eye during the interview to "see" the total discount that was not part of her sensuous and imaginative experience in the first part of the interview.

DISCUSSION: QUANTITATIVE & QUALITATIVE ANALYSIS (QUAN+QUAL PHASE)

The focus of the present study is the learning of percentages across different school levels and the metaphors underlying students' reasoning. We concentrated on the transition from lower secondary school to high school. The INVALSI results from high school are particularly significant in highlighting the student's difficulties in operating with fractions. The impact of the result is reinforced by the semiotic resources high school students can resort to. We analyze item 25 of 2012 whose Rasch model and distractor plots show the emergence of a macro phenomenon at the level of the Italian educational system: 24,25% of correct answers, 48,01% of the students answered distractor D with a "humped performance trend".

The quantitative data tells us not only the presence of a macro phenomenon but also the difficulty faced by the students also hit those at medium levels of competences. These quantitative results push towards a better understanding of the cognitive processes behind the learning of percentages and the related teaching practices.

The qualitative analysis allows us to dig into the possible origins of such a macrophenomenon. The four protocols we analyzed allow us to pinpoint four possible attitudes towards the learning of percentages based on the metaphors that inform their reasoning.

Miriam (who gives the wrong answer) draws on a metaphor for percentages whose source domain is the connection of gradient and up-down image schemas. This approach leads to sum 30% and 10% without any semiotic transformation in terms of treatments and conversions. The metaphorical thinking does not grasp the percentages as an operator on a given quantity that changes after each discount.

Sarah (who gives the wrong answer) draws on the same metaphor with gradient and up-down image schemas as source domain. She operates a conversion in arithmetical symbolic language and sums the two percentages. Prompted by the teacher to calculate the discount on a given original price she chooses, 60. At this point, she uses the percentage as an operator first on the original price (60) and then on the discount (18), not on the remaining quantity. She performs a series of treatments that have no background of meaning and she looks trapped in the cognitive paradox, using signs in mechanical transformation. We underline the inconsistency between her metaphoric thinking and her semiotic one. An appropriate metaphor does not sustain her use of percentages. This is a possible explanation for the mechanical use of semiotic transformations.

Mia (which gives the correct answer) draws on the same metaphor as Miriam's and Sara's. She considers percentages as operators on a given quantity and she is also aware that after the first discount the new percentage operates on another quantity. Conversions and treatment are correctly handled to arrive at the final price. Nevertheless, when asked to calculate the total discount her metaphorical thinking does not work anymore, and she is stuck in the cognitive paradox. At this point, missing an appropriate domain of meaning she starts performing a proportion as a series of treatments that turn out to be a cul-de-sac. Prompted by the teacher she completes her original source domain (gradient and up-down image schemas) also including the part-whole image schema that triggers the subtraction between the original price (total) and the final price (part) she calculated before. Eventually, Mia also offers the connection of container, full-empty and part-whole image schemas as the source domain for another metaphoric understanding of percentages and discounts.

Ava (which gives the correct answer) brings in the discussion a neat and concise example of the strong coherence between a suitable metaphoric reasoning and the correct semiotic functioning that allows the student to carry out the task successfully. Ava immediately casts the solution at a high level of abstraction performing conversions and treatments that involve the algebraic symbolic language and shows the metaphor behind her semiotic transformations. Her source domain is the correlation of the container, full-empty, splitting and part-whole image schema as the metaphor's source domain sustains her algebraic thinking.

We conclude the *QUAN+QUAL phase* highlighting the consistency between the quantitative and the qualitative analyzes. Firstly, the structure of the item we have analyzed did not provide any specific number for the discount and did not explicitly ask to carry out specific calculations related to percentages. A task that is different to the ones usually provided in school, especially high school, where the students are pushed to perform many calculations, resorting to treatments and conversions with more than one semiotic system. For example, the algebraic and/or the arithmetic symbolic languages are usually involved. Thus, in and of itself, the item orients the students immediately to metaphoric reasoning. Secondly, distractor D spotlights within the LSA a macrophenomenon regarding the learning of percentages that involves students up to medium levels of competences calling for an understanding of the possible causes of such behavior. The metaphoric interpretation we discussed in sections 4 and 5 gives a possible interpretation consistent with the item's structure and distractor D and the quantitative results.

CONCLUSIONS AND FURTHER ISSUES

We have all the information to answer the research questions.

RQ1. Looking at INVALSI data what type of difficulties do students encounter when facing tasks regarding percentages? What are the levels of competences mainly affected by such difficulties?

A1. In the transition from different school levels, specifically from lower secondary school to high school, students cannot handle percentages as operators on a specific and variable quantity. Given the quantitative data and the Rasch model, it is a macro phenomenon that involves students up to medium levels of competence. The humped form of the characteristic curve tells us the students' difficulties are accounted for by the specific cognitive functioning in the teaching and learning of percentages. The answer to **RQ2** elaborates on this point from a qualitative point of view to outline the didactical mechanisms behind the macrophenomenon.

RQ2. What image schemas and metaphors do students presumably use to conceptualize percentages? What forms of sensuous cognition with its related use of semiotic means of objectification give rise to such image schemas?

A2. The qualitative analysis shows that the scale, container, space, and unicity/multiplicity categories of image schemas play a prominent role in the metaphorization of percentages and discounts in incorrect and incorrect reasoning. It comes out that the container and unity/multiplicity image schemas play an important role in an effective metaphorization for the correct learning of percentages as operators on a given quantity. When students lack such source image schemas the semiotic activity does not support correct reasoning and it turns out a mechanical and meaningless manipulation of signs. The absence of suitable metaphorizations can be the cause of the emergence of the macrophenomenon highlighted by the quantitative data. Our qualitative data do not give important information about forms of sensuous cognition. However, the available information shows that the role of gestures, the use of icons/drawings and natural language play an important role in the domestication of the eyer, and the emergence of the image schemas mentioned above in kinesthetic and more imaginative forms of action.

RQ3. How do students handle semiotic transformations (treatment and conversion) in conceptualizing percentages? How does the coordination of semiotic systems relate to sensuous cognition and emergent image schemas?

A3. There is a strong link between metaphoric reasoning and the semiotic transformations in learning percentages, both in correct and incorrect conceptualizations. Where there is a link to correct metaphors, there is a background of meaning that sustains correct treatments and conversions. Where such a link is missing, students transform signs mechanically without reference to the mathematical object they represent.

The results of the present study can inform the school practice in the teaching and learning of percentages. There is a need for appropriate processes of objectification in sensuous cognition that domesticate the students to see suitable patterns and archetypes that can be recognized as image schemas for the correct metaphorization of percentages. The interiorization of mathematically correct metaphors can support students' learning and upcoming activities such as problem-solving and the encounter of percentages at higher levels of abstraction. Students also have strong bases for learning the subsequent mathematical knowledge related to percentages.

The study needs further development both quantitatively and qualitatively. On the one hand, we need to analyze more than one item to get a more structured outlook on these learning issues. On the other hand, we need a more articulated research design to fathom the features of objectification and sensuous cognition in the domestication of the eye to recognize the appropriate mathematical patterns and the ensuing image schemas.

Italian LSA provides data concerning other interesting tasks similar to the one analyzed, which could be the starting point for a wider analysis of students' difficulties in operating with percentages. In a previous study (Giberti, 2018) we focused on this and other INVALSI tasks: the quantitative analysis based on the Rasch model highlighted similar difficulties which could be deeper investigated through a mixed method approach as in this study furthermore, significant results emerged in terms of gender differences in these tasks in favor of males.

In addition, topics such as the one addressed are relevant as feedback to students and teacher training. Many studies show the importance of doing teacher training on the topic of LSA to interpret macro phenomena from the quantitative and qualitative points of view (Santi et al., in press).

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