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Research Article

The changing role of number facts in the development of arithmetic skills: A cross-sectional study on students' views and performance

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ABSTRACT

Students' performance in mathematics is a major focus of education research. One of the cornerstones of mathematical performance is arithmetic skills. The system of arithmetic skills is a complex one, the components of which can play different roles in the developmental processes. This study investigated number facts, general arithmetic skills, and students' views on the importance of knowing multiplication number facts to better understand the possibly changing role of these components. Participants were 3rd, 5th, and 7th grade students (N = 254). The results suggest that even in their early years of schooling, students possess strong beliefs about the value of knowing multiplication number facts by heart. However, many continue to struggle with recalling answers, such as 7 times 8, within three seconds. This cross-sectional development study also reveals the evolving role of number facts in students' arithmetic performance.

Keywords: number facts, arithmetic skills, multiplication table

INTRODUCTION

Students' performance in mathematics is a major focus of education research. During the elementary school years, mathematical skills are one of the pillars of the three Rs (aRithmetic, Reading, and wRiting). Arithmetic skills are the defining components of mathematical skills, and they form a complex system, the components of which can play different roles in the developmental processes. During the elementary school years, gradual automation of arithmetic skills, both speed and accuracy, developed in tandem. Even laypeople can judge speed and accuracy with which someone performs simple arithmetic operations.

The role of number facts in the development of arithmetic skills has long been a subject of significant theoretical and practical interest. From a theoretical perspective, understanding the development and potential transfer of distinct arithmetic skill components warrants closer examination. Recent methodological advancements and theoretical models (e.g., Csíkos, 2022) have particularly highlighted the importance of exploring the automatic and strategic aspects of these skills. Empirical research using the term fact fluency revealed (Chong & Siegel, 2008) that deficits in the acquisition of number facts has not only long-term consequences but they seem to be persistent throughout the elementary years of schooling. From an educational perspective, our findings offer insights into the persistent question of *how* the acquisition of multiplication number facts is essential for arithmetic proficiency. We pay special attention to multiplication number facts since there are not very many in combinatorial terms (compared, for example, to the number of simple additions and subtractions possible in the number range of 100), and–as described in section 2.3.– it is a common curricular requirement during the elementary school years to learn them by heart.

In this research, a cross-sectional development study involving students from grades 3, 5, and 7 will address the question of how different components of arithmetic skills evolve and develop during the school years. Although key phases of development preceded formal schooling and can extend throughout life, this age range is particularly critical for acquiring foundational skills essential for both academic success and real-life applications.

THEORETICAL BACKGROUND

The Role and Importance of Arithmetic Skills

In our study the term 'skill' refers to largely automated processes, in contrast to strategies, which involve conscious, metacognitive efforts (Afflerbach et al., 2008). In other words, even if they are learned through conscious effort and initially with strong metacognitive support, the characteristic of skills is that they can be used effortlessly and automatically. Different nomenclatures emphasize different components of arithmetic skills, which is itself a complex system. Whether we look at fully automated components or more complex calculations involving conscious decisions, arithmetic skills form an essential component of mathematics learning. Nogues and Dorneles (2021) highlighted the long-term importance and role of working memory and early numeracy as measured in the early school years. This is in line with what Baroody (2010) claimed when saying children should achieve fact fluency in their preschool years. Shanley et al. (2018) underscore that arithmetic skills encompass different components of early numeracy skills, such as counting and simple addition and subtraction, which strongly predict school performance (Aunio & Niemivirta, 2010). Gagné (1983) argued that failure in computation may stem from either insufficient knowledge or not sufficiently automatized computational procedures. In Gilmore et al.'s (2018) study of arithmetic skills, lower primary students were administered several tests including 12 items on number facts (simple addition tasks), counting, conceptual understanding, and other measures. In our study, we follow Gilmore et al.'s approach in that the arithmetic skills test of our study consists of several different parts that target automated and strategic components to varying degrees.

Number Facts as Key Components of Arithmetic Skills

Number facts or arithmetic facts can be defined as declarative knowledge components that can be retrieved from long-term memory (Ashcraft, 1982). This definition is grounded in the distinction between procedural and declarative knowledge; a dichotomy that flourished in that decade, although Winograd (1980) had already warned that the distinction between the two was unclear Ashcraft (1982) posits that by grade 3, children begin to expand their procedural arithmetic knowledge with a progressively richer structure of declarative knowledge—namely, number facts. This distinction is a precursor to the model proposed by Csíkos (2022) that describes the growth of knowledge and performance as a dynamic interaction of different levels of thinking components. As Ter Heege (1985, p. 375) observes, in the developmental processes "The border between 'figuring out' and 'knowing by heart' will gradually disappear." In his work, Ashcraft (1982) employed a chronometric approach, using reaction time as the primary performance indicator. With this time-sensitive measure, computer-based assessment tools can offer objective, reliable, and valid insights into the presence of number facts in children's cognitive processes.

Subsequent research established a 3-second limit for measuring and identifying number fact retrieval (Gilmore et al., 2018; Jordan et al., 2003). Jordan et al. (2003) defined fact mastery as the ability to solve basic addition and subtraction tasks within this timeframe, acknowledging that even within such a brief period, students may employ a "covert calculation shortcut"—a quickly executable strategy rather than pure fact retrieval. In contrast, Ashkenazi and Silverman (2017) applied a 4-second time limit, which, although still stringent, contrasts with the considerably longer 10-second limit commonly used to measure two-digit mental arithmetic skills involving strategic calculation (McMullen & Siegler, 2020). Further studies, including research by Frenck-Mestre and Vaid (1993) on bilingual individuals, reveal that the Arabic digit format elicits faster response times compared to number words in any language, suggesting that some number facts emerge early, potentially as early as kindergarten (Bisanz et al., 2005).

Any tests that apply time constraints may cause anxiety for children (Boaler, 2014). With their time frame measured only in seconds, number facts items might especially be triggering for some, therefore, the circumstances of administering the test are decisive. In any case, it seems inevitable that we should also use time-limited testing methods if we are measuring the complex system of numeracy, since in the dual system of speed and accuracy, it is precisely the number facts that primarily manifest the speed dimension.

Number Facts in the Curriculum

Curriculum requirements for memorizing multiplication tables draw on both mathematical and psychological insights. Historical perspectives reinforce the value of memorizing multiplication tables. According to De Morgan (1854, p. 25), “it is usual to learn the product of numbers up to 12 times 12. All pupils … should slowly commit the products to memory as far as 20 times 20.” Similarly, Dunton (1891, p. 82.) insisted on that the must be present to the student “in the twinkling of an eye”, because this is the foundation of further multiplication division operations. From an educational and psychological perspective, appropriate timing based on age characteristics is one element of educational relevance. Another crucial aspect, particularly in today’s era of information and communications technology tools, concerns how and why students should learn multiplication tables. Isoda and Olfos (2020) argue that within Japanese traditions of teaching multiplication, students are also encouraged to extend the multiplication table independently. In this way, multiplication tables serve not only as memorized facts but also as a tool to foster higher-order thinking skills.

International curricular comparisons by Olfos et al. (2020) question why some countries segment multiplication learning across multiple grades while others do not. Generally, by the end of grade 4, students should master single-digit multiplications up to 100. In Hungary, the national core curriculum mandates fluency in the mental multiplication of single-digit numbers up to 100.

Children’s Views and Beliefs About Number Facts

Alongside the development of arithmetic fact systems and increasingly faster and more efficient calculation processes, various metacognitive knowledge components also evolve over time. By 3rd grade, children generally have the ability to reliably report on their calculation strategies (see, e.g., Reed et al., 2015), and their measurable beliefs strongly correlate with mathematics achievement (see Hidayatullah & Csíkos, 2023). In this study, the term ‘view’ is used as a synonym of ‘belief’ which has been well-defined for a long time (Pehkonen & Pietilä, 2003).

However, empirical research on children’s perspectives regarding the necessity of memorizing number facts is limited. One study by Carr et al. (2024) used pictorial options (such as emojis) to allow 4th grade students to express their attitudes toward multiplication. It’s clear that students’ beliefs and perspectives are shaped by both curricular expectations and their experiences with their own and their peers’ performance. By the end of 3rd grade, knowing the multiplication table by heart is almost an assumed requirement. As Ter Heege (1985, p. 377) noted, a positive attitude is essential, and “children must quite consciously want to do it.” In particular, a multiplication fact deemed unimportant to know by heart is unlikely to be memorized in the long term.

Research Questions

To empirically investigate the possibly changing role of number facts in the development of arithmetical thinking, the following research questions were formulated.

(1) What developmental patterns can be observed in mastering number facts?

(2) How do students’ beliefs about memorizing number facts relate to their arithmetic performance?

(3) What is the connection between students’ mastery of number facts and their arithmetic skill level?

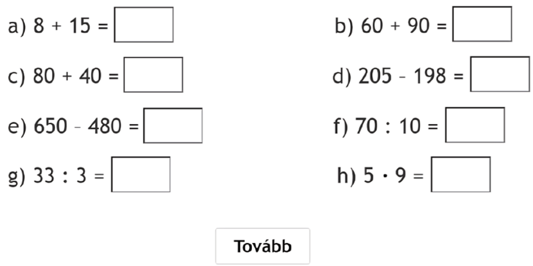

Figure 1. Arrangement and content of eight arithmetic skills items ("Tovább" means "Forward" in Hungarian) (Figure created by the authors)

METHODS

Participants

Convenience sampling included two schools from the capital city of Hungary. Having received parental consent and the school principal's support, one of the authors of the current study supervised the data collection process. The study followed a cross-sectional developmental study design involving 254 students from grades 3, 5, and 7. There were 94, 79, and 81 students involved from the three grades, respectively. Since both schools are inclusive for students living in the residential area, the gender and SES distributions can be considered random, thereby increasing the representativeness of the sample and the generalizability of the results. Data collection was entirely anonymous and no personal or sensitive information was stored, therefore approval from the institutional research ethics committee was not required.

Measures

(1) A larger test battery included three subtests for this study. The tests were administered on the eDia online platform (Csapó & Molnár, 2019). The number of fact items were then followed by Likert-type questions and arithmetic skill items. This online platform allows students to complete the tests anonymously, with individual data collection, while staying in their familiar classroom. Each number fact item was presented on a separate screen page. The other skill items were grouped. When using time constraint, the computer stored the final answer within the time limit, and after three seconds, the only option was to press the "forward" button. As for the other two item sets, we provided an overall time frame that students were free to allocate. However, after entering all the answers on a given page and clicking the "forward" button, students could no longer go back to that page to make any corrections. The structure of the test was the following: number facts. Ten multiple-choice items assessed basic arithmetic skills, including three addition, three subtraction, two multiplication, and two division problems. One of the multiplication items was an easy one (4 × 1), while the other was the one that is usually considered to be the most difficult among the one-digit multiplications (Ter Heege, 1985), 7 × 8. Students had to select the right answer within three seconds. The basic operations were displayed in Arabic number format in the upper half of the screen, and the four options were presented underneath.

(2) Students' views on multiplication facts. The question was: Does everyone have to know by heart how much is ...? The four multiplications to which the question referred were 8 × 7, 2 × 2, 10 × 10, and 20 × 20. The options were provided using a five-point Likert-type scale, measuring the level of agreement.

(3) Arithmetic skills. Comprising 24 open-ended items across the four basic operations, this subtest presented most problems in Arabic numeral format, with two division and two multiplication items displayed in number word format. Students had to type their answers in blank answer fields. Figure 1 presents the arrangement of the first eight open-ended items on the screen.

Procedure

The test battery was piloted on a small sample of children of the same age, and small refinements were made to the technical arrangement of the screen display. Students participating in this research solved the tests in their schools during math class in a classroom equipped with computers and internet access.

Table 1. Students' mean performance (SD in parentheses) on the number fact items according to grades

Item	Grade 3 (N = 94)	Grade 5 (N = 79)	Grade 7 (N = 81)
4 + 2	.91 (.28)	.96 (.19)	.95 (.22)
9 + 4	.80 (.40)	.89 (.32)	.91 (.28)
8 + 3	.85 (.36)	.89 (.32)	.84 (.37)
6 - 4	.79 (.41)	.95 (.22)	.91 (.28)
13 - 9	.39 (.49)	.62 (.49)	.64 (.48)
16 - 7	.80 (.40)	.85 (.36)	.79 (.41)
4 · 1	.88 (.32)	.91 (.29)	.94 (.24)
7 · 8	.66 (.48)	.75 (.44)	.72 (.45)
6 : 3	.86 (.35)	.96 (.19)	.96 (.19)
8 : 2	.87 (.34)	.92 (.27)	.93 (.26)

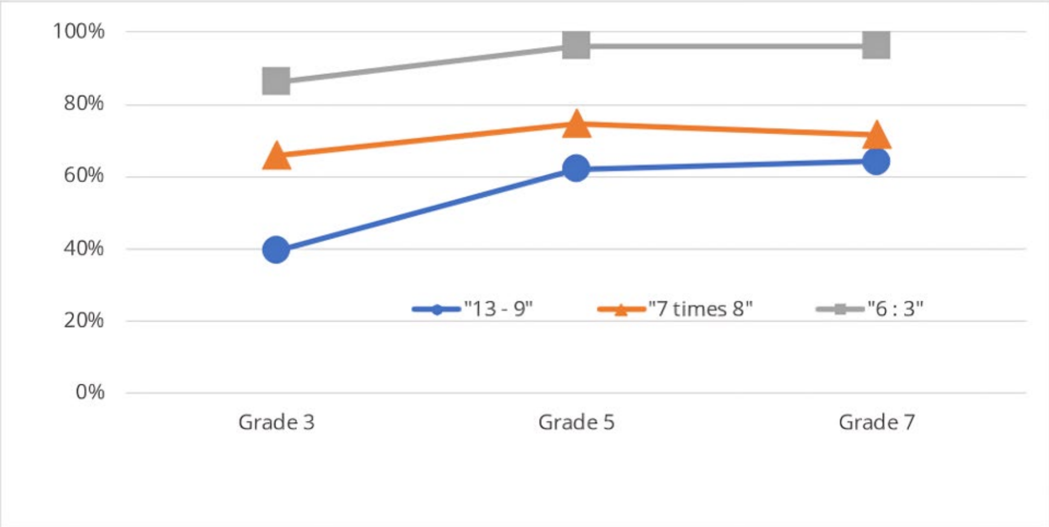

Figure 2. Students' performance on three items of the number facts subtest (Figure created by the authors)

At the beginning of the test session, warm-up items were solved by the children to get used to the 3-second time limit. These warm-up tasks were very simple, e.g., the instruction "click on the triangle" was followed by four simple geometric shapes: square, triangle, circle, and a general rectangle. Students had to click on the right solution within three seconds. After three seconds, a pop-up window appeared telling them that by clicking on the "forward' button at the lower right corner, the next task would appear.

RESULTS

Reliability

Cronbach's alpha coefficients for the number facts (8 items), views on number facts (4 items), and arithmetic skills (24 items) subtests were .57, .84, and .75, respectively. The low reliability of the number facts subtest can be attributed to the ceiling effect (see Garin, 2014).

Developmental Patterns of Number Facts

Table 1 presents mean performance (standard deviation [SD] in parentheses) by grade for the eight number fact items.

The results presented in **Table 1** suggest that the multiplication 7 × 8 is not readily available as a number fact for about one-third (grade 3) or one-fourth (grade 5 and grade 7) of the students. An ANOVA test revealed no significant difference between the three age groups: $F(2, 251) = .82$, $p = .44$. Besides the item 7 × 8, the subtraction 13-9 proved to be fairly difficult, indicating the need for slower mental calculation strategies (e.g., adding 1 to 9 and then adding 3 to 1). In **Figure 2**, these two difficult items are compared to an easy one across age groups.

Table 2. Students' mean (SD in parentheses) scores on the items about the importance of knowing multiplication number facts by heart

Item	Grade 3 (N = 93)	Grade 5 (N = 79)	Grade 7 (N = 81)
7 times 8	4.15 (1.08)	4.56 (0.66)	4.27 (0.92)
2 times 2	4.63 (0.93)	4.95 (0.22)	4.86 (0.52)
10 times 10	4.65 (0.93)	4.91 (0.33)	4.80 (0.70)
20 times 20	3.87 (1.13)	3.99 (1.06)	4.01 (0.89)

Table 3. Correlations between number fact item performance (p in parentheses) and performance on the arithmetic skills subtest. Significant values are marked by * (p < .05) and ** (p < 0.01)

Item	Grade 3 (N = 94)	Grade 5 (N = 79)	Grade 7 (N = 81)
4 + 2	.04 (.69)	-.03 (.80)	-.03 (.82)
9 + 4	.16 (.12)	-.15 (.20)	27* (.01)
8 + 3	.13 (.21)	.04 (.71)	-.05 (.68)
6 - 4	.11 (.30)	-.05 (.67)	,32**(.00)
13 - 9	.21* (.04)	-.00 (.99)	.44** (.00)
16 - 7	.30** (.00)	-.02 (.86)	.09 (.44)
4 · 1	.10 (.35)	.01 (.91)	-.01 (.95)
7 · 8	.14 (.18)	.06 (.60)	-.07 (.51)
6 : 3	.12 (.25)	-.00 (.98)	.02 (.88)
8 : 2	.23* (.03)	-.14 (.23)	.06 (.62)

The division 6:3 can be considered a mastered number fact from grade 3. The multiplication 7 × 8 does not show a developmental pattern, and performance is under the cut-off performance level of 80%. As for the subtraction 13-9, there is a significant development between grade 3 and grade 5 (Games-Howell post-hoc comparison gives p = .008).

Students' Views on the Importance of Multiplication Facts

Students were asked about the importance of memorizing some multiplication number facts by heart. By grade 3, they are usually required to know by heart three of them: 8 times 7, 2 times 2, and 10 times 10, while there is a slight chance that they can retrieve 20 times 20 from their memory. The judged importance of these multiplication number facts is presented in **Table 2**.

The results suggest that the three multiplications required by the curriculum are judged to be important to know by heart, with average values above four on the five-point Likert scale. There is one anomaly deserving special attention: the judged importance of knowing 7 × 8 by heart is higher in grade 5 than in the other two age groups.

The connection between the judged importance of the "7 × 8" item and the actual performance on that item showed a significant correlation in grade 7 (r = .25, p = .03), while there were non-significant correlations in grade 3 and grade 5 (r = .02, p = .87; r = .05, p = .66, respectively).

Number Facts and General Arithmetic Skills Performance

We examined the connections between students' views on number facts and their actual performance on the arithmetic skill subtest, and also the connection between their performance on the number fact items and the arithmetic skills subtests.

The correlation between the judged importance of knowing some multiplications by heart and the level of arithmetic skills proved to be significant for the item "7 × 8" (r = .24, p < .001) and non-significant for the other three multiplications.

Table 3 presents how students' performance on the number fact items correlated with their overall arithmetic skills performance. Remarkably, none of the correlation coefficients in grade 5 are significant.

To evaluate the overall contribution of the number facts items to performance on the arithmetic skills subtest, linear regression analysis was conducted with arithmetic skills as the dependent variable, and number facts and views on number facts as independent variables. The regression coefficients are presented in **Table 4**.

Table 4. Linear regression analysis with arithmetic skills as dependent variable

Grade	Independent variable	β	t	p
3	Number facts	.37	3.79	< .001
	Views on number facts	.11	1.13	.26
5	Number facts	-.05	-0.41	.68
	Views on number facts	.21	1.91	.06
7	Number facts	.22	1.99	.05
	Views on number facts	.07	.62	.54

Table 4 suggests that views on number facts does not have a significant contribution to performance on arithmetic skills, however, in grade 5, the explained variance is almost (marginally) significant ($p = .06$). The number facts scale has significantly positive explained variance on arithmetic skills both in grade 3 and grade 7. R^2 values for grades 3, 5, and 7 were .14, .05 and .06, respectively.

DISCUSSION

Main Findings

This research is a cross-sectional development study of the mastery of number facts and the connection between number fact mastery and arithmetic skills. Additionally, having investigated students' views on the importance of knowing multiplication facts by heart, specific metacognitive knowledge components were assessed. To evaluate the developmental level of number facts, both reaction time and mastery approaches were employed. According to Guskey and Anderman (2013), a typical mastery cut-off score is set at 80%. On a dichotomous scale, this can be interpreted as the percentage of students able to solve an item correctly. *Across all age groups, however, the proportion of students who solved the 7 × 8 multiplication item did not reach the 80% threshold.* In line with what Chong and Diesel (2008) found about the persistence of deficits in number fact mastery, beside those differences that can be considered between individuals, there are number facts that prove to be more challenging for both struggling and typical students (Ter Heege, 1985).

Our second research question concerned students' views on the importance of knowing different multiplication number facts by heart. Their opinion about the importance of knowing 7 × 8 by heart positively correlated with their achievement only in grade 7. This may be due to students' increased awareness of their own performance (Muncer, 2020), while those who perceived themselves as less successful may have viewed the memorization of this multiplication fact as less crucial. Nonetheless, it is notable that students form strong beliefs about multiplication facts from an early age, partially independent of their actual performance, aligning with Csíkos' (2022) multi-phase, two-level model of arithmetic skill development.

Our third research question focused on students' performance on the number fact items and its connection with their performance on the open-ended arithmetic skills items. In the latter, no time constraint (other than a general, overall time allocation) was used, while the number fact items did require quick solutions. Our results suggest that it is the mastery of certain number facts that positively correlate with performance on the open-ended items, while other number facts (more precisely, the non-mastery of these other items) indicate poorer performance on the open-ended items. Furthermore, these two kinds of connections may also be related to developmental changes as well. According to Björklund et al. (2021), there are individual differences in the level of awareness about the process of practicing arithmetic skills, and this may explain why certain number facts can or cannot contribute to arithmetic skill performance beyond the simplest number fact retrieval.

Novelty

Our research combined a modestly complex test system measuring various psychological components of arithmetic skills and a cross-sectional developmental design. This research design allowed us to support a recent theoretical model on the various roles fulfilled by metacognitive and non-metacognitive components of arithmetic skill development in addition to examining a developmental process important for educational practice.

Limitations

The first factor of limitations concerns the sample. With a larger sample, the statistical significance of more and clearer trends may have been revealed. With a more heterogeneous (and thus more representative) sample reliability issues might have been eliminated, and we could have achieved greater generalizability of the results. The second factor concerns our measures, especially the number fact items. There were only two items administered for each of the four basic arithmetic operations. Since students' views were asked about multiplication number facts, a greater share or emphasis of multiplication number facts within the items that were administered under time constraints.

Educational Implications

From an educational point of view, it is remarkable that students' performance on the number fact items has a time-varying role in the development of their arithmetic skills. There are different number facts in the different grades that significantly correlate with the arithmetic skills test score. Since students' arithmetic skill was measured by providing ample (sufficient) time, besides the quick retrieval of number facts and quick processing of certain strategies, other components of arithmetic skills could have been measured. These various components may belong to both the meta-level or the object-level of students' thinking, and similarly to what scholars in the field of reading research found (see Walczyk, 2000), compensatory mechanisms allow students to achieve well in the arithmetic skills test despite the lack of automation of number fact components. With the aim of expanding teachers' specialized pedagogical content knowledge (as defined by Ball et al., 2012) we summarize the pedagogical lesson as follows. *In an earlier phase of arithmetic skills development, the mastery of certain number facts is an indicator of good performance, while in a later phase, lack of mastery of other number facts may indicate poor performance in arithmetic skills*. Teachers can use simple tests like ours to reveal the specific individual differences in their classroom.

Future Research

Future research can extend in at least two directions. Replacing the cross-sectional design with a longitudinal one would enable the exploration of causal relationships and the transformation of internal cognitive structures. Additionally, enhancing the study's variables—such as by expanding the views on number facts scale and including a more comprehensive range of basic arithmetic operations on the number facts scale—could yield further insights. Integrating additional psychological constructs, like those measured by Pittalis et al. (2018) or Gilmore et al. (2018), may also help clarify their contributions to arithmetic skill development.

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Declaration of interest: The authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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