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Research Article

Teachers' leading whole-class discussions in a mathematics lesson study: From initial understanding to orchestration in practice

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ABSTRACT

This study aims to identify how mathematics teachers understand whole-class discussions and to know the influence of lesson study on the way they orchestrate these discussions. This is a qualitative study, conducted with two groups of middle school teachers. We analyze data concerning three teachers, Patrícia, Marta, and Diana, collected through initial individual interviews and observation of three research lessons. Data are analyzed by discourse analysis, establishing a relationship between the discourse about the teachers' understanding of whole-class discussions in the initial interview and their subsequent orchestration of this lesson moment. From this relation, we consider the influence of the lesson studies on the teachers' orchestration practice. The teachers' discourse in the interviews suggest the existence of a tension between their understanding of the goals of a whole-class discussion and their practice in orchestrating the discussion, causing several challenges. The lesson study contributed to lower these tensions and challenges through the definition of a fluid lesson structure and the appreciation, selection and recording of the students' activity, enriching the discussion. However, carrying out effective questioning and managing time proved to be complex challenges that deserve attention from further research.

Keywords: whole-class discussion, mathematical communication, lesson study, teacher education

INTRODUCTION

As a consequence of their communicative nature, whole-class discussions in mathematics lessons constitute a social practice in which a discursive practice unfolds. During a mathematics lesson, discourse is produced and received by the participants–teachers and students. Fairclough (1992) states that discourse is a social dimension that interacts with other dimensions and occurs in the form of oral and written language and in symbiotic systems, like images or gestures. Accordingly, discourse contributes to the construction of different dimensions, such as social identity, social relations and also to the construction of knowledge and

meaning systems (Jorgensen & Phillips, 2002). From a socio-constructivist perspective, these dimensions underlie the teaching and learning of mathematics.

Whole-class discussions in mathematics may be perceived differently depending on the communication patterns promoted and of the roles of the various participants and their purposes. Considerable research has been dedicated to whole-class discussions in mathematics lessons (Bishop & Goffree, 1986; National Council of Teachers of Mathematics [NCTM], 2014; Stein et al., 2008), in some cases in lesson studies (Clivaz & Takahashi, 2020; Gomes et al., 2022), and their findings point to the need of more investigation on the role of teachers' questioning, explaining, and systematizing and on how they may support student engagement and knowledge development (Franke et al., 2007; Menezes & Nacarato, 2020).

The goal of lesson study, a professional development process first developed in Japan (Fujii, 2018), is to create a collaborative and reflective environment in which teachers feel challenged, but also supported, to explore and reformulate their practice to improve students' learning. Thus, throughout lesson study sessions, teachers discuss aspects such as learning aims, tasks and questions to pose to students, anticipate their strategies and potential difficulties, and consider the lesson structure. Research has shown that lesson study may contribute to gradual, but meaningful, changes in teachers' practice and, consequently, to the quality of students' learning in mathematics (Fujii, 2018).

As suggested by Menezes and Nacarato (2020), orchestration of a whole-class discussion needs to be understood *in loco*. Still, identifying teachers' perceptions of whole-class discussions may foster access to evidence enabling

(a) to understand of how teachers perceive whole-class discussions in terms of their goals, orchestration and challenges,

(b) to link, by means of discourse analysis, an interpretation of teachers understanding and their orchestration practice, and

(c) to prepare lesson study sessions to increase their contribution to the orchestration of whole-class discussions.

The aim of this paper is to contribute to these issues by identifying how mathematics teachers understand whole-class discussions and to know the influence of lesson study on the way they orchestrate these discussions.

Whole-Class Discussions & Social Construction of Mathematical Learning

Within an exploratory approach, whole-class discussions foresee students' participation in the social construction of their learning (Ponte, 2005). It is a moment in which there is "far more than an exhibition of different solutions advanced alternately by different students" (Canavarro, 2011, p. 17). On a mathematics whole-class discussion students may actively engage with each other's ideas (Kooloos, 2022). Hufferd-Ackles et al. (2004) indicate that a whole-class discussion is linked to the development of a math-talk learning community in which "the teacher and students use discourse to support the mathematical learning of all participants" (p. 83).

Thus, students are expected to be active participants in the discourse, reflecting on the mathematical ideas involved and progressing in their mathematical knowledge. Therefore, effective whole-class discussions occur when the students present, explain, argue and actively listen to their own and their peers' mathematical activity. By using or creating a common mathematical language, they can construct new knowledge under the monitoring and guidance of the teacher. This perception of whole-class discussions is anchored on a social constructivist view of teaching and learning mathematics, like in the exploratory approach, where the aim of the teacher is "not to explain everything, but to leave a significant part of the work of discovery and knowledge construction for the students to carry out" (Ponte, 2005, p. 13).

Several authors, such as Ponte (2005) and Stein et al. (2008), suggest a possible organization of an exploratory lesson in three moments:

(1) introduction of the task,

(2) students' autonomous work, and

(3) whole-class discussion and synthesis.

In this approach, the task, and its features, as well as the management of the moments of the lesson, play a key role in the students' engagement in a mathematical activity. A task in which students are encouraged to find their own method to solve a problem rather than replicating a method previously provided by the teacher may, for example, "lead to a diversity of strategies that can be compared and assessed, giving rise to interesting mathematical discussions in the classroom" (Quaresma & Ponte, 2015, p. 299), which has a positive impact on students' learning (Swan, 2018).

However, research indicates various challenges experienced by teachers in this moment of the lesson such as, for example, time management, the diversity and unpredictability of students' mathematical activity (Gomes et al., 2022), the questioning and going behind "show and tell" (Ball, 2001; Mason, 2000), the selection, sequencing and filtering of students' ideas (Sherin, 2002) and maintaining the cognitive demand (Stein & Smith, 1998).

These challenges may arise or persist because orchestrating a whole-class discussion that is rich on students' discourse is unusual for teachers, as noticed by Kooloos (2022), and it requires a deep integration into teachers' practice, so that it becomes sustainable. On a similar vein, Takahashi (2021) states that to prepare and lead a lesson that includes a whole-class discussion, requires a change in teaching methods and the development of pedagogical skills.

To reduce some of the challenges associated with orchestrating whole-class discussions in mathematics, Stein et al. (2008) indicate five practices to support their preparation and orchestration:

(1) anticipating students' strategies and difficulties,

(2) monitoring their mathematical activity,

(3) selecting students' strategies and representations to consider in the discussion,

(4) sequencing students' interventions, and

(5) making connections between different students' solutions.

Issued during a period of curriculum change in Portugal, the documents *Aprendizagens Essenciais* [*Essential Learning*], produced by the Ministry of Education (Canavarro et al., 2021), seek to support teachers in this regard, suggesting a range of actions to promote the development of various competencies. For example, in the case of mathematical communication, this document suggests that teachers acknowledge and value students as communication agents, that they ask questions for different purposes and encourage the exchange and discussion of mathematical ideas and processes. The significance of communication is also highlighted in international documents, such as NCTM (2014), which recommends actions to be taken during discussions, both from the teacher, such as facilitating discourse among the students, and from students, such as listening carefully.

Thus, the preparation and orchestration of whole-class discussions in an exploratory mathematics lesson is a complex activity. It assumes the consideration of different aspects such as the task, the different moments and purposes of students' mathematical activity and the teacher's role at that particular moment.

Lesson Study & Teachers' Practice

Lesson study has several particularities, among which the continuous collaboration and reflection made by a group of teachers working towards the common goal of promoting students' learning (Fujii, 2018). To accomplish this common goal, Fujii (2018) underlines the importance of accessing students' mathematical activity. This requires that tasks must be carefully selected, and the lesson duly managed to promote students engagement and the discussion of their ideas. In a similar perspective to Ponte (2005), regarding the student-centered exploratory approach, Takahashi (2021) states that "it is better for students to learn new mathematical concepts by trying to solve problems on their own rather than by just imitating the work of others" (p. 3). In Portugal, lesson study has been carried out seeking the planning of lessons based on an exploratory approach, considering the preparation and orchestration of whole-class discussions (Gomes et al., 2022; Quaresma & Ponte, 2015), identical to other European countries (Clivaz & Takahashi, 2020).

Fujii's (2018) presents the lesson study cycle with five phases:

(1) setting a goal,

(2) planning a research lesson,

(3) teaching and observing the research lesson,

(4) making a post-lesson discussion, and

(5) making a final reflection.

The second phase of this cycle is usually the longer in a lesson study. In this phase, when the teachers have already defined the learning aim, they study curriculum documents, explore tasks and select the one they intend to use in the lesson. The teachers also anticipate students' difficulties and strategies, and ways of supporting the students in the different moments of the lesson. To foster learning, the task and the management of the lesson should promote student engagement and participation (Fujii, 2018).

Although the current curriculum changes reinforce the role of the student in the learning process (Canavarro et al., 2021), and despite the volume and quality of research on teaching, Leuverink and Aarts (2021) indicate that actual changes in teachers' practice are slight. Takahashi (2021), in this regard, alerts to the gaps in continuous teacher education, resulting largely from the absence of work around problem-solving, thus reinforcing teachers' continued preference to prepare and conduct lessons from a transmission perspective. Since problem-solving is a foci of the work promoted by lesson study, as are collaboration and reflection, this professional development process appears to offer considerable support to teachers innovate their pedagogical practice. On the same hand, Clivaz and Takahashi (2020) acknowledge that having all the participating teachers observing the research lesson contribute significantly to their professional development too.

METHODOLOGY

Participants

This study is based two lesson studies conducted in Portugal with two groups of mathematics teachers, between January and June 2022. The first author was the facilitator, and, for the teachers, it was their first participation in a lesson study. This research respected educational ethical principles, such as voluntary participation (AERA, 2011).

Four teachers participated in group A, all from the same school, and nine in group B, from five different schools, all based in the same city. The data analyzed in this paper was collected in the initial interviews of Patrícia (P)[1] (group A) and Marta (M) and Diana (D) (group B) and in the research lessons led by these teachers in 6th grade classes. Given the purpose of this study, the participants were selected because they led a research lesson. Pragmatic discourse of the interviews and the constative discourse of the discussions they orchestrated in their classes were considered. Even though the research lesson was led by these teachers, there was a shared responsibility within each group, since the lessons were collaboratively planned, observed, and discussed.

Patrícia and Marta are elementary and middle school mathematics and natural sciences' teachers, but most of their professional career has been dedicated to mathematics. Diana graduated in biology, began teaching in lower secondary school, but quickly moved to teaching mathematics and natural sciences in middle school. All three teachers have more than 25 years of experience. The educational project of the school of teachers of group A gave value to cooperation among teachers. Marta and Diana, from group B, who worked in the same school, also were used to cooperate with each other. In both cases, cooperation was about sharing materials, and did not include planning or discussing the preparation of lessons.

Lesson Study

The lesson study was structured similarly in both groups, with 12 two-hour sessions, following Fujii's (2018) model (with some adaptations), as shown in **Table 1**. Only one research lesson was led in group A, while two were held in group B. In this group, Marta was the first teacher to lead the lesson. Following the reflection on that lesson, the task and planning were adapted, according to the observations made by all participant teachers. Diana also led the lesson with her class. In the case of group A, there was a session to share experiences with another group of teachers from the same city who were also participating in a lesson study.

[1] All names in this paper of teachers and students are fictitious.

Table 1. Lesson study structure in group A & group B

Lesson study phases	Sessions	
	Group A	Group B
Learning goal definition	Sessions 1, 2, & 3	Session 1 & session 2
Planning research lesson	Sessions 4, 5, 6, & 7	Sessions 3, 4, 5, & 6
Leading & observing first research lesson	Session 8/lesson led by Patrícia	Session 7/lesson led by Marta
Post-lesson discussion	Session 9	Session 8
Leading & observing second research lesson	-	Session 9/lesson led by Diana
Post-lesson discussion	-	Session 10
Sharing experiences with other colleagues	Session 10	-
Follow-up	Session 11	Session 11
Final reflection	Session 12	Session 12

The individual interviews were held at the beginning of each lesson study, before planning and reflecting on the whole-class discussion. In the planning phase, in addition to working on the tasks, content and aim of each lesson, excerpts of whole-class discussions from open-source videos were analyzed and *essential learning* document (Canavarro et al., 2021) was examined with regard to the development of students' competencies, namely communication.

In both groups, several tasks were collaboratively explored, and the lesson was planned throughout the sessions with the contributions of all teachers. In group A, the aim of the lesson and the task was to *recognize direct proportion situations and to indicate the proportion constant, explaining its meaning* (**Figure 1**). In this task, the students were challenged to explore the possible time and prices of using a bicycle in two different companies. It was possible to establish a proportion between these variables in one case (Ciclotour), while in the other it was not (Ybike).

BICYCLE RIDE

Pedro and Margarida went for a ride to Parque das Nações and decided to rent bikes. In Parque das Nações there are only two companies that rent bicycles: Ciclotour and YBike, whose price lists are as follows:

CICLOTOUR	
Time (minutes)	Price (euros)
10	
20	
30	3
40	4
60	6
70	
90	9

YBIKE	
Time (minutes)	Price (euros)
10	
20	1,5
30	
40	4
60	6,5
70	
90	10

1. Complete the tables.

2. Pedro and Margarida want to ride their bicycles for 120 minutes. Can they predict how much they will pay at each of the companies? Justify.

Figure 1. Task led by Patricia (group A) (Adapted from Ponte et al., 2010)

In group B, the lessons and tasks led by Marta and Diana sought to *recognize the direct proportion ratio between the perimeter and the diameter of a circle and to designate π as the proportion constant* (**Figure 2**). In task 1, the students were challenged to identify a ratio between the perimeter and diameter by informally measuring perimeters and diameters using streamer strips. In task 2, by using a tape measure, the students were challenged to identify the proportion constant of this previously identified ratio.

Task 1: Serpentines and circles

Steps:

1 - Use the streamer to measure the perimeter of one of the bases of your object;

2 - Cut out this piece of streamer and glue it to the given sheet;

3 - Use a streamer of another color to measure the diameter of that base;

4 - Cut that piece.

5 - Repeat the 3rd and 4th steps until you get close to the length of the perimeter of the base, as shown in the picture.

6 - Glue the various pieces of serpentine representing the diameter, parallel and without overlapping, to the serpentine representing the perimeter.

Question: Compare the length of the perimeter and the length of the diameter. What can you conclude?

Task 2: Discovering π and circle circumference

1.1 1.1 Using a tape measure, measure the circumference and diameter of the base of some cylindrical objects and fill in the following table with their measurements.

Name of the object	Diameter (d)	Radius (r)	Perimeter (P)	P÷d
...	...	...	...	...

1.2 Compare the length of the radius and the length of the diameter. What can you conclude?

1.3 Compare the length of the perimeter and the length of the diameter. What can you conclude?

Figure 2. Task led by Marta & Diana (group B) (Adapted from DGIDC, 2009)

Data Collection & Analysis

We made initial individual interviews to all 13 teachers. From those, we selected for this study Patrícia, Marta and Diana, the three teachers that led research lessons and, therefore, orchestrated a whole-class discussion. Consequently, three lessons were observed by all the participating teachers, including the first author in the role of lesson study facilitator.

For data analysis, the individual interviews of the three teachers were transcribed. The lessons conducted by Marta and Diana were recorded and transcribed. The lesson led by Patricia was observed without video recording (as there was no agreement from the parents), hence the respective data was extracted from the field notes.

Given its aim, this study follows the interpretative paradigm, with a qualitative approach. A discourse analysis (Fairclough, 1992) was carried out. For this analysis, excerpts were selected seeking to illustrate three key aspects of whole-class discussions–the goals, the orchestration and the teacher's challenges–at two moments: in the initial interview (**Figure 3**), prior to the lesson study, and in the lesson led, during the lesson study. The interview excerpts were identified with the names of teachers, Patrícia (PI), Marta (MI), and Diana (DI) and the same for whole-class discussions in the lessons (with PL, ML, and DL), and numbers (such as in PI1 or MI1) are used to identify different discursive marks, synthesized, and related in **Table 2**. To ensure trustworthiness in the interpretations the three authors discussed the results when necessary to reach consensus.

Teachers' whole-class discussion understanding	Gather information about the whole-class discussion goals	Do you consider that in your classes there are whole-class discussions? In your opinion, what are the goals of a whole-class discussion in mathematics?
	Gather information about the whole-class discussion orchestration	What aspects do you take into consideration to start a whole-class discussion? Do you identify students' strategies and difficulties before or only during the whole-class discussion? Do you consider it important to ask questions during the whole-class discussion? What kind of questions do you think are important and who can ask them? Do the students propose, on their own initiative, to share their strategies or is it always the teacher who selects the students who will participate? Do you use the board during the whole-class discussion? If yes, for what purpose and what aspects do you take into consideration?
	Gather information about the challenges teachers faces during the whole-class discussion	How do you manage the appearance of strategies with different levels of effectiveness? How do you manage the appearance of wrong or incorrect resolutions? How do you manage disagreement between students during the whole-class discussion? Do you find orchestrating whole-class discussions challenging for teachers? Why?

Figure 3. Clipping from initial interview script (Source: Authors' own elaboration)

Table 2. Synthesis of discursive marks of Patrícia, Marta, & Diana in interview & orchestration of whole-class discussion

	Performative practice (interview)	Observed practice (whole-class discussion orchestration)
Goals	Promoting students' mathematical communication (PI1 & MI1)	Some exchange opportunities among students converged towards Marta, who reiterated their words (ML1). Patrícia fostered students' exchanges & asked several questions (PL3 & PL4).
	Promoting students' participation in social construction of their learning (PI2, MI3, & DI1)	Patrícia asked students to clarify explanations and to argue on the regularities and numerical relationships identified (PL3 & PL4). Instead of giving the students the formula, Marta elicited and steered their contributions, making the symbolic writing a whole-class product (ML2). Diana prioritized the conjecture of one of the groups (the "missing" bit) to challenge the class to explore its validity (DL1 & DL2).
	Identifying students' difficulties (MI2, DI2, & DI3)	Diana selected values to be logged in table (DL4) to guide students' exploration of regularity towards finding constant. This difficulty was also experienced in the lesson led by Marta, where values were not selected.
Orchestration	Limiting time to interpret, solve, & discuss (PI3, PI4, & MI5)	Patrícia extended autonomous work, reducing time for discussion (PL1). Marta did the same, however, she provided time for the students to interpret and discuss among themselves (PL4).
	Questioning, mainly to clarify questions & correct (PI5 & MI6)	Patrícia questioned to support students' explanation and to engage them in the discussion (PL3). Marta used questioning to elicit and steer the contributions so that the symbolic writing was constructed by the whole-class (ML2). Diana questioned them clarify their explanations (DL1) and to challenge them (DL2).
	Logging without planning & absence of monitoring (PI4, DI4, & DI6)	Diana used the board in task 1 to display the ratios found by the students, and in the discussion of task 2 she selected the values to log in the projected table (DL3, DL4, & DL5). Patrícia initiated the whole-class discussion by looking for a volunteer among the students to start the exchanges (PL2).

Table 2 (continued). Synthesis of discursive marks of Patrícia, Marta, & Diana in interview & orchestration of whole-class discussion

	Performative practice (interview)	Observed practice (whole-class discussion orchestration)
Challenges	Promoting students' active participation (PI6 & DI6)	Patrícia promoted students' active participation by asking for clarification and by fostering whole class's verification of validity of numerical regularities (PL3 & PL4). Diana used the conjectures formulated by students as a starting point to challenge the class to explore them further (DL1).
	Managing time, unpredictability, & diversity (PI3, MI4, MI7, & DI7)	Despite shortening the time allocated to the whole-class discussion, Patrícia and Marta promoted diversity in the students' exchanges (PL4). Diana monitored the students' work by selecting and sequencing what was to be shared, thus reducing the unpredictability (DL4).
	Maintaining cognitive challenge (DI5)	Diana challenged the students to go beyond what was requested in task 1 based on one of the groups' contributions, adapting the cognitive challenge to the heterogeneity of the class (DL2). In task 2, she selected the values of the ratios that were closest to π for discussion and sequenced the exchanges so that the last group to intervene was the one to identify the constant (DL4 & DL5). Thus, she promoted the students' identification of the constant through the table, maintaining the initial challenge of this task.

Discourse analysis was used to interpret and relate two communication event types, namely the interviews and orchestration of the discussions. According to Fairclough (1992), communicative events consist of moments in which language is used, involving the production of a text (oral, written, images or combination) in which a discursive practice occurs (production, distribution and consumption of the text), within a social practice associated with a particular context. This author sees discourse as "a mode of action, a way in which people can act on the world and especially on others, as well as a mode of representation" (p. 91).

Thus, the excerpts of the initial interviews are considered performative expressions with a declarative appearance. However, at no point does this analysis consider them true or false. These excerpts are deemed to be intentional or to show intended practice, whereas the excerpts of the whole-class discussions are considered constatations, therefore, observed practice. The excerpts of the whole-class discussions are taken as an intertext, influenced by each teacher's own understanding prior to participation in the lesson study and by the work developed across the orchestration of whole-class discussions throughout the sessions. Throughout the discourses we seek to identify discursive marks, expressing key ideas from participants concerning the whole-class discussion, both in their performative and observed practice.

Figure 4 illustrates how Fairclough's (1992) discourse framework is mobilized in this study. Establishing the relationship between the communicative events interview and whole-class discussion allows us to understand if there were contributions from the lesson study by comparing a performative practice with an observed practice.

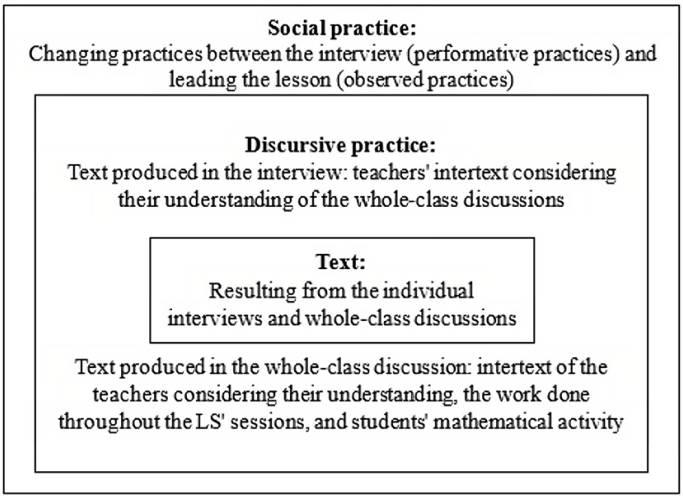

Figure 4. Analysis of teachers' discourse according to framework proposed by Fairclough (1992)

RESULTS

Whole-Class Discussion Understanding

Goals

As shown in **Figure 3**, in the initial interview, teachers were asked about the goals of the whole-class discussions. Patrícia states the following:

> That they [the students] manage to explain and develop the ability to communicate (PI1), respect the ideas of others … I think another more participatory type of learning should be fostered rather than me arriving and it just being about my idea (PI2).

Although Patrícia mentions the development of communication as a goal of the whole-class discussions, she highlights the challenge in defining this competence, which may give rise to challenges in promoting it:

> It is difficult for me to understand what communication is, even after reading, even having in writing what is expected so I can establish whether it is good or bad (PI1).

In Marta's response to the same question, she states that the whole-class discussion is a moment when the students can intervene "so that they can speak, share their ideas, their strategies" (MI1), which may be related to the development of their mathematical communication, as long as it is not solely a presentation. In this interview, Marta identifies goals from both the teacher and students' point of view:

> Working on that is also trying to see what questions they have, although not directly asking them we can perceive it and work on it (MI2). And improve mathematical communication (MI1). Right, and work on their ease with the subject (MI3).

From the students' point of view, Marta mentions the sharing of different ideas, the development of mathematical communication, as does Patrícia, and improving the students' relationship with the subject. From the teacher's point of view, she mentions identifying students' questions and working on them.

Diana also highlights goals from the students' point of view, as does Marta, such as their engagement, especially regarding those who struggle more with the subject:

> Well, first of all, I think we should try somehow to get them all to participate, to integrate them … And, if possible, in some way get them to reflect, even clarify, because as I said, sometimes some of them have a very determined position and will not give in, right? … The aim is always to get them to participate so that they feel that mathematics is also within their reach, especially the ones who struggle (DI1).

Diana also appears to attribute an evaluative function to the whole-class discussion, from the teacher's point of view, when she specifies that "sometimes whole-class discussions are triggered out of the blue", considering them important "just to see where they stand" (DI2) in the development of their knowledge.

Orchestration

In the initial interview (**Figure 3**), questions were asked about the orchestration of whole-class discussions, namely in terms of time management, participation, and the diversity of strategies, questioning and use of the board.

When questioned on orchestration, Patrícia emphasizes the fact that students "do not have time to reflect because there is too much to teach in the short period of time" (PI3), appearing to confront *essential learning* features with the scarcity of time to promote effective whole-class discussions. Regarding this, she reflects on her role in management of time and the curriculum, considering the impact this might have on the quality of the discussions and on students' learning:

> Maybe it's also our fault, now and in previous years … But perhaps we do not give them enough time to interpret, [if] nobody knows what to do and I tell them you must do this … We are often too quick to give this answer and then complain that they do not know (PI3).

By adopting this strategy, Patrícia may be conditioning the frequency and quality of the whole-class discussions. With the lack of time to interpret the tasks and the teacher's exemplification, the students' mathematical activity may be limited to reproduction. Thus, the diversity of mathematical strategies and ideas to enrich both the discussion and students' motivation is likely to be undermined.

Regarding the decision of when to initiate a whole-class discussion, Patrícia says "I cannot really answer that now because I have not done a task like that for a long time" [PI4]. Still, she appears to be influenced by time management and by the students' mathematical activity, but she highlights nothing related to monitoring the students' mathematical activity:

> Sometimes I really have to give them time to finish. Then [if there are] groups that are halfway through they end up hearing the others and completing the task … So, I tend to manage things by sometimes doing it one way in one class and in another [class] another way (PI4).

Regarding questioning during the discussion, Patrícia underlines the role of the teacher in asking focused questions:

> I think it should always be the teacher [questioning]. The task is given for a certain purpose, right? So, the teacher must always be a moderator, right? … We can even clarify questions and answer the questions that arise, but we should not lose the initial focus (PI5).

When asked about orchestrating the whole-class discussion, Marta displays features of managing different strategies. Generally, she appears to select a strategy she considers as being more formal to present to the students:

> I'm a bit fixed on making them all do it in a certain way, after they have understood and shared their ideas, I always get them to do it in a formal way (MI4).

To initiate the whole-class discussion, like Patrícia, Marta requires the students to finish the task, indicating: "I usually wait for most of the students to [finish]" (MI5). Marta also explains how she manages questions prior to the discussion:

> The questions, well this also depends a little on the task we are doing, because if there are lots of questions, we then probably start a whole-class discussion. And no, I do not wait. If I see that the work is flowing and that they're managing to do it, then I give them more time and save a little bit for the discussion … (MI5).

Thus, Marta appears to adjust the time attributed to each moment of the lesson considering the students' work. By moving on to the discussion, if questions regarding interpretation of the task arise, she will be introducing a moment of whole-class work without the students having the opportunity to think, thereby conditioning their contributions to the discussion. If she reduces the discussion time to "just a little bit", she may be limiting this moment to the presentation of a valid strategy.

Unlike Patrícia, Marta seems to include the students in the questioning, yet her focus appears to be on correction:

> I question the students. I try to see what is missing, what is wrong. But I always give room for the others to correct it first (MI6).

When asked about when to initiate a whole-class discussion, Diana stresses that it depends on the purpose, but mentions that when a discussion begins she tries to "gauge what they know and then, during the discussion, see if they can relate what has emerged in the meantime" (DI3), reinforcing the discussion's evaluative function. Regarding the use of the board, she claims to always use this resource and refers to the challenge of clarifying some representations:

> I use it to leave a record of something, because it helps them when they visualize it, yes … Sometimes a diagram, you know. Normally, when using diagrams, it's complicated because I ask– "ok, but how"? (DI4).

Challenges

The last set of questions asked in the interview (**Figure 3**) were related to the potential challenges experienced by the teachers when managing different strategies and when errors or disagreement occur among the students.

Patrícia mentions that whole-class discussions are challenging for everyone, students and teachers alike, and she justifies this by reflecting metaphorically on the potential of discussions, despite their related challenge:

> We must be able to argue, but to argue with what the others say, we have to listen, we also have to process, reason, do some mental work, be it in mathematics or another kind of discussion. And I think they get it then, they get it more than when they are just there as spectators. When you watch a movie, it does not have the same impact as when you read the book, because it's different. When we read a book there are other mechanisms of the brain that … We do not have the image that is triggered. And I think in a discussion it's the same thing … (PI6).

Marta's speech, on the other hand, shows that the challenge is more associated with the teacher and is related to the unpredictability and diversity of the students' strategies when she says, "It's challenging especially because sometimes they present thoughts or strategies that maybe we had not thought of, right?" (MI7).

Throughout the interview, Diana enumerates other challenges associated with orchestrating whole-class discussions, such as the number of students and managing their enthusiasm:

> Sometimes they get really noisy when they participate and that's the most difficult part to manage for me, the enthusiastic answers. Then I always turn to the weaker ones and the others get fed up (DI5).

The balance between integrating the students who struggle and challenging the others, that is managing the heterogeneity of the class, is one of the most difficult challenges experienced by Diana. It also seems to be a crucial element for the depth with which she explores certain aspects during the discussion and how she manages the cognitive challenge:

> The main issue is that sometimes I feel sorry that I cannot perhaps give the better ones so much practice and challenge them in that regard. Usually, it's more so that the weaker ones feel included. And then, sometimes, I end up exploring some things, but I soon get stuck there because the others do not think they can follow the reasoning any further (DI5).

Finally, Diana states: "I tend to go around the class, but I do not interfere too much in the [students'] work" (DI6). This practice during autonomous work appears to be related to a challenge associated with time management and strategy diversity. She goes on to say "I go to them if they ask me to or if I see something wrong …. (DI6). And what happens? Sometimes I feel there is not enough time and sometimes managing their participation is challenging" (DI7).

Throughout the interviews, each teacher highlights goals that are in line with the idea of whole-class discussion in an exploratory approach, such as the development of mathematical communication, the social construction of knowledge and the identification of students' difficulties. However, there appears to be tension between the intended orchestration practice and these goals, which may contribute to an increase in the challenges perceived by the teachers, synthesized in **Figure 5**. An example of this is the management of different strategies and the participation of students during the discussion since this challenge may be fueled by the lack of anticipation or monitoring.

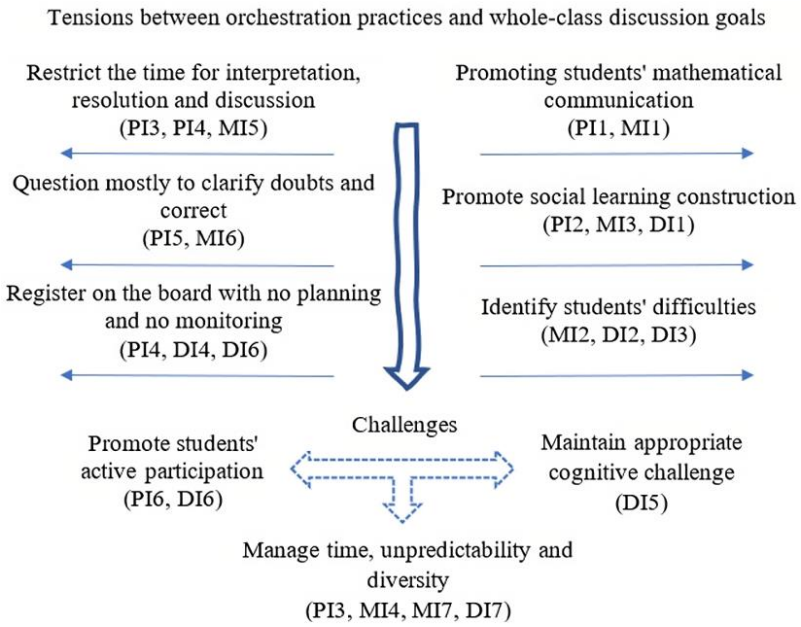

Figure 5. Tensions & challenges between orchestration practice & goals of whole-class discussions in mathematics classes identified in initial interviews (Source: Authors' own elaboration)

Whole-Class Discussion Orchestration

As mentioned, the planning of the research lesson occurred between the initial interview and Patrícia, Marta and Diana's orchestration of the lesson in which whole-class discussion-related aspects were explored and prepared.

Patrícia

In the lesson planned by group A, Patrícia decided to extend the time for students' autonomous work (PL1), which significantly reduced the time for the whole-class discussion. In the post-lesson discussion, she explained that despite the students' engagement in group work, she made this decision because they did not have many records of the ratios, numerical regularities or their conjectures.

To initiate the whole-class discussion, Patrícia projected the tables being explored by the students during their autonomous work and asked if anyone wanted to start sharing their solution (PL2). Rui volunteered and filled in the table on the board corresponding to the company Ciclotour, also adding the following ratio: *price=time/10*. Patrícia asked the rest of the class if they had any questions for Rui, or if they had thought the same. Since Rui's peers agreed with the solution, the teacher posed another question:

> Patrícia: I've got a question for Rui. How did you arrive at that ratio? What did you analyze? (PL3)
>
> Rui: I looked at the figures that were already filled in and saw that the price is a tenth of the time.

Thus, Patrícia first sought to engage the students in the discussion. Given the absence of questions and comments, she questioned Rui about the ratio written on the board. The student was asked to clarify the process by which his group had identified that the ratio between price and time is constant.

The teacher then invited Luís to share his group's solution to the YBike company table. This group filled in the table by looking at potential half and double ratios between the price and time. One of the students asked to speak and addressed Luís:

> Student: 40 is double 20, but four is not double 1.5.
>
> Patrícia: Luís, would you like to comment? (PL4).

> Luís: Right, we had already reached that conclusion. In this one [Ciclotour] it works, but in this one we could only get the same price up to five [YBike].

In this example, Patrícia requests a comment from Luís, giving him the opportunity to reflect on his peer's comment regarding the strategy used by his group. By doing so, Patrícia is not the one to validate or invalidate the conjectured relations, since the students do this themselves in dialogue during the discussion.

Patrícia represented another table on the board, corresponding to the YBike company, and invited Bruno to share his group's solution. The student said that they had considered that the price for 10 minutes would be 0.50€. One of the students disagreed with Bruno stating that at 20 minutes of use it would no longer be possible to validate that ratio. Patrícia also asked Lino to share the ratios found in his group. In Lino's group, four different ratios had been established, which they referred to as rules. During the discussion, another student intervened and said it was not possible to use different rules in the same table, recalling that for the Cyclotour case they had identified a numerical ratio that was always valid (PL4). The lesson ended at this point, with no conclusion to the discussion. However, Patrícia promoted the discussion of different numerical relations and regularities among the students, giving them time to interpret and argue with each other.

Marta

The following episode is related to the whole-class discussion of the second task led by Marta. While one of the groups logged one of the values obtained for the ratio between the perimeter and diameter on the board, Lina, one of the students, called Marta and raised a question to her directly about the number of decimal places that could be in the ratio:

> Marta to Lina: We'll analyze that shortly.
>
> Marta to the class: Lina, is here … I was going to ask if you have anything to say when you look here. Lina said that the values that were here … Some have two decimal places and the other one here has one. She was making some speculations … Well, let's wait a little to look at a few more objects and see if we arrive at a conclusion (ML1).

In this episode, Marta decided to share the question raised by Lina with the class. Despite informing them that Lina already had some conjectures regarding the number of decimal places, the student was not challenged to pose her question directly to the class.

The next episode appears in the whole-class discussion of the second task and illustrates how the teacher tried to elicit and mobilize the students' contributions to represent the ratio using symbolic language (ML2):

> Marta: Now, instead of writing in words, I would like you to try to write the formula with those letters using P for perimeter. What operations do we have to do? Basically, we have to take what's there and put it into letters and operations … Let's do it as a group. Let's try to put what we have here in words into letters. How do I write the letter of the perimeter?
>
> Students: P.
>
> Marta: The perimeter will be equal to … What do I need from the circle to calculate the perimeter?
>
> Students: The diameter.
>
> Marta: But you said that it was …
>
> Students: Three times the diameter.
>
> Marta: This is how I start: $P=3\times d$. Is my formula correct?
>
> Students: Yes.
>
> Marta: Maria, is my formula correct?

> Maria: A little bit more …
>
> Marta: So, where is the little bit?
>
> Maria: We put three dots teacher, three commas and then three dots.

The teacher pointed to the constant they had identified through the table:

> Maria: Ah! The perimeter is equal to π times the diameter.

These episodes of Marta's orchestration of the whole-class discussion show, on the one hand, that providing room for students to communicate their conjectures and questions is challenging. On the other hand, the teacher tried to manage the lesson time in a way that ensured space for discussion and sought to engage the students in the symbolic writing of the requested ratio. An example of this is Marta's decision to log the relationship between the perimeter and the diameter as told by the students using symbolic writing, and thereafter to question Maria directly about its validity.

In the post-lesson reflection session of the lesson conducted by Marta, adaptations to the first task and management of the lesson emerged: how task 1 should be introduced, what to select for discussion, and also when and what to log on the board during each of the discussion moments.

Diana

The following episode is related to the first whole-class discussion led by Diana. Here, the teacher challenges the student to offer a more in-depth explanation:

> Ivo: We did almost the same thing as them, but they left a space in one part, and we added the bit that was missing.
>
> Diana: And that bit that was missing, do you know more or less how much it is?
>
> Ivo: More or less a third or a quarter …
>
> Diana: What did you do to understand whether it was a third or a quarter? You cannot just get to this point and think that's what it is (DL1).
>
> Carlos: We had one of those [pointing to the strip representing the diameter] and we looked at that bit.
>
> Diana: I'd gathered that, but what do you think this bit is in relation to this? (DL1).
>
> Carlos: Well …
>
> Diana: You did it by rule of three?
>
> Carlos: No, if we roughly multiplied the smallest bit by three we have the size of the diameter.

João, a student from another group, intervened and asked Diana if they should have looked for that relationship since it had not been requested in the steps of task 1, but the discussion moved forward with the exchange of other groups. When the teacher began to log the operation with symbolic language, she returned to the question raised by the group of Carlos and Ivo and challenged the class, more directly João, to check whether the decimal value of the ratio would be close to a third or a quarter, taking the diameter as the unit of reference (DL2). Two students arranged pieces of a streamer representing a diameter unit, one of them dividing it into three equal parts and the other dividing it into four equal parts. On the board, the teacher compared these different parts, where the unit of reference was divided and showed that one quarter would come closer to the decimal value of the constant (DL3).

Before initiating the discussion of the second task, Diana went around all the groups and selected the object whose ratio between the perimeter and diameter was closest to π (DL4). When she began logging the operation on the board, she told each of the groups, which object they should consider for the table (DL5).

During the students' autonomous work on the second task, Diana selected the content to be shared, considering the aim of the lesson, and thus fostered the students' easier identification of the intended regularity. In the discussion, the teacher challenged the group of Carlos and Ivo to go beyond what was requested in the task, which prompted the formulation of conjectures that were later explored by the class during the whole-class discussion.

DISCUSSION

By means of discourse analysis, a connection was sought between the teachers' discourse during the interviews, before lesson study, and the practice of orchestrating discussions during the lesson study, under the influence of this process (see **Table 2**).

In the three teachers' cases, the planned time for the whole-class discussion was shortened because of the extra time given to the students' autonomous work, one of the challenges noticed by Gomes et al. (2022). When questioned in the interview on when to initiate the whole-class discussion, Patrícia explained that in some situations she waits for the students to complete the task. She also mentioned the lack of time usually allowed for the students to interpret what is being asked or shared. Despite having reduced the time allocated to the whole-class discussion, Patrícia did not rush the discussion itself, namely in relation to the ratios and regularities identified by the students. Thus, the discussion led by Patrícia was anchored on the students' prior autonomous work and enhanced by their interventions and the time provided to interpret them. Throughout the lesson study sessions, considerable thought was given to how to support students without diminishing the challenge of the task and the importance of students' interventions in the discussions (Canavarro et al., 2021; Sherin, 2002; Stein & Smith, 1998). Even though the same attention was given to when to end the autonomous work during the lesson study sessions, time management remained challenging.

In the interview, Marta also mentioned that she "gets" the students to reach a solution in a "formal way", which appears to indicate the teacher's presentation-led practice and the students' subsequent reproduction. In Diana's case, according to the initial interview, the unpredictability challenge is influenced by the absence of monitoring, selection and sequencing of the students' strategies, and by the prevalence of her corrective interventions. This performative practice stated in the interviews are in line with Kooloos (2022) and Takahasi (2021) observations that orchestrating whole-class discussions is unusual for teachers, and it requires a change in teaching methods. However, although in the discussion orchestrated by Marta none of the groups managed to write the relationship in symbolic language, contrary to what she stated in the initial interview, she did not formalize this relationship by herself. By exploring a table and the natural language of the students, she elicits their contribution to the symbolic writing of the formula, a practice that acknowledges and values students as agents of communication as suggested by NCTM (2014) and by the current Portuguese curriculum (Canavarro et al., 2021). As for Diana, she reduced the unpredictability of the whole-class discussion by selecting and sequencing the students' interventions and board entries, integrating this practice in the preparation of the discussion, as suggested by Stein et al. (2008).

Diana referred to the time management and student participation associated with not knowing what they would bring to the discussion as challenges she identified with. During the lesson study sessions, in collaboration with the other teachers, she anticipated the potential relationships students might establish and the difficulties they might experience in both tasks, one of the five practices from Stein et al. (2008). In Diana's case, especially as her lesson was held after her observation of Marta's lesson, the importance of selecting the values of the perimeter and diameter ratio to log in the table during the whole-class discussion was evident. This episode illustrates how Diana's role changed considerably during the autonomous work, which influenced the effectiveness of the discussion's orchestration, namely by selecting and logging entries on the board. In this sense, observing and reflecting on Marta's orchestration seemed to have contributed to Diana's practices of preparing and orchestrating a whole-class discussion (Stein et al., 2008).

The discussion orchestration considering the students' contributions was another widely reflected aspect in the lesson study sessions. This had clear repercussions in Marta's lesson as she questioned the students

for their input rather than corrected them (Mason, 2000), as stated in her initial interview. In the case of Patrícia, who, in her initial interview, also underlined the prominent role of the teacher in questioning, she asked questions in her whole-class discussion to maintain the fluidity of peer communication and to access students' thinking (Mason, 2000). Marta's practice was in line with Takahashi's (2021) idea that in this kind of lessons, students should not just imitate the work of others, or end up reproducing something demonstrated or validated by the teacher. In this regard, both Marta and Patrícia questioned the students to engage them in the discussion, to challenge or support them in achieving the lesson's aim while minimizing the corrective character of their questioning, as highlighted in the initial interviews.

In her interview, Diana referred to students' learning and reflection and clarification as one of the goals of the discussion. Furthermore, one of the challenges she also mentioned was managing the heterogeneity of the class. This challenge may potentially condition learning development if the teacher reduces the level of challenge of the discussions too often to integrate the students who struggle (Stein & Smith, 1998).

In the episode, where Diana validates the conjecture of Carlos and Ivo's group, she chooses to give all the groups the opportunity to share their conjecture about the relationship between perimeter and diameter. After, she redirects the attention of the class by stimulating the students as a group to explore the validity of what was said by their peers. The actions of acknowledging and valuing students as agents of communication and questioning with this purpose of challenging, going beyond the initial task but still contributing to the specific aim of lesson, are set down in *essential learning* document (Canavarro et al., 2021) and NCTM (2014).

During the interview, Marta and Patrícia mentioned the promotion of students' mathematical communication, as an aim of the whole-class discussion (Canavarro et al., 2021). In some moments of the lesson, Marta's role was quite prominent, not using some of the exchange opportunities, disagreement and questioning among students, and using indirect speech. To promote communication, the importance of Lina's question (posed in ML1) and conjectures could have been praised. On her side, despite having considerably shortened the lesson time allocated to whole-class discussion, Patrícia promoted the debate among the students on the observed regularities and conjectures. The students made use of direct speech, valuing their mathematical activity, and the validation or exclusion of the regularities and numerical relations emerged from their interactions. Contrary to what she had stated in the initial interview, Patrícia promoted opportunities and time for students to share their mathematical activity and for their peers to interpret what was being communicated.

Promoting students' communication during whole-class discussions is a complex challenge for which participation in more than one lesson study may display greater contributions. As advanced by Menezes and Nacarato (2020), various dimensions of the discussion, such as questioning, explanation and the systematization of knowledge are a complex endeavor. As a development process close to an inquiry into one's own practice, lesson study enables teachers to access and reflect on these dimensions of teaching practice, both in the preparation phase of the lesson and in the post-lesson discussion.

CONCLUSIONS

Some features of Patrícia, Marta, and Diana's discursive framework in the initial interview were in tension with the orchestration of effective whole-class discussions. By analyzing the coherence of the interview excerpts (Fairclough, 1992), it may be noted that, in addition to a somewhat superficial reference to the goals, some performative illustrations stated by the teachers are in tension with these goals, contributing to the emergence or complexity of the challenges perceived by the teachers (**Figure 5**). The work carried out during the lesson study sessions, namely during the planning phase, appears to have contributed to changes in practice, reducing the challenges associated with this moment.

In line with Clivaz and Takahashi (2020) and Gomes et al. (2022), conducting exploratory lessons, particularly whole-class discussions, was found to be a major challenge for the teachers, even when participating in a lesson study. Still, for Patrícia, Marta, and Diana, participating in a lesson study had a positive impact on the management of several challenges.

The whole-class discussions themselves are taken as an intertext (Fairclough, 1992), influenced by each teacher's own understanding prior to lesson study and by the work developed throughout the lesson study.

Therefore, encouraging active participation, as observed in the three teachers through their questioning, managing unpredictability through monitoring and selecting students' work, and maintaining the cognitive challenge during the discussion, as observed in Diana, are main features of teaching practice promoted in the lesson study. Thus, the work collaboratively done during the lesson planning, that focus on these aspects, supported these meaningful differences between their performative practice, known from the interview, and the observed practice.

Although the three teachers reduced the time allocated to whole-class discussion by extending the time for autonomous work, they sought to respect the fluidity between the lessons' moments (Ponte, 2005; Stein et al., 2008), as discussed in the lesson study sessions and structured in a lesson plan. Anticipating students' strategies and difficulties and the role of the teacher during the whole-class discussion seem to have contributed to better accomplishment of the practices suggested by Stein et al. (2008). The type of questioning (Mason, 2000) and the construction of communication in the discussion based on the students' contributions (Canavarro et al., 2021; Hufferd-Ackles et al., 2004) show that the teachers also sought to promote students' communication during the discussion.

Nevertheless, teacher's questioning is a challenge that still deserves further attention, namely how are questions linked to teacher's understanding of whole-class discussions and its impact on promoting students' participation. Even though significant research has been done on teacher's questioning, changing their practice regarding to this may need to take into consideration how they understand, at first, whole-class discussions in terms of their goals, orchestration, and challenges. This research already identified how mathematics teachers understand whole-class discussions and anchored on this, lesson study can be framed to better support teachers to overcome challenges related to their questioning during whole-class discussion. Another aspect that deserves attention is how to enhance the participation of students, creating moments of disagreement and argumentation (Wood, 1999). Also it might be studied what resources can be used for effective management of the lesson time, for example, the lesson plan itself or the anticipation of the organization of the board, in order to support teachers to promote the flow of the lesson granting that there will be enough time for the whole-class discussion.

The main limitation of this study is that we draw only in a lesson study with three teachers at a set school level. Other studies, with other teachers from the same and other school levels, may indicate further tensions and challenges and provide insights about how to overcome them.

Identifying teachers' perception of whole-class discussions made it possible to clarify tensions between their practice of orchestrating this moment and the goals associated with the discussion. Throughout the lesson study sessions, several aspects related to the tasks, the structure of the lesson, the students' activity and the teacher's role were collaboratively discussed and decided upon, which proved to have contributed to a change in practice and possible reduction of the challenges perceived by the teachers in their orchestration of whole-class discussions. Therefore, lesson studies may contribute to reflection and change in the practice of conducting exploratory lessons and, specifically, whole-class discussions. Further studies may consider how lesson study may contribute to further minimize these tensions.

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