European Journal of Science and Mathematics Education, 2026, **14**(1), 85-103
ISSN: 2301-251X (Online)

OPEN ACCESS

Research Article

Sustainability competencies in mathematics education: insights from individual and collective modelling

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Citation: Guerrero-Ortiz, C., Camacho-Machín, M., & Hitt, F. (2026). Sustainability competencies in mathematics education: insights from individual and collective modelling. *European Journal of Science and Mathematics Education, 14*(1), 85-103. https://doi.org/10.30935/scimath/17625

ARTICLE INFO

Received: 8 Jul 2025

Accepted: 28 Nov 2025

ABSTRACT

Education for sustainable development (ESD) involves addressing complex problems that require the development of mathematical abilities and general competencies. In the context of secondary mathematics teachers training, this challenge should be supported by a holistic process that considers theoretical frameworks on sustainability from a mathematics education perspective. In this work, we analyze and adapt general ESD through a particular approach from mathematics education. Key characteristics relevant to the development of mathematics teaching activities are identified and proposed. Then we present experimental results in a teacher training course, identifying the sustainability competencies that emerge and their relationship with mathematical abilities. The findings highlight the importance of generating more activities that reinforce the anticipatory competency in pre-service teachers for future sustainability scenarios.

Keywords: mathematics education, mathematical modelling, sustainability competencies, teacher training

INTRODUCTION

Education for sustainable development (ESD) involves transforming the way citizens think and act. This means enabling them to make informed and responsible decisions based on social effects (such as consumption, business practices, culture, and lifestyle), economic factors (business ethics, fair trade, improvements in the management of human, financial, and natural resources), and environmental considerations (use of natural resources, pollution control, ecological footprint, environmental management) for both current and future generations (Alsina, 2022). Addressing these challenges requires a range of abilities that allow us to tackle complex situations from both local and global perspectives (UNESCO, 2017). According to Alsina (2022), this issue cannot be approached solely through mathematics; rather, it requires the integration of knowledge from other disciplines and teaching methods that promote the development of essential competencies needed in students.

It has been proposed that to achieve the sustainable development goals (SDGs), education must be holistic, inclusive, and transformative. This involves integrating sustainability into curricula and promoting teaching and learning environments focused on action, communication, and based on interaction and exploratory learning. Competencies such as critical and systemic thinking, anticipatory thinking, strategic thinking as well as collaborative decision-making, should be encouraged, thus taking responsibility for both current and future generations (Rieckmann, 2012). Empowering students to transform both themselves and society is crucial, as they learn to formulate critical questions, clarify values, and foresee a sustainable future (UNESCO, 2017; Vásquez et al., 2021). Thus, teachers are central to fostering the competencies and knowledge that students need to respond the challenges of today's world (Çibik & Boz-Yaman, 2025). This represents a significant challenge, as most teachers lack solid training in sustainability (Lamanauskas & Malinauskienė, 2024). In addition, a disconnection between sustainability and mathematics education is evident in the literature (Hui-Chuan & Tsung-Lung, 2021), therefore, in this research based on empirical evidence we offer an approach to address this issue.

To develop sustainability-related competencies (Rieckmann, 2012), this paper explores the following research questions within the context of secondary mathematics teachers training:

1. *What types of activities and problems are necessary to develop mathematical abilities and sustainability competences in pre-service teachers?*
2. *Which mathematical abilities and sustainability competencies emerge when pre-service mathematics teachers work with a task presented in sustainability context?*

The first question is approached from a theoretical perspective, as it seeks to conceptualize the kinds of activities that can foster the development of such competencies, while the second question is addressed empirically through the analysis of participants performance when solving a particular task. In the following sections, we initially present some concepts related to ESD. Subsequently, we provide a state of the art concerning sustainability and mathematics education, from which the conceptual framework guiding the research is derived. Finally, we present the research methodology, the results, and the conclusions drawn from this work.

EDUCATION FOR SUSTAINABLE DEVELOPMENT

ESD aims to provide individuals with the knowledge, skills, values, and capacity to address global challenges such as climate change, biodiversity loss, the unsustainable use of resources, and social inequalities. ESD also seeks to empower learners to make informed decisions and act both individually and collectively to transform society and care for the planet. Climate change and water scarcity are among the highest priority issues in international forums and environmental agendas. The unsustainable use of water resources is depleting aquifers and deteriorating water quality, prompting warnings of an imminent "water crisis" that threatens food security and ecosystems. Interventions are needed to manage these resources sustainably, meeting human demands while protecting ecosystems. Understanding the consequences of humanity's appropriation of resources requires analyzing their availability and consumption.

From this perspective, it is crucial for current and future generations to develop sensitivity towards global issues. With this aim, and from a mathematics education standpoint, we propose a teaching activity and explore how pre-service mathematics teachers approach a situation focused on the problem of the water crisis. Specifically, the activity analyzed in this research is an adaptation (following ideas of a guided reinvention, Freudenthal, 1991) of the one presented in the introduction of a secondary school textbook (Boucher et al., 2007) titled "blue gold".

MATHEMATICS EDUCATION FOR SUSTAINABILITY

Despite the interest of several organizations in sustainability education, this area remains challenging. Notably, there is a lack of genuine integration of sustainable education within teacher training and mathematics teaching, which is also reflected in the limited research related to this area (Hui-Chuan & Tsung-Lung, 2021). In the context of mathematics teacher training, it has been found that the presence of sustainability-related competencies in the mathematics education curricula for teacher training in compulsory secondary education in Spain averaged only 25% (Moreno-Pino et al., 2021). Highlighting studies that explore teachers' and students' perceptions or attitudes regarding topics such as sustainability competencies, the

SDGs, and ESD (Bulut & Borromeo Ferri, 2025; Çibik & Boz-Yaman, 2025; Lamanauskas & Malinauskienė, 2024; Muñoz-Rodríguez et al., 2020). However, few studies delve into the types of tasks that integrate sustainability and mathematics (Alsina & Vásquez, 2024; Vásquez et al., 2023), and even fewer address students' performance when engaging with such tasks. For example, in a STEM project that integrated mathematical modelling, students recognized modelling as a useful tool for identifying current problematic situations and for predicting changes and possible future solutions (Suh & Han, 2019). Although the participants used Markov chains and modelling elements, the researchers' analysis focused on identifying perceptions and impressions without delving into exploring the mathematical work and its connections to the SDGs.

Elsewhere, specific activities have been designed to emphasize the role of modelling in helping students understand today's world (Garfunkel et al., 2021). For instance, in a competition problem presented at the international mathematical modelling challenge, participants were asked to use mathematical modelling to answer the question, what is the earth's carrying capacity for human life? To address this question, they had to identify the factors impacting the earth's carrying capacity and understand how these factors interact with one another. Garfunkel et al. (2021) clarify that, although the mathematics used by participants to tackle the problem was relatively simple, they invested a significant amount of time in determining the most relevant factors. This supports an understanding of the situation and fosters the development of critical thinking, while also relying on systemic thinking to identify relevant factors and relationships, thus further encouraging its development. Our approach to these issues is essential, as it seeks to place greater emphasis on the development of both mathematical skills and competencies related to sustainability.

The work of Wiegand and Borromeo Ferri (2023, 2024) shows significant progress in teacher training through the development of a seminar that integrates mathematical modelling and ESD. In this seminar, students solve tasks involving mathematical modelling in different contexts, adapt textbooks tasks, and create their own ESD-modelling tasks. Furthermore, in the implementation of an interdisciplinary mathematical modelling course based on the SDGs, a positive effect was observed on pre-service teachers' attitudes and self-efficacy beliefs towards sustainable development. In the same vein, other teacher education programs have shown significant results (Alsina & Vázquez, 2024; Çibik & Boz-Yaman, 2025). This is reflected in the mathematical knowledge and readiness to tackle problems through critical thinking and reflection (Çibik & Boz-Yaman, 2025).

In synthesis, our review of the literature revealed many studies that explore curricula and perceptions on a general level. While other studies show how professional development, mainly in basic education, can align with learning objectives for the promotion of ESD, highlighting the need to conducting research that contributes to improving the pedagogical knowledge of mathematics teachers (Alsina & Silva-Hormazábal, 2023; Su et al., 2023; Vásquez et al., 2021). Within the field of secondary teacher education, there is a notable lack of research that implements strategies aimed at analyzing how sustainability is integrated with mathematical knowledge. Therefore, it is still evident that the integration of sustainability and mathematics education faces significant challenges and, the question "what would and should ESD in mathematics education look like in the twenty-first century?" remains unanswered (Hui-Chuan & Tsung-Lung, 2021). Based on the insights provided by this review of the literature, the research presented in this paper aims to

(1) explore the abilities and competencies that secondary mathematics pre-service teachers employ when solving a task in a sustainability context and

(2) present a discussion on task characteristics that support the development of mathematical abilities linked to sustainability competencies in pre-service mathematics teachers.

THE KEY COMPETENCY FRAMEWORK IN SUSTAINABILITY AND SPECIFIC MATHEMATICAL SKILLS AS SUPPORT

Competencies in Sustainability and Competency in Mathematical Modelling

In this section, we initially define the competencies for sustainability. Subsequently, as we consider a holistic proposal for the integration of mathematics and the development of competencies, we argue how elements of mathematics education, particularly mathematical modelling, can be adapted to modelling in

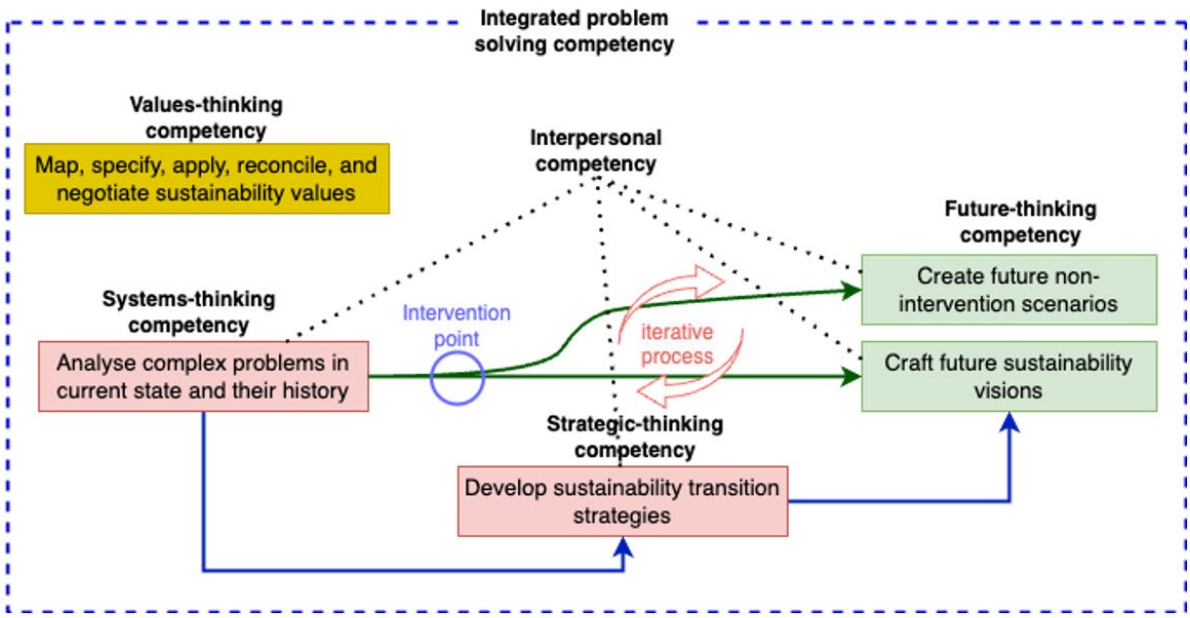


Figure 1. Key competencies in an integrated problem-solving competency (an adaptation of Brundiers et al., 2021 and Wiek et al., 2011, p. 206)

both natural and social sciences, thereby contributing to the development of specific abilities and competencies from a sustainability perspective.

Education for sustainability should enable students to analyze and solve related problems, anticipate and prepare for future challenges, and create and seize opportunities for sustainability (Brundiers et al., 2021; Weik et al., 2011). As Wiek et al. (2011) suggest, it encompasses "complexes of knowledge, skills, and attitudes that enable successful task performance and problem-solving with respect to real-world sustainability problems, challenges, and opportunities" (p. 240). Establishing a foundation based on this, Brundiers et al. (2021) and Wiek et al. (2011) propose a framework that considers competency as the ability to solve problems, specifically defined as competency in research and problem-solving for sustainability. This framework is structured into four modules: analyzing the complexity of current problems and their history, creating and crafting visions of sustainability ("the solved problem"), exploring less desirable future scenarios that could become reality without interventions, and developing and testing strategies to transition from the current state to sustainable states without veering towards undesirable pathways (critical intervention points) (**Figure 1**). Some of these modules are also considered desirable competencies to be developed, as described further on.

For this research, we adopt the key competencies described by Wiek et al. (2011), defined as follows:

1. **Anticipatory/future thinking competency:** The ability to collectively analyze, evaluate, and create detailed images of the future related to sustainability issues and sustainability problem-solving frameworks.
2. **Normative/values thinking competency:** The ability to collectively map, specify, apply, reconcile, and to negotiate values, principles, goals, and objectives for sustainability.
3. **Strategic thinking competency:** The ability to collectively design and implement transformative interventions, transitions, and governance strategies towards sustainability.
4. **Interpersonal/collaborative competency:** The ability to motivate, enable, and facilitate collaborative and participatory research in sustainability and problem-solving.
5. **Systems thinking competency:** Studying sustainability requires a comprehensive understanding of the systems involved, considering temporal and spatial domains and scales. It entails understanding cascading harmful effects and the transformation of individual and collective actions.

From our perspective in mathematics education, and in line with several authors (Blum et al., 2007; Maaß, 2006; Niss & Blum, 2020; Niss & Højgaard, 2011), mathematical modelling enables the interpretation of a wide range of real-world phenomena. English (2009) also suggests that mathematical modelling serves as a tool to recognize the applicability of mathematics in understanding and solving relevant real-life problems. Furthermore, English emphasizes the importance of conceiving and understanding the world as a set of interconnected complex systems, which is essential for making informed decisions both at the individual and collective levels.

In mathematics teacher training, mathematical modelling processes are a priority and, for this reason, drive the integration of sciences within the sustainability context. Hence, the importance of "modelling competency" highlighted by Niss and Blum (2020), which is defined by two interrelated components:

> The first component is the ability to actively *construct* mathematical models in various domains, contexts and situations, i.e., bringing mathematics into play in dealing with extra-mathematical matters.
>
> The other component is the ability to *analyze* given mathematical models, constructed by others or by oneself, and their foundation and critically examine and asses their scope and validity (p. 78).

From a sustainability perspective, it can be argued that modelling processes extend beyond mathematical modelling. In fact, physicists (e.g., Cohen-Tannoudji, 2002) and engineers consider *phenomenological models* essential, where cognitive processes must be closely connected to real-world experiences. This approach aligns with the perspective of realistic mathematics education, which suggests that mathematics learning should occur through progressive mathematization–starting from a real or conceivable situation for students, moving towards a level of schematization, and ultimately constructing an understanding of the general principles underlying a problem, enabling them to see the broader picture (Van den Heuvel-Panhuizen, 2003). Picavet (2002, p. 181) highlights the importance of analyzing the model constructed by participants in their interaction, including any contradictions that may emerge according to their conceptions.

All of this leads us to recognize a dialectical modelling process that includes both a mathematics education perspective and a sustainability perspective. This process requires consideration of modelling across various scientific and social disciplines, as previously mentioned. Thus, the actions undertaken in the mathematics classroom to address a sustainability issue prompt us to focus on the dialectical modelling process, which considers the interaction of participants, their contradictions, and the responses they provide–not only based on a mathematical model but also on the non-contradiction of the mathematical model with the phenomenon being studied. The primary contribution of our approach to examining sustainability issues through mathematical modelling lies in ensuring that once students transition to the mathematical model, the consistency of the model with the situation, from a sustainability perspective, *is not solely a matter of logical-mathematical consistency*. Instead, consistency is determined by the phenomenon being analyzed and the importance of providing a response to the issue at hand from a comprehensive and ethical standpoint.

In this way, the design of ad hoc activities also facilitates the development of value-based competency. Regarding the development of interpersonal competency, this is addressed through the ACODESA teaching method, which incorporates a sociocultural approach to learning (Hitt, 2007; Hitt et al., 2023), to be described in greater detail below.

Mathematical Abilities That Support Sustainability Competencies

Under this perspective of modelling in mathematics education and considering the sustainability competencies described above, we are interested in exploring which types of abilities in pre-service teachers provide a solid foundation for sustainability competencies.

Hitt et al. (2023) suggest that *visualization, anticipation and prediction, empirical validation, sensitivity to contradiction*, and *enquiry* are fundamental abilities in the development of mathematization processes. We argue how these abilities align with sustainability competencies, allowing us to construct a conceptual framework grounded in mathematics education. Mathematical visualization relies on individuals' intuitive ideas, often connected to context. Anticipation and prediction in mathematics converge with the notion of future thinking in sustainability. Empirical validation, whose processes are essential in mathematics learning, fosters sensitivity to contradiction in mathematics, which, in our theoretical approach, should extend to a

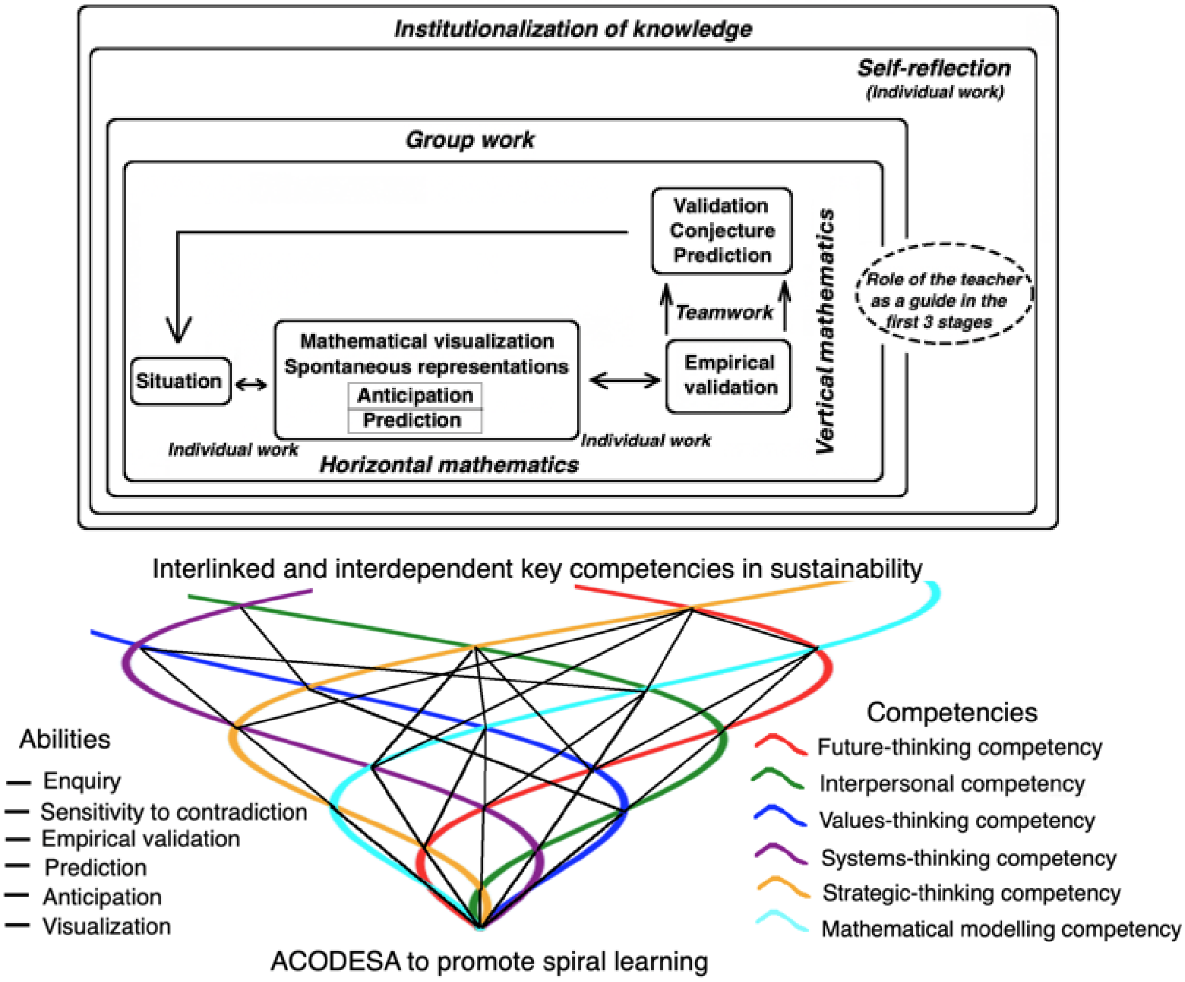


Figure 2. ACODESA to promote spiral learning and sustainability-related competencies (Authors' own creation)

non-contradictory stance on moral, ethical, and social judgment within the sustainability framework. Sensitivity to contradiction is manifested as an inner discomfort when faced with an unexpected and contradictory outcome from the problem-solver's perspective. A person is considered sensitive if they revisit their processes to reflect on (and possibly correct) their results, thereby achieving satisfaction with the outcome (cognitive contradiction, Hitt, 2004). The result may not necessarily be correct for a specialist but is satisfactory to the solver, as the contradiction may not be evident to the solver. In such a case, while a formal contradiction exists, the cognitive contradiction has dissipated.

Thus, our interpretation of the proposal by Wiek et al. (2011) and Brundiers et al. (2021), aligned with constructs specific to mathematics education, enables us, on one hand, to view sustainability competencies as complex interrelations of knowledge, skills, and attitudes that facilitate successful performance in real-world sustainability problem-solving. On the other hand, we propose a complex integration of abilities that combines not only competencies, but also capacities and attitudes related to mathematical abilities. Both abilities and competencies develop progressively as individuals engage in complex tasks related to problem formulation and modelling. Together, these perspectives form a cognitive network that supports a spiral-based approach to learning (**Figure 2**).

Therefore, education for sustainability from a mathematics education perspective should consider the holistic development of abilities that support the sustainability competencies, along with the modelling competency, due to its potential as a tool for understanding, analyzing, intervening in, and predicting various situations. As students gain experience in modelling sustainability situations, skills and competencies are developed in an interrelated manner. The ACODESA teaching method provides the methodological foundation for this research, as shown in **Figure 2**.

METHODOLOGY

Interpersonal Competency from the Perspective of Mathematics Education

A crucial aspect in education are the types of activities proposed in the classroom for developing competencies and mathematical learning. To guide the discussion in this section, we will address the responses to the question: What characteristics should a teaching situation have *to develop mathematical abilities and sustainability competencies in pre-service teachers training*?

According to the theoretical elements mentioned in the previous section, an initial approach to the question involves two fundamental aspects from the perspective of mathematics education:

1. The development of mathematical abilities directed towards sustainability competencies, and
2. The ACODESA classroom methodology to foster the development of mathematical abilities (Hitt, 2007; Hitt & Quiroz, 2019) and sustainability competencies (Brundiers et al. 2021; Wiek et al. 2011).

The first point has already been discussed in previous sections. Meanwhile, the ACODESA teaching method is based on collaborative learning in the classroom, focused on developing abilities across five stages:

(1) individual work,

(2) teamwork,

(3) whole-class debate,

(4) self-reflection (an individual reconstruction process of classroom production), and

(5) institutionalization process.

ACODESA encourages the development of mathematical visualization, anticipation and prediction, appropriation of the situation (enabling students to adopt the problem or situation as their own), sensitivity to contradiction, and further exploration (Hitt, 2004, 2007; Hitt & Quiroz, 2019; Hitt et al., 2023). In this way, classroom work supports the development of abilities that underpin the five competencies for sustainability, while individuals engage in a dialectical modelling process when dealing with sustainability-related situations (**Figure 2**).

In general terms, we seek the comprehensive development of the competencies shown on the right side of **Figure 2** by addressing a sustainability-related problem, a process that is supported by mathematical abilities such as enquiry, sensitivity to contradiction, empirical validation, prediction, anticipation, and visualization. Based on the previous arguments and with the aim of reinforcing the importance of mathematics education in sustainability contexts (in line with the initial question in this section), the tasks should have the following characteristics:

- They should involve situations linked to humanity's problems.
- The situations should be approachable in such a way that they can be framed as mathematical problems, without losing sight of the outcomes resulting from simplification.
- They should promote a dialectical modelling process that enables the interpretation of the phenomenon under study (e.g., identifying variables, covariation between variables, functional models, etc.) and testing from a phenomenological perspective.
- The model should allow for reflection beyond mathematics, recognizing future causes and effects of intervention or non-intervention, as well as the strategic design of potential interventions.
- The situations should enable classroom discussion of social issues within mathematics and be approached from a sociocultural perspective on learning.

Considering the characteristics just outlined, the task "blue gold" was selected, which is described in the next section.

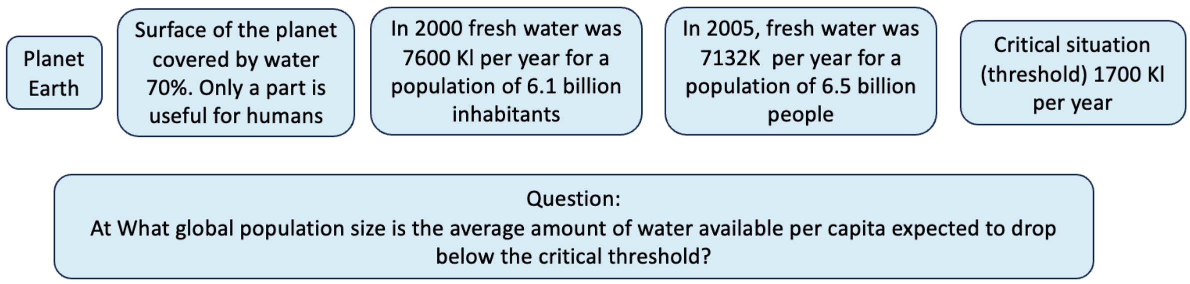


Figure 3. Information provided locally (Authors' own creation)

The "Blue Gold" Task and A Priori Analysis

An outline of the "blue gold" task is provided below.

> Water is abundant on earth. It covers approximately 70% of the earth's surface. However, only a small portion of this water is fresh water accessible to humans. According to the United Nations, the average amount of water available per capita was 7,600 kiloliters per year in 2000, for a world population of 6.1 billion people. In 2005, this amount was 7,132 kiloliters per capita, for a world population of 6.5 billion people. The global situation is critical if the amount of water available per capita falls below the threshold of 1,700 kiloliters per year. At what global population level is the average amount of water available per capita expected to fall below the critical threshold?

The question required to identify variables and use different representations of the situation and the mathematical models involved. After studying the situation, the participants were also asked to formulate new problems or questions related to it, in keeping with our problem-posing and sustainability approach.

Schematically, the information provided locally is as shown in **Figure 3**.

Several prompts were provided to support guided reinvention (Freudenthal, 1991), such as using different representations to understand the problem, organizing data to identify the quantities involved, selecting the relevant variables, or devising a strategy to address the question, among others.

To address the question while considering the problem-solving and future-thinking competencies, it would suffice to identify two significant variables: *the amount of fresh water* and *population growth*. Relating the time variable to the analysis of the situation requires, on the one hand, problem-posing skills that allow students to go beyond the requested response (as seen in the formulation of an additional question). On the other hand, it requires a certain level of future-thinking competency (**Figure 2**), considering the implications of the intervention. In other words, once the question about "the population level at which the human situation would become critical" is answered, a reflection is needed to induce the questioning of when this might happen. In summary, two questions should emerge once the situation is considered:

1. At what global population level is the average amount of water available per capita expected to fall below the critical threshold? (initial question)
2. When will this crisis occur (in what year)? (a question that students should propose and solve)

To answer the initial question, a functional model could be constructed where the dependent variable is the amount of water and the independent variable is population growth, thus providing a response linked to a mathematical model. For instance, with a linear model, the answer would be 11.14 billion people; with an exponential model, the answer would be 15.52 billion; and with an inverse variation model, it would be 27.27 billion (**Figure 4**). The best model is the one that accounts for population growth (the selection of which reflects sensitivity to contradiction). Its construction requires systemic thinking, as it involves considering elements that are interconnected and have an impact on the situation presented. The processes developed to answer this question are associated with modelling competency from a phenomenological perspective.

Once the initial question is answered, the second question should emerge among pre-service teachers (anticipatory/future-thinking competency). They could search for population growth data online and use a model based on the logistic function to predict when a crisis might occur. Answering the second question, Several models could provide an answer to the first question. The most appropriate model would need to account for variations in population growth.

Population (billions)	Water per capita
6.1	7600
6.5	7132
?	1700

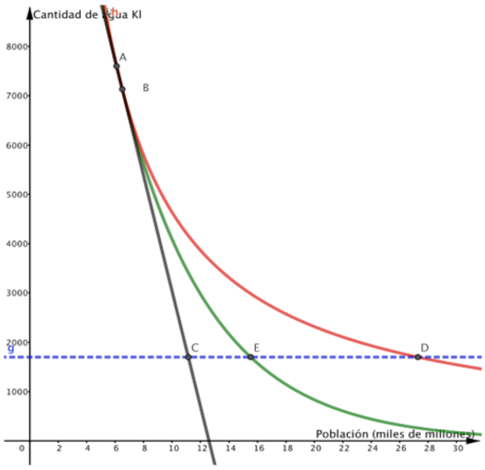


Figure 4. Consideration of population growth as the independent variable and amount of fresh water as the dependent variable (Authors' own creation)

The answer to the second question could be found using one of the models—linear, exponential, or inverse variation.

Time	Populati...
1910	1.75
1920	1.86
1930	2.07
1940	2.3
1950	2.52
1960	3.02
1970	3.7
1980	4.44
1990	5.28
2000	6.09
2010	6.84
2024	8.15

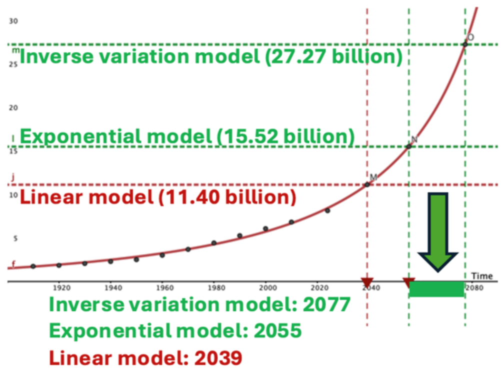


Figure 5. Inclusion of time as the independent variable and population growth as the dependent variable (Authors' own creation)

considering the chosen model (2039 with the linear model, 2055 with the exponential model, or 2077 with the inverse variation model) (**Figure 5**) requires adopting a critical stance on the use of mathematics to make a value judgment. This approach enables students to *craft future sustainability* visions and enhance their *future-thinking competency*.

The answer to the second question, therefore, is also connected to the development of the *systemic thinking competency* and, especially, the *future-thinking competency*. Even if a team arrives at different dates, this should prompt further research to select the most appropriate model and analyze the consistency between the model and its answer to the second question. This process would clearly establish the development of *sustainability competencies* in an interdependent and interconnected manner, in line with our model.

Participants and Data Analysis Methodology

The study was carried out with nineteen voluntary participants within the context of a master's program for mathematics teacher training. The participants hold a bachelor's degree in mathematics, which ensures a solid knowledge of mathematics. At the moment of the research, they were involved in a course that engaged them in problem-solving, mathematical modelling, and the use of technology for the teaching of mathematics. As part of their training, they completed several activities (Camacho-Machín et al., 2024) aimed at gradually develop abilities such as visualization, anticipation and prediction, empirical validation, sensitivity to contradiction, and enquiry. Prior to data collection, participants were informed about the study, assured of the anonymity of their participation, and reminded of their right to withdraw from the research at any time. By this way, informed consent was obtained in accordance with the requirements of the University's Ethics Committee.

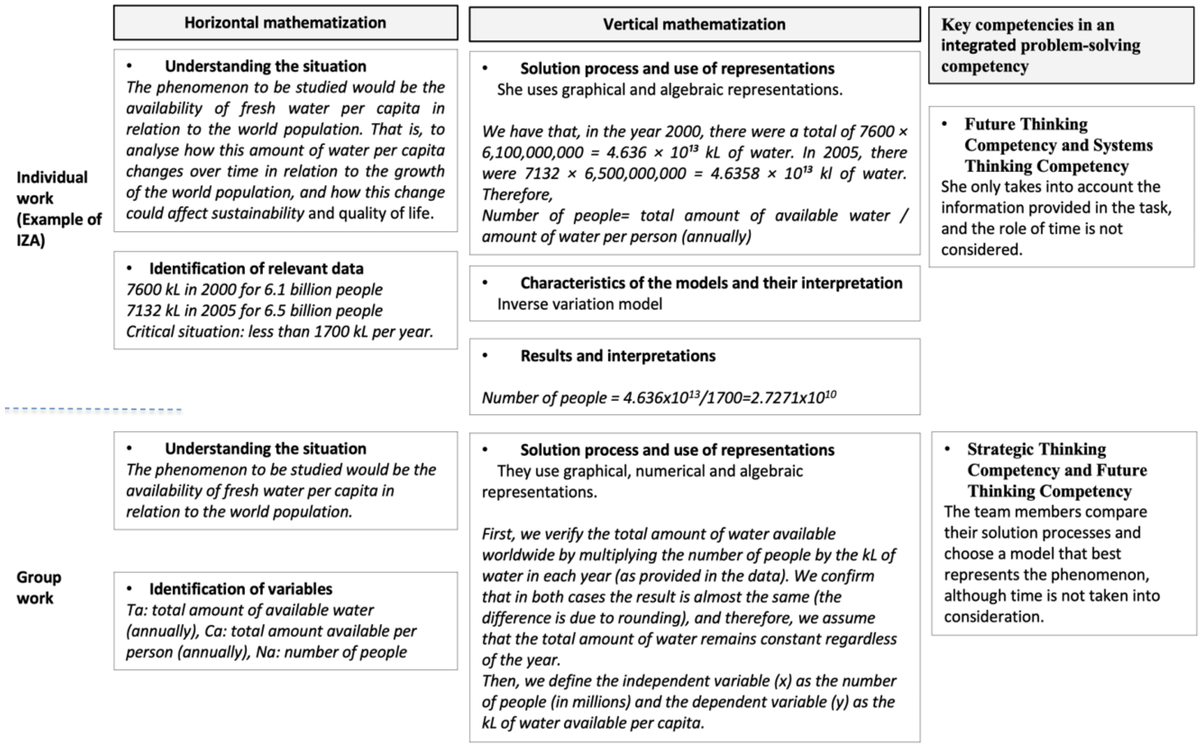
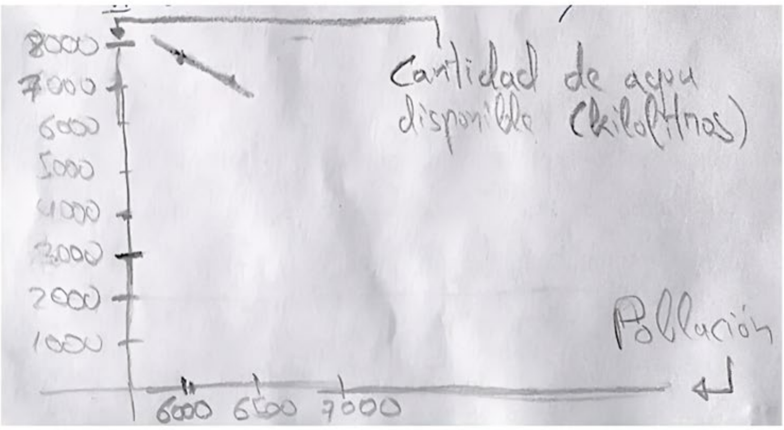


Figure 6. Example of individual and group coding (Authors' own creation)

The solution of the "blue gold" task followed a structured sequence. The participants initially worked on it individually, which enabled personal engagement with the situation. This was followed by group work (each team with two or three participants), allowing collaborative exchange of strategies. A whole-class discussion was then conducted to promote collective reasoning. This was followed by an individual and collective reflection, Finally, the process culminated in an institutionalization phase, during which key mathematical ideas and modelling strategies were formalized.

The data collection instruments consisted of two written reports in which, according to the ACODESA stages, the process of individual, group, and whole-class problem-solving, as well as interpretations and explanations, were recorded. The researchers' written notes, documenting the teams' work methods, were also considered. For data analysis, we categorized the written work according to content analysis approach (Kuckartz, 2019), particularly following a deductive analytical framework. Initially, one of the authors conducted a preliminary categorization considering the individual description of the phenomenon under study, types of representations, data organization, identification of the quantities and variables involved, strategy organization to answer questions, and formulation of new problems or questions. Then, the aim was to identify the manifestation of necessary abilities (described in **Figure 2**) to develop, as consequence, the key competencies for sustainability (Brundiers et al., 2021; Wiek et al., 2011). This categorization was discussed and refined with the other two authors of this paper. Subsequently, a similar procedure was applied to the group work, with a more precise focus on identifying text fragments that evidenced aspects related to the development of competencies in connection with the abilities (related to a horizontal and vertical mathematization). Reliability was ensured by comparing the categorizations made by each of the authors, which were then presented to a group of experts for final validation. In cases where a discrepancy was deemed too great, or when a response showed potential to belong to two categories, the entire solution process was reviewed once more, carefully identifying the interpretations developed by the participants.

An example of the coding process can be seen in the following image, which shows the coding of the individual work completed by Iza, a member of group 5 (**Figure 6**). The coding of the group work can also be observed. In this case, no major discrepancies were observed between the individual and group work. This form of coding allows us to ensure the representativeness of the final coding.



To be in a critical situation, the population must increase from 6,500 million to nearly double [...] we must assume that the world population continues to grow over the years, while the amount of water decreases.

Figure 7. Marcus described a lineal relationship between population and quantity of water (Authors' collected data)

RESULTS AND DISCUSSION

In what follows, we focus on the analysis of the responses to the "blue gold" task. As the experiment was organized using the ACODESA collaborative learning method, our analysis is centered on individual and collective process of solutions, highlighting elements of mathematical abilities and sustainability competencies. Due to space limitations, we presented the analysis of the full process related to the first team and then we summarized the results for the others teams. Team 1 was selected to illustrate the analysis due to the variety and clarity of the solution processes evidenced in their work. The members of this team demonstrated strategies that reveal different forms of reasoning and decision-making throughout the task. Moreover, their work presents a coherent and explicit structure that facilitates the interpretative analysis of their procedures. Team 1 can be regarded as a typical case, as it representatively reflects the trends observed in the other teams, although with a level of explicitness that allows for a more detailed examination of the strategies employed.

Team 1 (T_1): This team was composed of Marcus, Henry, and Any. During the previous activities and in the process of solution of the "blue gold" task, Henry took the role of leader, this is particularly important because the decisions they make as a group will be influenced by the leader. In below a synthesis of the individual productions of each member of the team T_1 is showed. Then the final decision of the team is discussed.

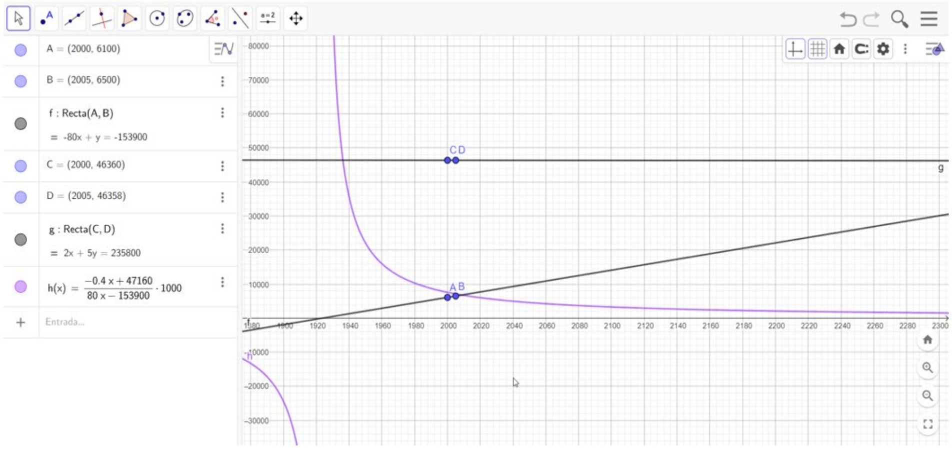
Marcus's Solution Process

He identified three variables of relevance to the situation (per capita water quantity, world population, and time) and recognized that the variables water quantity and population are most closely related. Then graphed population versus water quantity (**Figure 7**), calculated a linear function $1{,}700 + 468x - 5{,}894{,}800 = 0$, and found that the intersection with 1,700 gives 11,143 million people. Marcus further anticipates that a critical situation would emerge if the population reached approximately 13,000 million.

As can be observed in the synthesis of Marcus's work, he assumes a linear behavior for the situation, presuming that population growth might develop in the same manner.

Henry's Solution Process

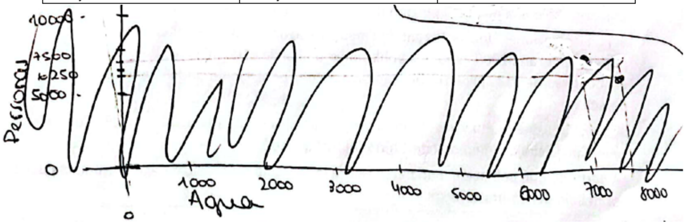
He calculated two linear functions, $f: -80x + y = -153{,}900$ with points A = (2000, 6,100), and B = (2005, 6,500), to represent the relationship between world population vs. time, and $g: 2x + 5y = 235{,}800$ with points C = (2000, 46,360) and D = (2005, 46,358) to represent the total amount of water available for each year. On the same graph (**Figure 8**), Henry constructed a rational function, $h(x) = g(x)/f(x) * 1{,}000$ and calculate the intercept with $y = 1{,}700$, $F = (2263.86, 1{,}700)$. Then, he calculated $g(2263) = 27{,}140$ million people.



In the defined function h(x), the value 1,700 is reached when x is between 2263 and 2264, so it will be in the year 2263. f(2263) = 27,140 (millions of people).

Figure 8. Graphical representations of Henry (Authors' collected data)

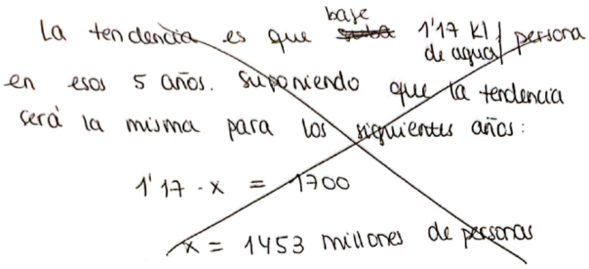
Year 2000	Year 2005	Unknown
7,600 Kl/year	7,132 Kl/year	1,700 Kl/year
6,100 millions	6,500 millions	X millions



$\bar{x}_{water} = \frac{7600-7132}{6500-6100} = \frac{468}{400} = 1.17$ *Kl/person per year has decreased in 5 years.*

With more data, we could determine the trend in the number of liters of water per inhabitant.

Figure 9. Amy organizes the data and calculates the rate of change (Authors' collected data)



*The trend is for water consumption to decrease by 1.17 kiloliters per person over those five years. Assuming the trend will remain the same for subsequent years. 1.17*x = 1,700, x = 1,453 million people.*

Figure 10. Amy rules out a first solution strategy (Authors' collected data)

Although Henry considers an inverse relationship between population growth and the amount of available water, there is no explicit mention of the behavior of population growth.

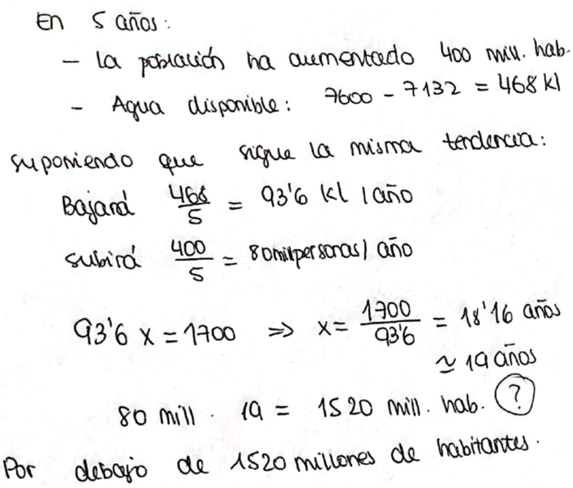
Any's Solution Process

Any organized the data into three columns with respect to time and determined the rate of change of per capita water per year. She also expresses the need for more data, perhaps in order to make a more accurate prediction (**Figure 9**).

She then attempts to calculate, by means of a direct proportional relationship, the number of people corresponding to 1,700 kiloliters per year. However, she eventually abandons this calculation (**Figure 10**).

Accordingly, Any assumes a linear behavioral trend and applies an alternative method to address the task. Through this approach, she estimates that the critical situation will occur in 19 years, when the population is projected to reach approximately 1,520 million inhabitants (**Figure 11**).

We now discuss the individual mathematization processes of the three participants, contrasting them with the collective decisions made by the group during their solution process.



In 5 years:

- *The population has increased to 400 mill. hab.*
- *Water available: 7,600 – 7,132 = 468 kl.*

Assuming the same trend continues:

It will decrease by $\frac{468}{5} = 93.6$ *kl/year.*

It will increase by $\frac{400}{5} = 80$ *million people/year.*

93.3x = 1,700 =>x = $\frac{1,700}{93.6} = 18.16$ *years, ≈ 19 years.*

*80 million * 19 = 1,520 million inhabitants (?)*

Below 1,520 million inhabitants.

Figure 11. Any's solution process (Authors' collected data)

Horizontal mathematization: Translating natural language into algebra and the notion of variable:

- Anticipation leads Marcus to identify three variables: time, water quantity, and population. However, when constructing the predictive model, the team omits time as an explicit variable, treating population as the independent variable and water quantity as the dependent one. This reflects a conceptual simplification that does not fully correspond to the structure of the real-world phenomenon.
- Henry, the team leader, also engages in anticipatory reasoning and identifies the same three variables. On the other hand, the team correctly assigns time as the independent variable, with population and water quantity as dependent variables. Unusually, they represent all three variables within a single graph, which may suggest an integrative intention; however, the approach lacks methodological clarity.
- Any organizes the data in a table using time as the reference for three columns: the year 2000, the year 2005, and a third column labelled "unknown." This structure shows an explicit temporal organization of the phenomenon and suggests an intention to extrapolate into the future.

In this process of horizontal mathematization, the students, through a process of visualization, anticipation, and prediction, are engaging with the task. For example, in organising the data while considering time as the main variable, Any's productions implicitly involve processes of visualization, along with anticipation and prediction.

Vertical mathematization: Once the variables were identified, each participant engaged in a distinct algebraic process:

- Marcus used only the two variables previously identified as dependent on time. Taking population as the independent variable. He constructed a linear function and intersected it with *y = 1,700*, obtaining a critical population of 11,143 million people.
- Henry, by employing a "graphical representation of the three variables", constructed two linear functions, *f* and *g*, and subsequently built a rational function by combining both lines: $h(x) = \frac{-0.4x+47,160}{80x-153,900}$. The time is implicit in this function, and it was multiplied it by 1,000, without providing any justification. The team adopted this approach as their collective solution process. At this stage, the participants encountered a contradictory situation given by two inconsistencies:
  (1) Amy and Marcus assumed linear behavior throughout the entire solution process.
  (2) The division of the two functions and the subsequent multiplication of 1,000 were not accompanied by any conceptual explanation.
- Amy, by calculating the average annual decline in water (93.6 kL/year), concluded that a crisis would occur in 19 years, at a population of 1,520 million. Notably, if the prediction was based on data from 2005, adding 19 years leads to 2024–the actual year in which the activity was conducted. Remarkably, the team showed no reaction to this coincidence.

In synthesis, we observe that Marcus, by selecting a linear model, was in a state of implicit contradiction. He didn't recognize that such a model was inadequate for the situation, as the decline in potable water is linked to global population growth, which does not follow a linear trend. Moreover, although he provides a numerical answer to the original question, he did not attempt to determine the year in which this situation would actually occur. Similarly, Henry–although employing an inverse proportional relationship–still considered, as in the work of Marcus and Any, a linear behavior throughout. While any was also in a state of contradiction; however, she could potentially resolve it. By stating that the critical moment would occur 19 years after 2005, that is, in 2024–the actual year in which the experiment was conducted–she aligns her result with the real timeline. This suggests that, by comparing it with real-world data (e.g., via the Internet), she could recognize that her population estimate of 1,520 million is inaccurate and, therefore, adjust her reasoning accordingly.

As observed, during the group work, the initial questions were addressed using the solution proposed by the team leader. However, when responding to the question "Did the group discussion give rise to new questions?", the team stated:

> "Yes, since each member approached the problem differently, it becomes inevitable to ask which of the solutions offers a better approximation of the situation we are trying to model. To this end, we attempted to identify the weaknesses in each individual proposal."

What stands out in this response is that, despite acknowledging divergent results, they did not interpret them as mathematical contradictions. Instead, they framed them as "different approximations" and ultimately adopted the leader's model as the final answer–assuming, perhaps, that they did not perceive the mathematical contradiction arising from the nature of the phenomenon being modelled. Given that the three productions are different, the participants missed the opportunity to engage in a deeper inquiry that could have allowed them to determine which model best aligned with the phenomenon, a step that would have enabled them to progress towards a more meaningful stage of vertical mathematization. This leads us to identify a key issue at the group level:

- A lack of sensitivity to mathematical contradiction among all three members, stemming from a lack of phenomenological sensitivity–an essential component from a sustainability perspective. These issues suggest that the team failed to develop three key sustainability competencies: *the future thinking competency*, *the strategic thinking competency*, and *the systems thinking competency*.

Table 1 presents a comparative summary of the work produced by the members of the five teams. As can be observed, most of them managed to establish the relationship between the amount of water and the population. Only three participants explicitly incorporated the temporal variable. Linear and inverse variation models predominate, and in general, the timing of the critical situation was not explored.

Table 1. Summary of individual productions

Teams	Number of members	Relates amount of water vs. population	Relates amount, population, and time	Mathematical model	Result (billions)	Critical year	Individual performance
T_1	3	Marcus	Henry Any	• Linear, • Linear & inverse variation • Linear	11.14 27.14 15.20	**?** 2263 19 years	Contradictory responses
T_2	4	Joao Sandy Ana Lore	-	• Inverse variation	27.27	**?**	Answers coincide
T_3	4	Nancy Tere Manu	Rosa	• Inverse variation • Linear and inverse variation • Inverse variation	27.26 11.14 & 27.27 27 27.27	**?** **?** 2376 **?**	Critical year not discussed except by Ana
T_4	4	Javy Martha Dany Lucy	-	• Description • Exponential • Inverse variation • Description	**?** 15.52 27.26 **?**	**?** **?** **?** **?**	Critical year not discussed
T_5	4	Vince Sasha Mary Iza	-	• Linear • Inverse variation • Linear • Inverse variation	11.14 27.27 11.14 27.27	**?** **?** **?** **?**	Critical year not discussed

From **Table 1**, we can observe that the participants' responses mostly align with what was expected in the a priori analysis. It can also be extrapolated from the individual analysis shown in **Table 1** that the diversity of participants' responses reinforces the validity of the results.

Collective Results

Following a similar analytical procedure, we examined the other teams (team 1 to team 5), from which we can synthesize the following group results, as presented in **Table 2**.

Table 2. Summary of the analysis of the five teams' work on the "blue gold" activity

Team	Horizontal mathematization	Vertical mathematization	Sustainability competences
T_1	Correct identification of key variables. Failure to adequately emphasize the important role of time in relation to the phenomenon under study.	Realize linear and rational models, without critical verification. The final model selection does not consider mathematical contradictions or real implications.	Strong formalism, lacking connection to reality or critical reflection. Sustainability competencies such as strategic thinking and future-oriented thinking were not mobilized.
T_2	Good identification of variables and recognition of the possibility of multiple models (linear and inverse).	Consistent and well-structured mathematical processes were developed at the individual level. However, in the teamwork, the differences between models were not critically analyzed.	High potential for collective integration. Sustainability was left behind due to a lack of phenomenological and ethical sensitivity.
T_3	Adequate identification of variables and organization of the phenomenon, although without collective integration.	Various models were developed with time estimates and even references to real data, but in isolation between members.	High individual potential, lacking coordination. No systemic vision or collective strategy emerged; sustainability competencies were weakened by a lack of dialogue.
T_4	The basic variables were identified, but with explicit exclusion of time and without articulation between students.	Simple models, isolated graphs, instrumental use of GeoGebra. Lack of critical evaluation without reflection on the real phenomenon.	A disjointed team, lacking leadership or discussion. Lacking the development of critical thinking skills or a systemic approach.
T_5	The key variables were clearly identified. There were attempts to personalize the problem and connect it to reality.	Various models were presented, some of which were critically compared. Although group discussion was lacking, efforts were made to validate and justify the models.	The team demonstrated progress in strategic thinking and an emerging critical and phenomenological awareness. High potential if collaborative work and contextualization are strengthened.

Overall, horizontal mathematization was generally adequate across all teams, although several failed to incorporate the temporal dimension or to contextualize variables with enough sensitivity to the real-world phenomenon. Regarding vertical mathematization, individual technical proficiency was evident, yet there was limited comparison of models or collective validation. A formal approach tended to prevail over an interpretative one. Competencies such as future thinking, systems thinking, and critical sensitivity were very limited, with partial exceptions in T_2 and T_5. Only one participant in group T_5 demonstrated an empathetic and ethical stance toward the phenomenon analyzed.

CONCLUSIONS

The experiment suggests that the proposed activities effectively foster abilities in mathematical visualization, anticipation, and prediction. However, the results indicate that developing sensitivity to contradiction was more challenging, both from mathematical and sustainability perspectives. Enhanced communication (through the development of interpersonal competency) could strengthen sensitivity to contradiction in mathematics, thereby supporting the acquisition of anticipatory or future-thinking competency. A methodological shift to facilitate this could involve modifying classroom practices to incorporate the 1-2-4 dynamic approach (Camacho-Machín et al., 2024), which consists of individual work, pair discussions (aiming for consensus), discussions between pairs (to facilitate consensus among groups of four), and finally, a whole-group discussion.

Participants also demonstrated weaknesses in enquiry skills, affecting their ability to further their investigation, both from a mathematical and sustainability perspective. For instance, when asked to pose new questions, some participants reflected on whether their solution process was correct, while others raised questions about the variation in water quantity over time. They did not answer their own questions. In terms of sustainability, posing additional questions was crucial to gain insight into future scenarios. Weaknesses in anticipation and prediction skills were evident, indicating a need for more practice in formulating new questions and envisioning more complex scenarios. This aspect needs greater emphasis in teaching to ensure that students understand the social and ethical implications of their mathematical analyses. While some groups were able to design strategies to build their mathematical models (mathematical modelling competency), they struggled to align their results with a future-oriented perspective.

The mathematical modelling work was formally correct for the most part, but it failed to transform mathematical knowledge into a critical tool for sustainability (a phenomenological model). There was a lack of connection to the real context, in-depth collaborative work, and an authentic investigative attitude.

The results of the experiment show that, although future secondary school and high school teachers have a solid background in mathematics, developing skills related to sustainability, such as systems thinking, anticipation, and sensitivity to contradiction, through phenomenological modelling processes, is more complex than expected. Building models that coherently represent real-life situations linked to socio-environmental problems requires not only mathematical skills but also a critical, ethical, and contextualized approach. Despite these difficulties, participants showed significant progress, responding to the first research question and partially to the second, highlighting the need to more frequently incorporate integrative tasks such as the "blue gold" task. These types of activities allow progress toward teacher training that is more consistent with current challenges and fosters the development of a critical and proactive awareness regarding sustainability issues.

Finally, from the perspective of mathematics education, we recognize that the modelling competency is supported by systemic thinking, since understanding how different variables interact within a system is essential when modelling complex situations. The creation of mathematical models makes it possible to visualize the impact of decisions and actions (both individual and collective), facilitating future-oriented thinking and strategic decision-making.

Limitations and Implications

A limitation of the study was the small sample size, as it considered only nineteen participants involved in a specific teacher training program with particular cultural setting, which may limit the generalizability of the findings. The study used a single modelling task and although it was adequate for exploring the development of sustainability competencies and mathematical knowledge, it does not capture the diversity of modelling contexts.

Despite these limitations, the findings provide relevant implications for both teacher education and future research. Regarding teacher education, the results suggest the need to design and implement more open tasks that approach sustainability problems through mathematical modelling, promoting reflection on social aspects. For future research, it would be worthwhile to carry out longitudinal studies to explore how sustainability competencies develop over time through successive modelling experiences. It would also be meaningful to examine how these kinds of tasks can be integrate in the curriculum and guiding mathematics teachers training.

Author contributions: CGO: conceptualization, methodology, formal analysis, investigation, writing – original draft; **MCM:** conceptualization, methodology, formal analysis, investigation, supervision, validation, writing – review & editing; **FH:** conceptualization, formal analysis, investigation, supervision, validation, writing – review & editing. All authors approved the final version of the article.

Funding: This study was carried out with the support of project PID2022-139007NB-I00 funded by MCIN/AEI/10.13039/501100011033/ FEDER, EU.

Ethics declaration: This study was approved by the Pontifical Catholic University of Valparaíso, Chile, with reference number BIOEPUCV-H 370-2020 and verified by National Agency for Research and Development, Fondecyt/Iniciación No. 11200169, Chile. All participants were informed of their voluntary involvement in the study. Once the data had been collected, the information was securely stored under lock and key, and pseudonyms were assigned to ensure the anonymity of the participants.

AI statement: As the authors are not native English speakers, AI was used to support the review of the translation into English.

Declaration of interest: The authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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