European Journal of Science and Mathematics Education, 2024, **12**(1), 21-37
ISSN: 2301-251X (Online)

OPEN ACCESS

Research Article

Problem-solving in a real-life context: An approach during the learning of inequalities

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Citation: Rocha, H., Viseu, F., & Matos, S. (2024). Problem-solving in a real-life context: An approach during the learning of inequalities. *European Journal of Science and Mathematics Education, 12*(1), 21-37. https://doi.org/10.30935/scimath/13828

ARTICLE INFO

Received: 26 Apr 2023

Accepted: 18 Oct 2023

ABSTRACT

This study was conducted while 9th grade students learn to solve inequalities and seeks to understand their approach to solving problems with a real-life context. Specifically, the aim is to understand: (1) What are the main characteristics of the students' approaches to the proposed problems? (2) What is the impact of the real context on the students' resolutions? A qualitative and interpretative methodology is adopted, based on case studies, with data collected through documentary collection and audio recording of discussions between a pair of students while solving problems. The main conclusions suggest a trend to approach problems without establishing immediate connections with what was being done in the classroom, with students' decisions being essentially guided by criteria of simplicity. The real context of the problems seems to have the potential to develop in students a more integrated mathematics, focused on understanding and not so much on the repetition of mechanical and meaning-independent procedures. The students' familiarization with the context in question is one of the aspects highlighted by this study.

Keywords: problem-solving, real-life context, inequalities

INTRODUCTION

The teaching of inequalities tends to be approached focusing on the reproduction of procedures, often disconnected from meaning, and assumed by the students as useless and uninteresting. The procedural approach is characterized by Gerasimova et al. (2023) based on the central role assumed by the teacher, where the teacher tells, and the students practice the rules, formulas, or facts. According to Pólya (2003), a rule-centered approach to mathematics teaching will prevent students from understanding its interest, resulting in a loss of interest in learning. Besides this, Kilpatrick (2010) points to the integration of problem-solving in mathematics learning as a way to involve students and awaken their interest for the discipline. And Rocha (2022) emphasizes the relevance of a global development of the students, instead of promoting a mathematical learning based on a set of disconnected knowledge. Real-life context is also assumed as important to promote learning. However, as highlighted by Chapman (2006), previous research concluded that the students often tend to ignore that context, suggesting this is a consequence of their mathematical experience in the classroom. As so, it is important to deepen the understanding about the influence of a real-life context in the students approaches to a problem. And this is the main contribution of this study.

The present study focuses on inequalities at the 9th grade (14 years old students), seeking to understand the students' approach to problem-solving in a real-life context. Specifically, the aim is to understand:

1. What are the main characteristics of the students' approaches to the proposed problems?
2. What is the impact of the real context on the students' resolutions?

PROBLEM & PROBLEM-SOLVING

The understanding about what a problem is, has evolved over the years. Based on the work of Pólya (1975), Kantowski (1977) considers a problem to be a situation to which there is no immediate way of responding. In turn, Lester (1983) defines a problem as a task in which there is a need to find a solution, but in which a defined procedure is not known to allow finding that solution. To this understanding Blum and Niss (1991) add the intellectual challenge posed by the task.

The distinction between problem and exercise was also the focus of attention. Schoenfeld (1985), following the work of Kantowski (1977), Lester (1983), and Pólya (1975), establishes the difference, considering an exercise a situation in which a familiar and routine procedure is used, even if it is difficult, to arrive at the solution; while in a problem a way to arrive at the solution is not already known. Pólya (1981) clarifies the understanding of a problem when he states that solving "a problem is to find a way out of the difficulty, it is to find a way around an obstacle, to obtain a desirable end, which is not immediately available through appropriate means" (p. 1). Pólya (1981) emphasizes here the issue of difficulty, a dimension of defining problem that, as stated by Christiansen et al. (1986), in the initial attempts to define problem had not been placed.

Based on a literature review, Yeo (2017) analyzes the understanding of problem, considering that over time there have been essentially two perspectives. One focuses on the individual nature of what is a problem, i.e., what is a problem for one person may not be for another. And Schoenfeld (1985) highlights this aspect by stating that this characteristic of being or not being a problem is somehow external to the task. The other perspective mentioned by Yeo (2017) relates to the nature and purpose of the task. According to Yeo (2017), a problem must have a specific intention, which cannot be training or practicing a procedure. It must require some thinking and sometimes even some creativity.

Problem-solving has long been considered an important aspect of mathematics, both for teaching and learning (Liljedahl et al., 2016). According to Bruder (2016), in the 20th century the theme was the subject of attention, mainly in the 1960s and 1970s. Several studies addressed problem-solving processes, emphasizing the importance of heuristic strategies. Correia (2005) considers that heuristics are "mental operations" that can be applied to a comprehensive set of problems. Bruder (2016) clarifies that "heuristic strategies, principles and tools would provide students with an orientation in problem situations and that this could thus improve students' problem-solving abilities" (p. 3). The well-known Pólya's (1975) model stems from his interest in understanding how to teach problem-solving. It is a model that includes the following phases:

1. Understand the problem: Identify what is intended. Although it may seem obvious, a superficial reading is often done without really understanding what is being asked.
2. Create a plan: Find a way through similar problems. The objective is to find connections between the problem data and unknown; prior knowledge and experience on similar problems is very important.
3. Execute the plan: Difficulties may appear at this stage, but it is important to persist. If necessary, go back to the previous step and find a new plan.
4. Check if the solution and method respond to the intended and, if possible, generalize – if applicable in solving similar problems.

Authors such as Guzmán (1986) and Lester (1983) proposed adaptations to the Pólya's (1975) model, but without significant differences. Guzmán's (1986) adaptation addresses the issue of learning through problem-solving experience. The author mentions that sometimes you learn more from a failure than from many successes, thus valuing the process more than just the final product.

For Lompscher (1975), to solve a problem, mental agility is more important than mathematical knowledge. And by mental agility the author refers to the intuitive reduction of a problem to the essential, the flexibility of the lines of thought, the visualization of the problem in different perspectives and the transfer of one procedure to a new situation. It should be noted that this author also mentions that anyone who does not have this intrinsic mental agility can acquire it through heuristics. But what Pólya (2003) emphasizes most in all phases of his heuristic is the constant interrogation. It is therefore important to develop a proper posture, which also enhances the role of the teacher. And Rocha (2020) emphasizes the role of the teacher when the students are facing an unfamiliar problem. In the case of this study, we will not only have problems, but problems with a real context and within the scope of learning algebra and, more specifically, inequalities.

The context is assumed as important to develop mathematical meaning and understanding and to promote students' motivation for learning (Chapman, 2006). However, students' difficulties related to the context in problem-solving have been documented. Students tend to ignore the context of the problem, focusing on the procedural approach. What Schoenfeld (1991) and Verschaffel et al. (2000) call a suspension of sense making. As Chapman (2006, p. 213) refers "students come to believe that executing mechanical calculations is more important than considering the real-life meaningfulness of their actions in mathematics activities". The potential of the use of problems with contexts from the real-life and the difficulties identified in achieving that potential, turns the inclusion of real contexts in the problems used in this study very relevant. It is important to better understand how the students deal with the context when solving problems. And, as mentioned before, this is the main contribution of this study to the field.

METHODOLOGY

Given the characteristics of the research questions, the study adopts a qualitative and interpretative methodology, based on case studies (Yin, 2003). Three pairs of students were analyzed, and this article presents the analysis of one of the case studies. This is the case of the pair who was more talkative and tend to discuss more about what they were doing. Data collection took place in a 9th grade class (14 years old students) while the theme inequalities was being taught.

Four tasks including problems with a real context were proposed to the students (three in the classroom and one outside). These tasks were selected to create a sequence of progressively demanding tasks. The context of the task was carefully considered to include situations from the students' daily life and situations not so familiar. The students involved in the study worked on the tasks in pairs and audio recorded their discussions. The last task was done outside the classroom to permit a more detailed monitoring of the students' work, allowing the investigator present (the third author of this article) to ask questions and seek for justifications. These discussions were also audio-recorded. Besides this, all the written resolutions made by the students were collected.

The participants in the study were chosen among the students achieving level 4 or level 3 in mathematics who volunteer (the grades in the Portuguese basic schools use a scale of one to five, where five is the highest level), valuing the most communicative students, due to the ease with which they could share information. The pairs were organized considering the personal relationships between the students.

Data analysis was based on the interpretation of the collected information, considering the research questions and based on the analysis categories defined in **Table 1**. For each of the proposed problems, the students' resolution and the process experienced by them were analyzed, seeking to identify the choices made and the underlying reasons. In relation to the first research question, the approach to each problem was analyzed seeking to identify stages. First, we look to the stage where the students were trying to understand the problem, then we look for the plans considered to go forward, after that we gave attention to the decision about the plan to implement and then to its implementation, finally we attended to the reflection about the results obtained. These categories are inspired by literature and the stages for solving a problem, but they also result from our analysis of the students' involvement in the problems. As so, it was assumed as important to give a specific attention to the decision about the plan to implement.

For each category some subcategories were considered, as described in **Table 1**. Once again, these result from the literature and from a preliminary analysis of the data. The second research question focus on the impact of the real context. As so, the category and subcategories defined were used to analyze the data in each of the stages of the students' approach to the problems identified when addressing the first research question.

Table 1. Categories & sub-categories of analysis

1st research question	Categories & sub-categories of analysis		2nd research question
Characteristics of students' approaches to problems	**Understanding of problem** Actions taken by students: - Read problem - Discuss understandings - Register information (including using draws, diagrams, or other representations) - Relate problem with problems solved previously	**Role of real context in students' choices** - Familiarity or not with context–impact on ability to solve problem - Reality of context present in problem–impact on ability to solve problem	Impact of the real context on the students' resolutions
	Definition of one or more than one possible plan Elaboration of plan(s) using: - An approach with a newly learned process (inequalities) - An approach using knowledge previously learned by students - An approach using a reason developed by students to address problem	**Role of real context in students' choices** Idem	
	Choice between plans & reasons for it Reasons do decide between plans: - Easy to implement - Faster to implement - Teacher expectations or similar	**Role of real context in students' choices** Idem	
	Plan implementation Important points: - Difficulties faced - Points discussed	**Role of real context in students' choices** Idem	
	Analysis of result obtained Central points: - When is result obtained analyzed or not - What reasons contribute to decide to analyze or not the result - Time constrains - Familiarity with context of problem	**Role of real context in students' choices** idem	

RESULTS

This section presents and analyzes the data collected from one of the pairs of students participating in the study: the pair AB, formed by student A and student B.

The investigator/researcher is represented by "I" when taking part in the dialogues.

Characterization of the Pair

Student A is one of the best students in the class, she is devoted, with an appreciation for mathematics, and very hardworking. She is intuitive showing no difficulty in understanding new content. Her participation in the classroom work is regular, posing pertinent questions, showing mathematical knowledge and agility in solving any proposed task. She has maintained a level 4 (on a scale of one to five, where five is the highest grade) for the first two terms of the current school year.

Student B started the school year with a classification of level 4, but in the 2nd term he only achieved level 3. Although he finds it easy to learn the mathematical content being taught, he is easily distracted by conversation with colleagues. He has critical spirit, is intuitive and manages to generalize results, but does not spontaneously participate in class.

Analysis of Task 1

This task included three problems with a real context and was proposed to the students in the lesson, where the inequalities were introduced.

Problem 1–T1

> **Music CD's:** Vicente loves rock music CDs. Knowing that he only has €74.81 and that each CD costs an average value of €12.35, what is the maximum number of CDs he can buy? (adapted from https://brainly.com.br/tarefa/7466023).

In Problem 1–"Music CDs"–, as soon as it was proposed, pair AB established and executed a plan: they carried out a division (**Figure 1**).

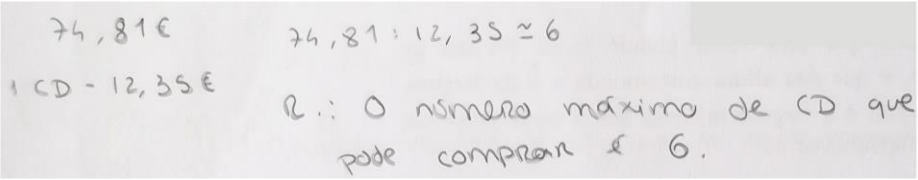

Figure 1. Resolution of problem 1–T1 by pair AB (Source: Authors)

The researcher tried to understand if the students had considered using an inequality. Their answer pointed to the simplicity of the chosen approach as the reason for the choice:

> B: Xiii! That would be more difficult. Making this division is much easier.

It is not clear if the students considered the use of inequalities, but it is obvious that, when they think of it, they did not consider this to be the most appropriate approach. A decision probably related to the characteristics of the problem proposed. Deciding how much they can buy with the money they have is a situation very familiar to the students. As so, the context helped them to decide about what to do.

The analysis of the students' resolution shows that they understood the problem, established a plan, and executed it. The verification of the solution found was not carried out, but this has to do with the level of simplicity that the students seem to have felt in the problem and to the level of confidence they have in their resolution.

Problem 2–T1

> **Skate:** Valentine received €60 from his grandparents on his birthday. He earns €16 a week handing out advertising leaflets. Since his birthday, he has already saved more than the €180 needed to buy the skateboard he wants. How many weeks ago was his birthday? (adapted from Ponte et al., 2009, p. 170).

In problem 2–"Skateboard"–, the pair AB discussed the problem, pondered which strategy they should use, and analyzed the solution to respond to the problem. Regarding the strategy, after the initial understanding and simplification of the problem, they thought of two options: focusing on division or addition. They considered checking which value multiplied by sixteen would be closer to €120 (180-60), or successively adding 16 until they reached €120. In their own words:

> B: We must see how many '16 a week' he needs to reach €180.
>
> A: Yeah! But if he received 60€, I think I have to subtract these 60 from the 180 he needs.
>
> B: Yes. We see what value it gives, and we look in the 16-multiplication table for the closest value to 120 … the closest above 120.
>
> A: So, 180-60=120. We can do 120 divided by 16 or we can do 16+16+16 …
>
> B: Yes, but it will take longer.

Speed thus turns out to be the decisive factor in the choice they made and, once again, the familiarity with the situation guides them in the decision about how to approach the problem. After executing the plan, however, they realized that the solution is not immediate and that there was something else to consider, since the value obtained was not an integer: 7.5 weeks. Although they are familiar with the situation, and this familiarity helped them to decide how to approach the problem, the truth is that the problem is different from what that are used to face in their life. They usually know how much money they received in their birthday and then think about how many weeks they will need to get all the money they need to buy what they want. So, they are used to look to the future, and the problem asks them to look to the past. Realizing the difference, the students discussed what should be the appropriate answer, and presented their answer (**Figure 2**):

> A: Valentine's birthday was seven and a half weeks ago.
>
> B: Calm down, as it is weeks we cannot put seven and a half. As they are asking for weeks, we must put a whole number, so it was eight weeks ago.
>
> A: You're right! You're right! Seven weeks ago, he did not have enough money to buy the skateboarding.

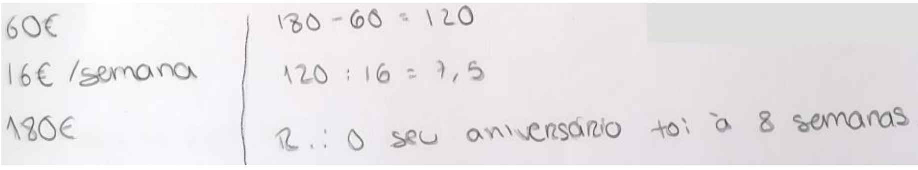

Figure 2. Resolution of problem 2–T1 by pair AB (Source: Authors)

As the dialogues illustrate, the pair AB explored several alternatives to solve the problem. The strategy used was the calculation of the value that, multiplied by sixteen, would be closer to 120. For this calculation, the students associated multiplication with division, dividing 120 by 16. Later, they discussed the solution, reflecting on the context of the problem and in the question posed by the problem–number of weeks. It is clear that the students: understood the problem, established a plan, executed it and contextualized the solution within the scope of the presented situation.

Problem 3–T1

> **Dimensions of a suitcase:** Airlines often impose restrictions on the number, weight, and dimensions of bags that each passenger can carry. A Portuguese airline does not allow bags exceeding 158 cm in the sum of their dimensions (height, width, and depth). Consider one large suitcase is 30 cm deep, and its height is $\frac{2}{3}$ of its width. What is the maximum width that the suitcase can have, to be accepted by this airline? (adapted from Gomes, 2017, p. 175).

In problem 3–"Dimensions of a suitcase"–, pair AB immediately considered that it was a more elaborated problem than the previous ones. After reading it aloud, they showed understanding of the problem, using the suitcase draw, present in the task, to register the relevant information.

Student A suggested writing an equation to represent the information in the problem. They agreed and student A wrote and simplified the equation (**Figure 3**).

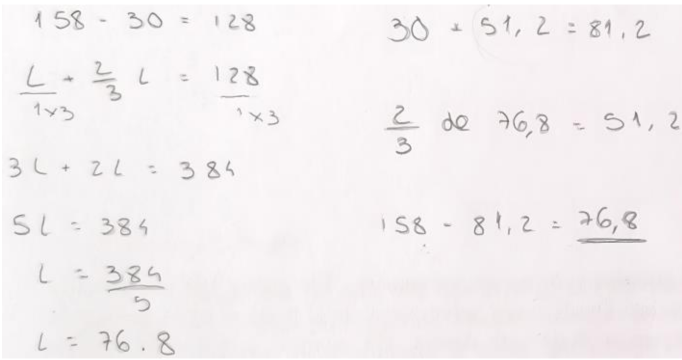

Figure 3. Resolution of problem 3–T1 by pair AB (Source: Authors)

The use of equations and the option to register the information of the problem by the suitcase draw is an approach different than the ones used in the previous problems, and closer to what they are used to do in the mathematics classroom. A difference identified in a problem that is also different from the previous ones in what relates to the context: this is a situation that is not familiar to students, and not faced in their real-life.

Fractional numbers caused some discomfort, as the following dialogue illustrates:

> A: So … Stay calm! How do you do this?
>
> B: Do this twice (referring to 76.8) and then divide by three.
>
> A: Is that how it is done?
>
> B: I think so. At least that's how I do it … And I always get it right.
>
> A: Ok! And we found the height.
>
> (…)
>
> A: Same result! What is the answer?
>
> B: So, it can only have a maximum of 76.8.

In this problem, pair AB showed several hesitations. They wrote an equation instead of an inequality, however they represented the information algebraically. It is possible to verify that they went through different stages, from the understanding of the problem, the elaboration of a plan and its implementation, to the analysis of the result obtained. And their reasoning, until reaching the final answer, is clear.

As the problems became a little more complex, pair AB showed the ability to consider different approaches and to choose the one to implement. Although they did not explicitly mention it, it is possible to identify criteria of simplicity and speed guiding the choice between the work plans considered. The students did not assume the topic being study in class–inequalities–as something they should use in their resolutions. They started considering inequalities as something too complex to use in solving the problem at hand–when they understand that there were simpler ways to solve the problem–to resort later, when faced with a slightly more complex problem, to an equation. Although the best way to translate the problem is using an inequality, the students relied on an equation, showing the ability to properly interpret the result to respond to the problem. That is, the students relied on the different mathematical tools available, integrating them into their work in the way it seems to them to be useful. A greater familiarity with equations, which had been learned previously, may also be present here, and this could be the factor that leaded students to opt for an equation rather than an inequality. But the truth is that the students started with an approach that is clearly disconnected from the content being studied in class, suggesting that this is not the reason that guides their choices. The real context of the problem seems to be fully assumed by the students and the fact that they focused on the situation may be one of the factors that leads students to seek to rely on their mathematical knowledge, as a whole, to solve the problem.

Analysis of Task 2

Task 2 included two problems, where students could relate equations and inequalities. In the classroom, the students had remembered the partial monotony of multiplication and the influence it has on the inequality sign when solving an inequality. In this task, the presented problems continued to focus on real-life situations, but the degree of difficulty was higher than in the previous task.

Problem 1–T2

> **Graduation trip:** Joaquim saved €1,250.00 for the graduation trip. Of this amount, €375.00 will be spent on tickets. The rest will be used to pay for meals and the hotel stay. Assuming that Joaquim expects to spend €30.00 per day on meals, and that the daily rate in the hotel is €75.00, how many days can he stay? (adapted from Gomes, 2017, p. 175).

Problem 1–"Graduation trip"–was related to money. Thus, pair AB immediately associated it with one of the problems in the previous task–problem 2 of T1: "Skateboarding". The students began by reading the problem and recording the relevant information. They identified the unknown–*x* is the number of days–and associated the amount spent per day with the unknown. The previous experience, when they organized the visit to Azores and discussed costs, was important to a better understanding of the problem. They wrote the inequality used (**Figure 4**) showing understanding about the sign of the inequality, as illustrated in the following dialogue:

> A: As it is money, it is the same as Valentine's problem (T1 problem 2).
>
> B: Yes! That is it! But first we must write all the information.
>
> (...)
>
> A: We must join the €30 with *x* and the €75 with *x* because *x* is the number of days.
>
> B: So, we can put 30 and 75 together because they both go with *x*.
>
> A: Yeah, yeah. And we know that if we add this all up with the 375 it must be less than 1,250.
>
> B: It must be less than or equal to … because he can spend exactly 1,250.

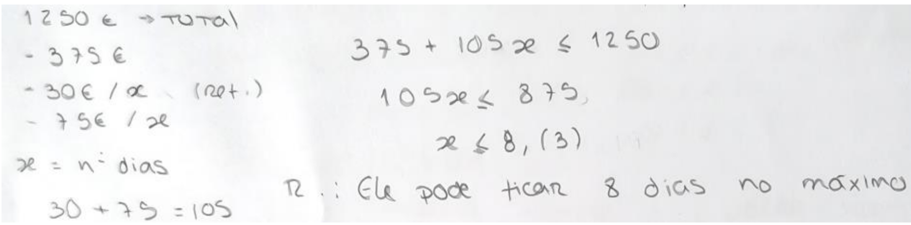

Figure 4. Resolution of problem 1–T2 by pair AB (Source: Authors)

Despite the understanding and confidence shown by the plan outlined by the students, the result achieved was not what they expected. The problem asked for the number of days, and the students were expecting to obtain a natural number as the result of the inequality. As in one of the previous problems, the situation is somehow different from what they experienced in the real-life. When they organized their visit to Azores, they knew how many days they want to stay, and they estimated how much money they need. In this case, they have some money and want to know how long they can stay. The decimal answer achieved made them wonder if there was something wrong. A reaction that can also be the result of the students' previous experience, where real situations are not common, and the questions posed by the teachers usually have an integer answer. Under the circumstances, they decided to confirm the result. To do so, they did not re-analyze their work, instead they used a different process:

> A: There is something wrong here. This is not the right number, it is 8.3333 …
>
> B: That is eight, but let's replace the *x* in the expression with eight and see if that is okay.
>
> A: So, it is 1,215 … Ok!

Problem 2–T2

> **Football field:** The rectangle in **Figure 5** represents the school's soccer field. The head of the school has 50 meters of fence available and wants to put a fence around the field. It is known that twice the difference between the available fence and the required fence is greater than 20 meters. The school head states that the field is five meters wide. Is he right? (adapted from Magro et al., 2015, p. 154).

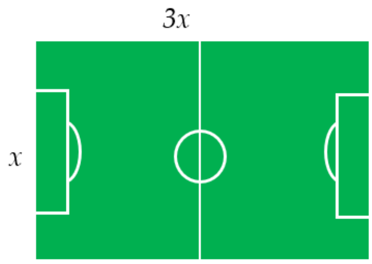

Figure 5. Football field for problem 2–T2 (Source: Authors)

In problem 2–"Football field"–pair AB began by noticing some similarities with a previously solved problem–"Dimensions of a suitcase":

> A: These are measurements, it's like the suitcase (T1 problem 3).

The way how the information is presented in the problem suggests a mathematics' problem and not a situation from the students' life. As so, the students try to rely on their mathematics to solve the problem. Student A reflected for a few moments and wrote an algebraic expression, trying to translate the information in the problem–twice the difference between the available fence and the necessary fence. Then the pair defined an inequality, solved it, considering what had been discussed in class regarding the impact of a negative coefficient of the variable on the inequality (**Figure 6**). In the process, they mentioned the relevance of the perimeter in this problem:

> B: We cannot forget that we must calculate the perimeter.

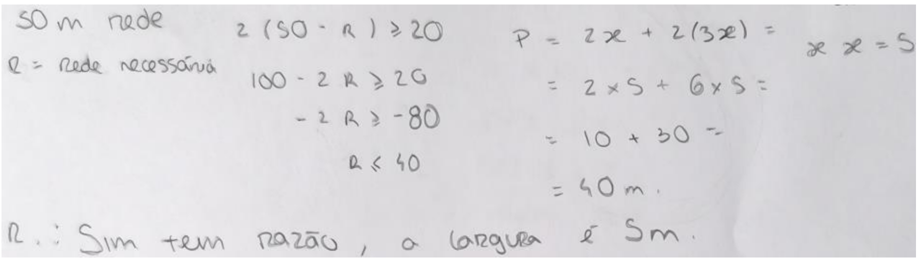

Figure 6. Resolution of problem 2–T2 by pair AB (Source: Authors)

They then wrote the expression corresponding to the perimeter of the soccer field and decided to calculate it in the case, where the width of the field is five meters. At this stage, they face some difficulty in interpreting the results they obtained:

> A: Here it is less than 40 and the perimeter is equal to 40. So, the School Head is right.

> B: No! Less than 40 is not equal to 40.

> A: Yes … In the problem it says greater than 20. So, this is all wrong!

Students tried to make sense of the calculations performed, trying to articulate the result of the perimeter for a five meters wide field and the result of the inequality. The real context of the problem, that in some of the previous problems helped them do define an approach, in this case is too unreal. This circumstance, together with the lack of time to conclude, made them adopt an approach used in previous problems, without a rationale for its adoption:

> B: Put greater than or equal to. We've always done like that.

> A: Ok, we do as usually here. Anyway, we do not have time to do it again.

This was the second problem in which the students used an inequality, being both problems of the second task proposed to the students. Apparently, the greater complexity of the problem was the reason that led to this option. So far, the real context seems to have been important for the students, helping them to make sense of the problems posed and to decide what they should do. And this is the first problem, where the pair AB showed some difficulty in reaching a conclusion and effectively solving the problem. It is interesting to note that this happens in a problem where the real context is unrealistic. If, on one hand, the measurements of the field do not correspond to the measurements of a soccer field, on the other hand, the information given–twice the difference between the available fence and the required fence is greater than 20 meters–does not correspond to the information we would expect to have in a real situation. The difficulties faced by the students in this problem (which had not occurred in any of the previous problems) suggest that the real context assumes an important role to define a strategy for solving the problem and for its implementation. In this problem, the students never refer to the context of the problem, an attitude very different from the one assumed in other problems, where it was possible to identify moments where the students options were being guided by their knowledge of the situation.

Analysis of Task 3

Task 3, like the previous one, included two problems, where the real context continued to be present. It was a task proposed by the end of the lesson in which the intersection and union of intervals of real numbers were remembered and where the association with conjunction and disjunction of conditions was made.

Problem 1–T3

> **How many tests?:** When discussing assessment, the mathematics teacher and his students agreed in having six tests during the school year. However, it was defined that those who had an average of not less than 70% in the first five tests do not need to do the last one. Catarina's results on the first four tests were 50%, 90%, 78% and 72%. What is the minimum score that Catarina can have on the fifth test if she does not what to take the last one? (adapted from Magro et al., 2015, p. 169).

This problem–How many tests?–, besides allowing an approach using inequalities, also involved other mathematical knowledge previously worked on by the students–the notion of average, studied within the Statistics theme.

Pair AB's approach to the problem showed its familiarity with the situation addressed. The students responded quickly, showing that they knew how to determine the average of the tests and using an inequality to solve the problem (**Figure 7**). Pair AB was confident about the solution found, not feeling the need to verify it. Familiarity with the situation involved was clearly decisive here for the quick and relaxed way in which the students solved the problem. However, it is possible that the option of using an inequality to find the solution was influenced by the context of the classroom and by the teacher herself.

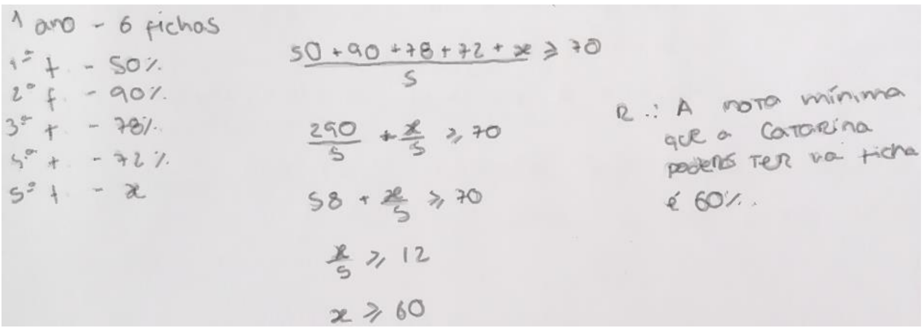

Figure 7. Resolution of problem 1–T3 by pair AB (Source: Authors)

Problem 2–T3

> **Taking a taxi:** The price to be paid for a taxi ride includes a fixed portion, called flag fall, and a portion that depends on the distance travelled. If the flag fall costs €3.44 and each kilometer driven costs €0.90, determine the distance that can be traveled with a value between €20.00 and €30.00, inclusive (adapted from Gomes, 2017, p. 175).

In problem 2 of this task–"Taking a taxi"–the proposed situation was not so familiar to the students, and their reaction was different. In this case, there was an association between the problem and the mathematical contents that had been covered in class–conjunction and disjunction of conditions–but also some hesitation about what to do:

> A: The flag fall is €3.44.
>
> B: We must multiply the 90 cents by x, which is the distance, and add it to the flag fall.
>
> A: I also think we must use that "and" thing.
>
> B: It's supposed to! And we must use inequalities.
>
> (...)
>
> (they wrote $20 < 3{,}44 + 0{,}9x < 30$)
>
> A: Now, I do not know how we do it. I do not know how we calculate this with three things, we must cut one. Which one do you want?
>
> B: Take the 20.

Not feeling confident, the students decided to check with the researcher what to do next:

> A: Teacher! How do we calculate this with three things? We do not know.
>
> I: So, what were we talking about in class just now? When we have $20 < 3{,}44 + 0{,}9x < 30$, it is equivalent to what? How do you read this expression?
>
> A: This is between 20 and 30. Two inequalities with the "and" sign. Is the "and" sign up or down?
>
> B: "and" is down (∧).
>
> I: Exactly. Then, can you calculate separately or not?
>
> A: Ahhh! I got it, I got it! We calculate this one and then the other one.
>
> I: Yeah!
>
> A: And we are left with values between so much and so much.
>
> I: Good, that's right!

The students solved each of the inequalities separately (**Figure 8**) and presented the corresponding solution set considering both conditions simultaneously. Even so, in the inequality they did not include the possibility of equality, but they consider it when presenting the solution set.

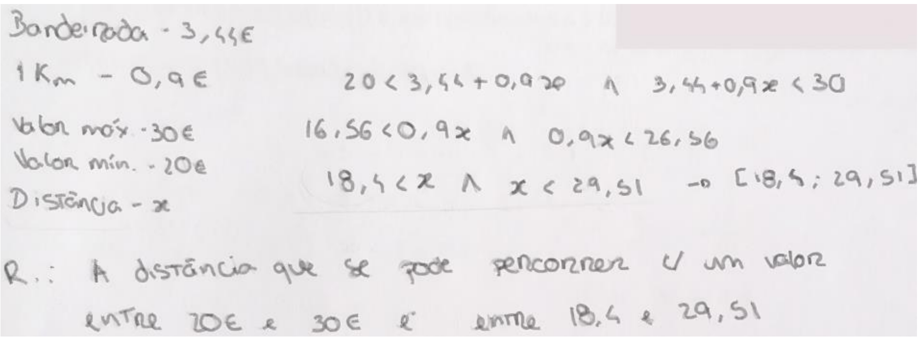

Figure 8. Resolution of problem 2–T3 by pair AB (Source: Authors)

Given the range obtained, the students feel some difficulty in finding the answer to the problem, and decided to ask for the researcher support once again:

> A: And now we have this solution set ([18,4; 29,51]) we subtract one from the other?
>
> I: What is the question in the problem?
>
> B: Determine the distance that can be traveled with a value between 20 and 30€.
>
> I: What is your unknown?
>
> A: It is the *x*.
>
> I: Which represents … what?
>
> A: The distance. So, the distance it can be traveled is 18.4 and 29.51.
>
> B: The distance you can travel is between 18.4 and 29.51 km.

Their difficulties clearly illustrate the lack of familiarity with the situation. Although this is a situation from the real-life, it is not part of the students' life. As so, while in some of the previous problems the context helped them to make sense of the situation and to go further, in this case it did not. When the students did not know how to do the mathematics, the only option was to call for the teacher's help.

Globally, in this problem, the students chose to use the conjunction of inequalities, something they had just learned. This option, together with the lack of familiarity with the context of the problem, seems to have created a distance from the situation presented in the problem, leading students to have difficulty moving forward and to feel the need for support. Something that had not happened with the previous problems. Even so, the support provided, in line with what Pólya (2003, p. 24) recommends, focusing on questions to make students ponder what was being considered ("always asking the same questions: What is the unknown? What do you need? What do you want to look for?"), seems to have been enough to allow progress.

Complementary Task Analysis

When proposing the complementary task–"The job"–, the students were already familiar with first degree inequalities, having completed their study. This task proposed a single problem, with a higher degree of difficulty than the problems presented previously. In addition to inequalities, the problem also involved percentages, a topic previously addressed by the students.

Complementary task

> **The job:** The employee of an enterprise receives a salary of €500 to which is added an amount referring to the extra hours worked per month. He receives €10 per hour overtime, plus an additional 5.0% on the sum of the base salary to the amount referring to overtime worked. The discount for governmental taxes is 8.5% on the total salary. How many overtime hours must he work in a month to receive a net salary of more than €1,000? (adapted from Gomes, 2017, p. 225).

In this problem, the strategy implemented by pair AB was the division of the problem in parts (**Figure 9**). While reading the problem, the pair AB described the steps to be taken to solve each part. The students used algebraic expressions, representing by *x* the number of overtime hours of work. Thus, successively, they wrote the algebraic expression corresponding to the amount of salary and overtime, a 5.0% increase on this amount, an 8.5% discount on the total amount to be received and, finally, the algebraic expression corresponding to the net amount to receive. Once that expression was found, they wrote and solved the inequality that would allow them to answer the problem.

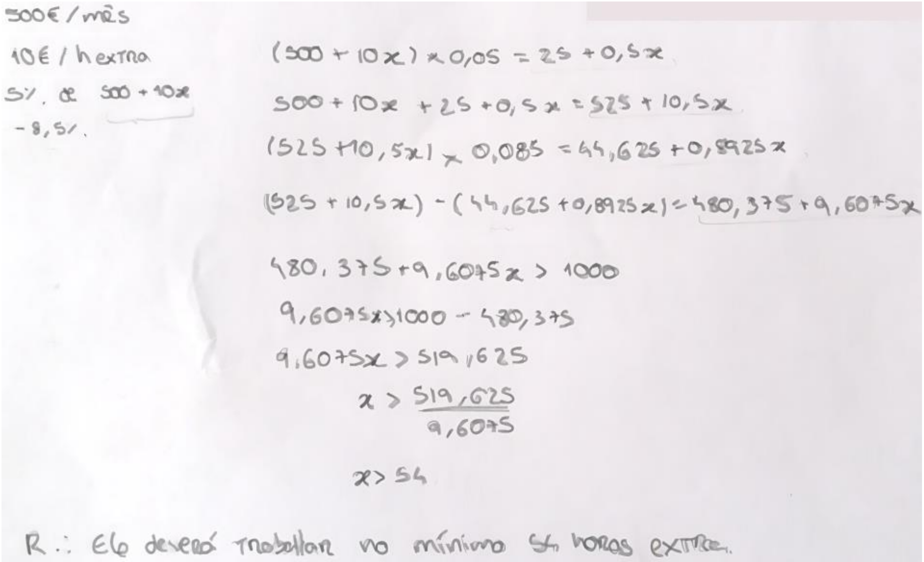

Figure 9. Complementary task resolution by pair AB (Source: Authors)

It is precisely the steps taken to solve the problem that the students explain, referring that they had to analyze the information provided, before they were able to write the inequality that allowed them to answer the problem:

> B: The unknown is *x*, and it represents overtime. It is how many hours he must work to earn a salary of more than €1,000.
>
> I: Did you know right away what you were going to do to write the inequality?
>
> A: No, first we analyzed the information a bit more … to know what we were supposed to do, or not. Then we write the inequality, but first we did it by parts …
>
> B: As we read, we wrote.
>
> I: What is the meaning of each part, or each expression, you wrote?
>
> A: We have the €500, which is the base salary and we added 10*x*, where the 10*x* was overtime. And we went to find out how much 5.0% of the salary plus 10*x* was. Then, we added it to the salary to find out how much he received.
>
> B: As we had to know how much the deductions were, we found out how much the 8.5% was and subtracted it from the salary.
>
> A: We simplified and wrote the inequality of the problem.
>
> I: Did you check the solution?
>
> B: No, we did not have time.

And at the request of the researcher, the pair AB refers how they usually proceed to solve a problem, describing the steps they follow. A description, where we can easily find parallels with the steps suggested in the literature, namely by Pólya (2003):

> I: What are the steps to solve a problem?
>
> A: First: read carefully; second: write the information or not, but it usually helps; third: write an expression that translates the problem; fourth: solve this expression; fifth: give the answer; at the end … discuss the results … Yes, that is how it is.

In what concerns the relevance of the context of reality in the problems, the students identify it as a positive aspect, which can help to facilitate understanding:

> I: What did you think of the task?
>
> A: It was difficult because it was different from what we had done before, and nobody likes percentages (laughs).
>
> B: But this and the other problems being about more common things in our day-to-day life help us to understand better.
>
> A: Yes, because if it were just mathematical expressions and mathematical language, I do not think we would know, where they apply, and it would be more difficult.
>
> B: This one was also very useful for adding more things to the inequalities. It was different because of the percentages.

Final Analysis

In the first set of problems, the students chose the simplest process, the most familiar for them. Problem-solving thus seems to have the potential to lead students to consider options and to choose the approach that seems the most appropriate in the circumstances, instead of simply use the last procedure that was discussed in class. However, with the increasing degree of difficulty of the problems and the increased understanding of the content–inequalities–, which continued to be worked on in class, the students began to consider the use of inequalities in their resolutions. In the first task, they started using division and then equations, showing agility in translating the information from verbal language to algebraic language. The translation into algebraic language was common in the options assumed by the students, and the inclusion of a context of reality seems to make a significant contribution to the development of meaning, facilitating the understanding and relevance of the expressions considered and the procedures used.

During the solving process, it is easy to identify stages in the students' work and points of contact with the problem-solving steps proposed by Pólya (2003). Students understood the proposed problems and implemented a resolution strategy according to their sensitivity. Sometimes they opted for the strategy they considered easier, but there were also situations in which the students identified points of contact or similarities with previously solved problems.

CONCLUSIONS

This study was conducted while 9th grade students learn to solve inequalities and seeks to understand their approach to problems with a real-life context. Specifically, it was intended to understand:

1. What are the main characteristics of the students' approaches to the proposed problems?
2. What is the impact of the real context on the students' resolutions?

Students' Approach to Problems

Pólya (1981) distinguishes different approaches to a problem according to the options taken by the student. He thus refers to an approach with a newly learned process; an approach with a procedure learned by the student, but with some decision-making added by the student; or an approach that combines two or more procedures and even possible derivatives of these. And these last two options are considered richer and more interesting by the author. In the case of the students involved in the study, there was a tendency to address problems without establishing immediate connections with what was being learned in the classroom. The students thus showed a tendency to reflect on the problem, trying to define an approach that seemed to be the most appropriate. Simplicity criteria are present among the reasons that guided the students' choices. This is evident in the decision not to use inequalities in certain situations, despite the fact that these are being worked on and that there is even some reference to them by the teacher. Although it can be discussed if the option for not using inequalities in the first problems could be due to some lack of confidence regarding the resolution of inequalities (once the students were in the initial stage of studying the topic), it seems to be the simplicity of the first problems and the progressive increase in their complexity, which determines an initial approach without using inequalities and a subsequent use of them.

The approach followed by the students was very similar to that suggested by Pólya (1981), going through the various steps suggested by the author. Reading the problem and writing the information, aiming at understanding the problem; going through the elaboration of a plan, which often included the translation of verbal language into algebraic, and its implementation, sometimes including the use of inequalities; until verification of the work carried out and the result achieved. It is interesting to note that this last step, verification, was not always carried out. Sometimes this was due to a lack of time for this purpose, but it also happened that the students' familiarization with the situation in question gave them the confidence not to feel that need (e.g., P1-T3 "How many tests?"). The opposite also occurred in which the doubt about the correctness of the result obtained raised the need to, rather than verify what had been done, find a different way to confirm the result achieved (e.g., P1-T2 "Graduation trip"). The students showed ease in understanding and translating verbal language into algebraic language. This was recognized from the first moment as a faster and more assertive resolution strategy.

Impact of Real Context to Students' Problem-Solving Approach

The use of problems with a real context is assumed to be promising for enhancing the development of mathematical concepts (Freudenthal, 1991), sometimes even being considered fundamental for learning mathematics (Doorman et al., 2007). However, it can also be the origin of some difficulties (Champan, 2006). The students involved in this study seem to have felt the real context as an enhancer of mathematical compression. Indeed, this context provided them with something that allowed them to make sense of algebraic expressions and even something that offered them evidence of the potential of the techniques taught in class. According to the results obtained in this study, solving problems with a real context seems to have the potential to develop a more integrated knowledge of mathematics, focused on understanding and not so much on the repetition of mechanical and meaning-independent procedures. These students were able to properly mobilize mathematical content, regardless of the topic being addressed. That is, they did not assume that they should use inequalities in all problems (since this was the topic under study), and they also proved capable of mobilizing mathematical knowledge previously studied (such as the notion of average or percentage). A conclusion in line with what was mentioned by Godino et al. (2015), who argues that continued work with this type of problems provides students with the ability to develop mathematical tools within the scope of understanding. What is interesting about this study is the way in which the context and the students' familiarization with this seems to have reduced the usual tendency of students to assume that they are expected to use what they have just been taught.

Final Comments

This study corroborates the idea that solving problems with a real context can contribute to mathematics learning, but the most significant result is related to the way in which meaningful contexts for students and close to their experiences can contribute to potentiate a use of the mathematical knowledge in a global, articulated and meaningful way. It would be important to deepen this dimension in future studies, namely seeking to better characterize the characteristics of the problems with potential for this.

Author contributions: HR: Conceptualization, literature review, methodology, data analysis presented, writing—original draft, writing—original first version, writing—review and editing, writing—review and translation; **FV:** data analysis presented, writing—review and editing; **SM:** conceptualization, literature review, methodology, data collection, data analysis presented, writing—original draft. All authors have read and agreed to the published version of the manuscript.

Funding: This article was supported by the national funds through FCT–Foundation for Science and Technology, I. P., in the context of the project 2022.03892.PTDC.

Ethics declaration: The authors declared that the study adhered strictly to standard ethical principles, ensuring the anonymity of those involved. Participants and their parents were informed about the study and how the data collected would be used for academic purposes, and voluntarily agreed to participate.

Declaration of interest: The authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the third author of this article on request.

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