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Research Article

Navigating the seas of the curriculum: A mathematics teacher's interactions with curriculum materials

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ABSTRACT

Curriculum materials are not a fixed object; they are an active partner in a teacher's practice. Teachers and curriculum materials interact in a participatory relationship where both the characteristics of the teacher and the characteristics of the curriculum material affect the instruction. Teachers interpret and tailor these materials to create particular instructional strategies, therefore, teachers can be considered as lesson designers. The study presented in this paper uses longitudinal case study design to investigate the relationship between a mathematics teacher and curriculum materials over a period of nine years. Using the lens of curricular noticing, the study examined what the teacher attends to, interprets and responds to while planning a lesson. Data collection occurred at four distinct time points, encompassing both teacher observations during lesson planning and interviews after lesson planning, as well as the teacher's lesson plans for the same topic at each time point. The obtained longitudinal data provided comprehensive insights into how the teacher's curricular noticing changed over time and how it affected the teacher's practice. The findings show that the teacher attended the national curriculum and textbook at all four time points, indicating the stability of these particular resources; however, the interpretation of the textbook elements and response to this interpretation changed significantly. The teacher's practice became more student-centered and inquiry-oriented. The implications of the study are also discussed.

Keywords: curricular noticing, mathematics teacher, longitudinal study, curriculum materials

INTRODUCTION

In mathematics education, materials used by teachers and students for learning have always been very important (Fan et al., 2013). Today, there are a wide range of materials, from traditional print media like textbooks to more modern electronic formats like e-textbooks, digital learning platforms, teacher-created content, lesson plans, worksheets for students, and all-inclusive assessment resources (Trouche et al., 2023). The materials that offer a learning progression for particular mathematical domains, corresponding to a course or pathway, are referred to as curriculum materials (Remillard & Kim, 2017).

Curriculum materials are not a fixed object; they are an active partner in a teacher's practice. This stems from the idea that curriculum materials are artefacts. The term artefact refers to the production or reproduction of human social activities (Wartofsky, 1979). According to Rabardel (2002), an artefact is a particular object with its intrinsic characteristics, designed and realized for the purpose of accomplishing a particular task. In that context, curriculum materials can be regarded as artefacts because they have the power to shape human activities (Brown, 2009). A consequence of that view is that teachers and curriculum materials interact in a participatory relationship where both the characteristics of the teacher and the characteristics of the material influence the outcomes in classroom practice (Brown, 2009; Remillard, 2005). For instance, in lesson planning, a teacher's interaction with curriculum materials is a dynamic and creative process. Teachers interpret and tailor these materials to create instructional strategies that lead to success in the classroom (Pepin et al., 2013). Therefore, teachers can be considered as lesson designers (Jukić Matić,

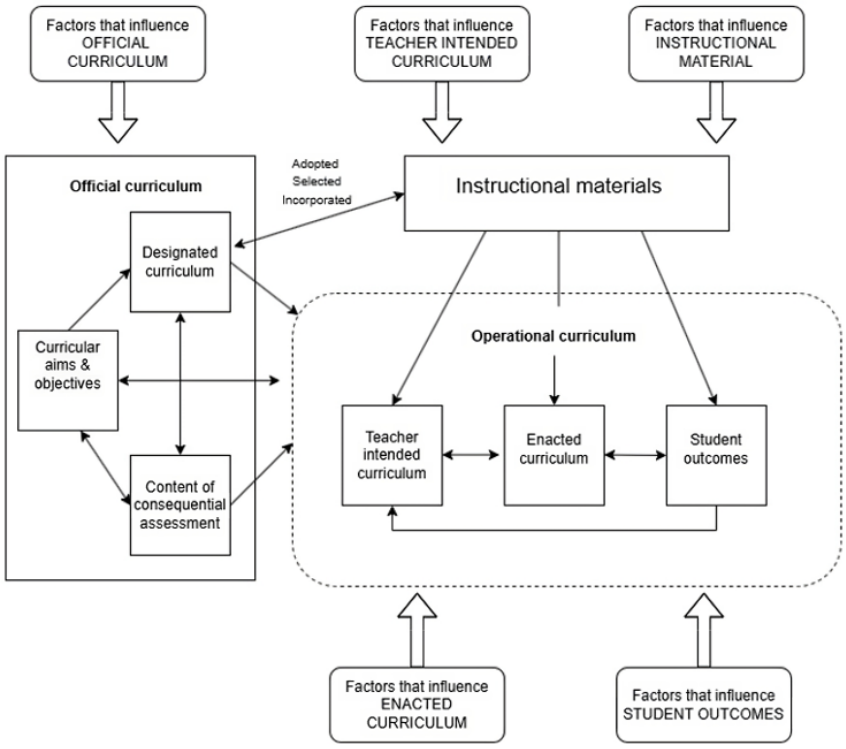

Figure 1. Remillard and Heck's (2014) model of the curriculum policy, design, and enactment system (Source: Remillard & Heck, 2014, p. 709)

2018). The design perspective assumes that adaptations to curriculum materials are inevitable, and modifications can be either productive or limiting for students' learning (Roth McDuffie et al., 2018a). This study delves into a teacher's engagement with curriculum materials, particularly textbooks, over a nine-year period using a longitudinal case study approach.

THEORETICAL FRAMEWORK

Teachers as Designers of Curriculum within a System

Among all curriculum materials, the textbook has always had a prominent role in teaching and learning mathematics (e.g., Fan et al., 2013; Rezat, 2024). Textbooks cover the topics students should learn during a given school year, therefore they serve as a form of consensus and support for uniformity within the educational system (Johansson, 2006). In this way, the textbook is used to translate prescribed curriculum and education policy into pedagogy (Pepin et al., 2013). The curriculum itself is often conceptualized through the tripartite curriculum model, which encompasses the intended, implemented, and attained curriculum (Schmidt et al., 1996; Valverde et al., 2002). Specifically, the intended curriculum outlines the objectives and goals of the educational system; the implemented curriculum consists of the actual lessons and activities conducted in classrooms; and the attained curriculum measures the knowledge and achievements gained by students. According to Valverde et al. (2002), this model serves as a foundational framework for modelling educational opportunities in school mathematics and science. In this context, textbooks function as practical templates for classroom action, ensuring alignment with the requirements of the intended curriculum. In light of this, Valverde et al. (2002) proposed a revised model with the textbook as a fourth component, the potentially implemented curriculum, bridging the intended and implemented curriculum.

However, in many countries mathematics textbooks are approved by local or state governing bodies, making them part of the intended curriculum (Pepin et al., 2013). This is one of the reasons behind Remillard and Heck's (2014) decision to expand the curriculum model. The main idea was to encapsulate the enactment process. Using the tripartite model as a starting point, Remillard and Heck (2014) designed a model of curriculum policy, design, and enactment systems, with a focus on the official curriculum and the operational curriculum (see **Figure 1**).

The official curriculum is akin to Schmidt et al.'s (1996) intended curriculum, but it is much broader: it includes not only curricular aims and objectives but also the designated curriculum and consequential assessments. Curricular aims and objectives are the specified learning expectations and outcomes often set or adopted by a national body, state or province, or school system. The content of consequential assessments refers to entrance or exit exams or tests employed as part of an accountability system; they are not typically intended as curriculum documents but indirectly specify competencies students are expected to acquire and, as such, represent intended outcomes. The designated curriculum refers to specific instructional plans authorized by educational governing bodies. Its purpose is to direct instruction towards the official curricular goals, offering a level of detailed instructional specificity that broader curricular objectives and assessment content cannot provide. The designated curriculum could be either an adopted mathematics textbook or one that has received approval. According to Remillard and Heck (2014), the designated curriculum takes the place of the textbook as the mediator between the intended and implemented curriculum.

The operational curriculum encompasses what actually occurs in practice during the enactment process. This part of the curriculum corresponds to the implemented and attained curriculum of the Schmidt et al. (1996) curriculum model. The operational curriculum includes teacher-intended curriculum, the enacted curriculum and student outcomes. Teachers create the teacher-intended curriculum by combining the designated curriculum with other materials to design instruction. This curriculum includes the interpretations and decisions that teachers make in order to envision and plan instruction. For instance, lesson plans, created by teachers, can be used as artefacts of a teacher's intentions. When designing curricular plans, teachers frequently use instructional materials that are not part of the official curriculum. The interaction between teachers and students in relation to the tasks in each lesson and the lessons that make up a unit of instruction represents the enacted curriculum. The teacher's intended curriculum, the instructional materials used, the students, various contextual factors, and the teacher's ongoing responses to these variables directly influence the enacted curriculum (Remillard & Heck, 2014). Teachers are likely to make revisions in their plans as the curriculum is being enacted. Student outcomes refer to student attainment or learning, but encompass other components such as students' identity, attitudes, motivation, and peer relations.

In this model, instructional materials are resources designed to support or supplement instruction. Instructional materials are situated outside the official and operational curriculum. However, instructional materials are considered to be part of the official curriculum to the extent that they are integrated into the designated curriculum through authorized selection or adoption processes (Remillard & Heck, 2014).

Using Remillard and Heck's (2014) model, one can position the teacher as a curriculum designer within the larger educational system (Roth McDuffie et al., 2018a). For the purpose of this study, the model will be used to examine how the teacher translates the official curriculum into the teacher-intended curriculum.

Teacher Noticing in Mathematics Teaching and Learning

The translation of the official curriculum into the operational curriculum becomes extremely important when linked to the notion of curricular noticing (Males & Setniker, 2019). Curricular noticing is an interactive process where teachers engage with curriculum materials through phases of attention, interpretation, and response (Dietiker et al., 2018). This approach is a subset of the broader concept of 'noticing' in teaching, which is essential for analyzing and enhancing teaching practice (Roth McDuffie et al., 2018b; Sherin et al., 2011). Despite slight variations in definitions among scholars, the essence of noticing in teaching revolves around teachers' observations, interpretations, and subsequent decision-making processes (Jacobs et al., 2010; Sherin et al., 2011; van Es, 2011). It is an active, dynamic process, molded by teachers' prior experiences and predispositions (Erickson, 2011). Sherin et al. (2011) emphasize that this process is interconnected and cyclical, significantly impacting student learning outcomes. Developing expertise in noticing goes beyond mere time and exposure; it demands intentional effort to focus attention and build interpretive frameworks (van Es & Sherin, 2008)

Integrating these ideas, Dietiker et al. (2018) developed the curricular noticing framework, which aims to describe how teachers recognize opportunities within written or digital curriculum materials, understand their affordances and limitations, and use strategies to act on them. Here, curricular noticing is a process that

involves sets of skills that unfold in the following phases: curricular attending, curricular interpreting and curricular responding:

- Curricular attending denotes "viewing information within curriculum materials to inform the teaching and learning of mathematics" (Dietiker et al., 2018, p. 525). This concept includes skills involved in searching, looking, locating, surveying and other ways of visually taking in materials prior to their interpretation. Curricular attending includes all aspects of curriculum materials: mathematical activities, mathematical content and strategic teaching advice.
- Curricular interpreting relates to teachers making sense of what they attended in the material. This concept includes connecting the ideas within the curriculum materials with the teachers' mathematical knowledge for teaching, making sense of what has come before and what will come afterward. Understanding the design of curriculum materials is an underlying factor in curriculum interpretation (Choppin, 2011).
- Curricular responding denotes making decisions based on the interpretation of curriculum materials. Here teachers decide what they want to do and how they want to do it. This is aligned with their interpretation of curriculum material and with their experience. The teacher might want to offload the material, adapt it, or can choose to improvise (Brown, 2009). Curricular responding includes both planning and enacting (Parrish et al., 2023).

The process of curricular noticing is not linear but rather iterative (Dieteker et al., 2018; Males & Setniker, 2019). It starts with curricular attending, as teachers can only interpret what they have noticed. The teacher's response to their interpretation might prompt them to revisit the attending phase, thus creating a continuous loop of engagement with the material.

The curricular noticing lens has proved to be extremely effective. Roth McDuffie et al. (2018b) investigated the practices of middle school teachers who use common core state standards when planning, implementing, and reflecting on math lessons. They found that teacher perceptions significantly influenced curriculum implementation and adaptation, leading to very different classroom enactments. Males and Setniker (2019) studied how prospective teachers' (PSTs) attention to curriculum varied while planning with two different sets of materials. Employing eye-tracking technology, they found that PSTs switched more rapidly between student and teacher materials when using non-embedded formats, indicating that the presence of embedded versus non-embedded teacher support significantly influences their focus. A more recent study by Parrish et al. (2023) examined task launch among PSTs. The lens of curriculum noticing was used to describe how PSTs interpreted the task and how they responded to their interpretations as they planned to elicit and develop their students' understanding of key aspects. Furthermore, de Guzman and Adamos (2020) used curricular noticing to examine the epistemological features of Philippine secondary school mathematics textbooks. The construct of curriculum noticing afforded the development of a textbook model which revealed how writers frame the mathematics curriculum in the design and development of textbooks.

Types of Curriculum Programs

The design of a curriculum program influences teachers' practice (Roth McDuffie et al., 2018b). The curriculum program is the whole set of resources assembled to guide teaching and learning. Such a program typically includes materials for students, such as textbooks, and for teachers, such as a teachers' guide, as well as additional didactical materials (Van Steenbrugge & Ryve, 2018). Choppin et al. (2015) characterized curriculum programs as either a delivery mechanism (DM) or a thinking device (TD); curriculum programs can be classified as DM if the content was developed from an expert performance perspective and delivered to novices. In contrast, curriculum programs are classified as TD when the primary goal is to elicit student thinking and provoke interactions that lead to understanding. This is in line with current research on mathematics teaching and learning. Roth McDuffie et al. (2018a) detected that teachers' interpretations and planning decisions varied depending on the curriculum type and personal approach to teaching. Teachers working with a DM-designated curriculum noticed curriculum from a direct approach to teaching (the teacher maintains primary intellectual authority along with the textbook by presenting an objective for a lesson, demonstrating how to complete problems, scaffolding students' practice, and evaluating to correct students). On the other hand, the teachers working with a TD curriculum noticed and organized their teaching to provide

students with opportunities to assert and justify claims, engage in carefully designed high cognitive-demand tasks, and sequence activities in ways that position students as autonomous learners. Thus, by examining the characteristics of the curriculum program, one can understand the influence it has on teacher's noticing.

Rationale of the Current Study

This study attempts to at least partially address the lack of longitudinal studies in the field of mathematics education. Moreover, investigating how teachers interact with the official curriculum and other curriculum materials over time is crucial for understanding educational dynamics. Specifically, how teachers attend to, interpret, and respond to these materials to craft the teacher-intended curriculum evident in their lesson plans. This process is not static; it involves ongoing reflection and adaptation to align the curriculum with the unique needs of students and broader educational goals (Tran & Reid O'Connor, 2023). As teachers move towards implementing this tailored curriculum, they not only apply various materials and strategies but also engage in professional development and continuous learning opportunities (Trouche et al., 2023). This research aims to understand how these interactions affect teachers' practice, particularly through the lens of curricular noticing – the ability to observe and respond to curriculum-related issues. Therefore, the following research questions were posed:

How does a teacher's curricular noticing change over time and why? Particularly

(a) How does a teacher's curricular noticing change over time in relation to the official curriculum?

(b) How does a teacher's curricular noticing change over time in relation to teaching practice?

METHODOLOGY

Participant and Research Design

The study presented in this paper was conducted in Croatia, and covers four time points in 2013, 2017, 2020 and 2022. Using a longitudinal case study design, we delve into the professional life of a female mathematics teacher, referred to as Sonia, who teaches at a lower secondary school in a Croatian city, covering grades 5-8. Sonia participated in an earlier study (Jukić Matić & Glasnović Gracin, 2015), and was willing to participate in follow-up studies. At the beginning of the study in 2013, Sonia had 12 years of teaching experience. From 2013 to 2017, she consistently employed the same textbooks. Following the change of curriculum in 2019, she switched to a different publisher, and continued to use those textbooks through to the later phases of the study in 2020 and 2022.

Longitudinal case study provides valuable insights into complex phenomena (Saldaña, 2003; Yin, 2014). The flexibility of this methodology allows for a multifaceted exploration of the matter in question, using a variety of tools and techniques. Notably, longitudinal case studies, which extend over significant periods of time, are particularly adept at addressing the critical *why* and *how* questions in educational research. This specific study spans nine years, adhering to Saldaña's (2003) criteria of extending beyond a year for social studies and at least nine months for educational projects. This kind of duration provides an opportunity for in-depth analysis through repeated interviews, observations, document reviews, and targeted questionnaires (Mills et al., 2010). While acknowledging the difficulty of generalizing findings from case study research, careful analysis and correlation with existing research may justify broader generalizations (Mills et al., 2010). Furthermore, the study contains detailed descriptions to facilitate the relevance and applicability of its comparisons for other contexts (Kvale & Brinkmann, 2009).

Official Curriculum

Following Remillard and Heck (2014), textbooks and the national curriculum are considered to represent the official curriculum. The Ministry of Science and Education (MSE) establishes and approves the mathematics curriculum in Croatia, which governs both primary and secondary education. In 2013, the mathematics curriculum included topic and content outcomes for each grade, but not the sequencing (Ministry of Science, Education and Sport [MSES], 2006). Teachers in primary schools were required to develop annual and monthly plans that delineated the sequence of teaching topics and units, the number of hours to be devoted to a topic, and student activities for each grade they were teaching (MSES, 2014). In 2019, schools

Table 1. Questions for interview

Category	Lesson planning
Lesson planning	Describe how you prepared the mathematics lesson.
Teacher guide	Did you use the teacher guide in lesson preparation? What did you look for in the guide? How did you use it?
Textbook elements	What did you attend to in the textbook (definitions, language, symbols, sequence, didactical approach, worked examples, figures)? Explain why.
Other materials	Which other materials did you use in planning? What did you attend to there? Explain why.

implemented the new mathematics curriculum. This curriculum includes general mathematical competencies, content outcomes and topics for each grade, and suggestions for evaluating students' knowledge levels (MSE, 2019a). However, the order of topics and content outcomes are not specified. Consequently, teachers are required to develop an annual plan for each academic year and grade that they teach, which specifies the order of topics and the content outcomes outlined in the national mathematics curriculum and the number of hours to be devoted to a particular topic (MSE, 2019b).

Textbooks, which are mandatory at all school levels, must conform to the national mathematics curriculum and fulfil specific standards. These standards include scientific, pedagogical, psychological, and didactic-methodological criteria, as outlined in the national textbook legislation (Act on Textbooks and Other Educational Materials for Primary and Secondary School, 2018). Textbooks are approved by the MSE, but teachers in Croatia are granted the autonomy to select textbooks from the approved options, with the chosen textbooks typically being used for a minimum of four years or until the MSE sanctions a new set of textbooks. A notable change was made in 2017, the textbooks themselves remained largely the same but the teacher guides accompanying them were revised to place greater emphasis on active teaching strategies (Jukić Matić & Glasnović Gracin, 2019). During the period of this study, in 2019, the national mathematics curriculum underwent changes that consequently impacted the textbooks used in schools.

The teacher guide is not a required component of the textbook set, which means that the textbook publisher may create a guide to accompany the textbook but is not required to do so. Moreover, unlike textbooks mandated by the Act on Textbooks and Other Educational Materials for Primary and Secondary School (2018), the inclusion of the teacher guide in the textbook set does not obligate the teacher to use or adhere to it. Furthermore, the publisher's website can host the teacher guide without requiring approval from authorities. Thus, we do not consider the handbook to be part of the official curriculum.

Data Collection

Data collected for this study revolve around the same topic – parallel lines and pairs of angles – in all four time points in 2013, 2017, 2020 and 2022. The content outcomes for this topic did not undergo any changes in the new mathematics curriculum (MSE, 2019a; MSES, 2006). Data were gathered from teacher observation during lesson planning, individual interviews with the teacher after lesson planning, and an analysis of the lesson plans created at the designated time points.

At each time point, an observer was present in the classroom to observe the teacher during the lesson planning phase but did not participate in the planning process. The observation focused on the quantity of materials utilized by the teacher during the lesson planning, the curriculum material that was employed, whether the teacher revisited the material, the frequency of her revisits, when revisiting occurred (which material was previously used), and the number of times she did so. The notes were recorded in the observation table that had been previously designed, with the aforementioned categories. Additionally, a 'miscellaneous' category included instances where the teacher expressed an opinion about specific curriculum materials to herself.

The interview was conducted following the lesson planning session at each time point. These interviews offered the teacher the opportunity to express her opinions on a variety of topics such as the integration of technology in educational environments, the adoption of digital textbooks, and ongoing professional development. The interviews were semi-structured, allowing for flexibility in response while still adhering to the primary objectives of the study, as delineated in **Table 1**. The main purpose of the interview was to clarify

Table 2. Characteristics of DM or TD curriculum programs

DM characteristics	TD characteristics
Problem-solving steps and procedures are described or provided through example, before students begin working on problems.	Problem contexts serve as the basis of exploration for multiple lessons
Problems may be set in context, but the contexts vary with each problem and prompts do not encourage students to reason analogically about the mathematics.	Students explore a problem and/or mathematical concept before concepts, procedures, and/or mathematical terms are formalized.
Formal definitions are presented before students use terms or constructs associated with specific terms, and precise use of language is considered essential to developing understanding.	Representational fluency is promoted through work with individual representations and the connections among representations.
Group work and/or seat work are used primarily to practice problems demonstrated by the teacher.	Mathematical practices, particularly problem solving, reasoning, and argumentation, are considered essential in teaching and learning
	Grouped work is collaborative, used for high level tasks, and often involves a group product and/or presentation.

the lesson planning process that was observed, investigate the teacher's rationale for utilizing specific curriculum materials, and acquire information about her perspectives and experiences. Interviews were audio-recorded.

The data in the study also included several curriculum materials at each time point: textbook content, teacher guides, national mathematics curriculum, and the teacher's written lesson plans. The focus was the topic of parallel lines and pairs of angles. The written lesson plans are indicative of the operational curriculum, specifically reflecting the teacher's intended curriculum (Remillard & Heck, 2014). The comparison of elements from both the official and operational curricula, augmented by data from observations and interviews, facilitated an investigation into changes in the teacher's curricular noticing over time.

Data Analysis

Qualitative content analysis was used to analyze obtained data. Content analysis is a method that may be used with either qualitative or quantitative data; furthermore, it may be used in an inductive or deductive way (Elo & Kyngäs, 2008). In this study, both the deductive content analysis and inductive content analysis were used as follows.

Using deductive content analysis, the textbook content for the topic *Parallel Lines and Pairs of Angles* was analyzed to determine if it has DM characteristics or TD characteristics. For this endeavor, we used the analytical framework presented in **Table 2**, developed by Roth McDuffie et al. (2018b). In 2013 and 2017, the teacher used mathematics textbooks published by Profil, but these two editions have some minor differences, and in 2020 and 2022, the teacher used textbooks published by Alfa. These two editions do not have any differences; therefore, the textbooks from 2013, 2017, and 2020 were examined. Categorizing textbooks as having TD or DM features can help in understanding how teacher's curricular noticing changed.

In order to ascertain the curriculum materials that were used during lesson planning, the observation reports were examined. Using inductive content analysis, codes were generated that correspond to each component of the material used during the analysis of the first observation report. These codes were supplemented with the new codes, i.e., newly identified materials, during the examination of the other observation reports. Then codes were organized into subcategories, which are illustrated in **Table 3**. Subcategories were subsequently combined into four categories: plan, textbooks, teacher guides, and other materials. Each interview was transcribed and analyzed to identify the curriculum materials that were mentioned and/or utilized. The categories in **Table 3** stayed the same after this process. In order to verify the reliability of the data collected through the observation and interview, the teacher's detailed written lesson plans were compared to the curriculum outcomes, textbook content, and teacher guide (for example of lesson plans, see **Appendix A**).

Table 3. Used/mentioned curriculum materials

Category	Subcategories
Plan	The national curriculum, annual plan, monthly plan
Textbook content	Textbook as a whole, definitions and rules, worked examples, exercises, exam preparation
Teacher guide	Guide as a whole, lesson plans, activities
Other materials	Older lesson plans, older textbooks, other textbooks, ICT tools/applications, colleague's materials, professional journals, material from professional development workshops, online platforms

The next step in data analysis involved deductive content analysis using the following codes: attend, interpret and respond. The codes follow from the curricular noticing framework (Dietiker et al., 2018), and their explanations are, as follows:

- **Attend:** Engaging visually with materials before interpretation, including searching, looking, locating, and surveying.
- **Interpret:** Engaging in the analysis, critique, and evaluation of ideas and approaches, as well as comparing and contrasting tasks, curriculum materials, strategies, and curriculum outcomes.
- **Respond:** Making decisions to offload the material, adapt it, or improvise using personal knowledge.

The codes attend, interpret, and respond were employed to analyze the transcribed interviews and observation reports. The reasons for the use or non-use of specific curriculum materials are documented when the code 'interpret' is employed. The lesson plans were exclusively coded with the code 'respond'.

The categories from **Table 3**, along with the text segments coded as attend, interpret, and respond, were placed into a matrix for chronological analysis. This approach allowed us to track the teacher's curricular noticing in different years. In the final phase of the data analysis, the conclusions were validated through a re-examination of the lesson plans, observation reports and interview data, ensuring a comprehensive and substantiated analysis and resulted with diagrams of curricular noticing presented in the *Results* section.

Ethical Considerations

The study presented in this paper adheres to ethical standards to ensure the protection of the participant, maintain the integrity of the research, and comply with all relevant legal and ethical guidelines (British Educational Research Association [BERA], 2024). Informed consent in the study was obtained from the participant at each time point. At each time the participant was fully informed about the purpose of the research, the procedures involved, and her right to withdraw without any negative consequences. The teacher was assured that her participation was voluntary and anonymous. To protect the identity of the participant, a pseudonym (Sonia) is used throughout the study. Any identifiable information that could reveal the identity of the teacher or the school has been omitted or anonymized.

RESULTS

Features of Textbooks

All three textbooks analyzed have DM features (for instance, see **Figure 2**). Specifically, each lesson begins with formal definitions for key terms. In addition, worked examples or models for procedures are provided at or near the beginning of the lesson. Tasks are sequenced from lower to higher levels, with repetitive skill practice coming before tasks that require the application of those skills in context. The tasks are mainly well-defined problems. More complex problems allow students to choose from multiple known approaches.

Interaction with Official Curriculum

The analysis of the observations, interviews, and the teacher's written lesson plans revealed a change in the teacher's curricular noticing over time, particularly in relation to the use of textbooks. In 2013, the teacher's lesson plan was primarily aligned with the textbook content. However, as time passed, the reliance on the textbook for planning diminished. By 2022, the textbook primarily served as a resource for exercises. In contrast, the mathematics curriculum has played a consistent role as a reference over the years. Both the

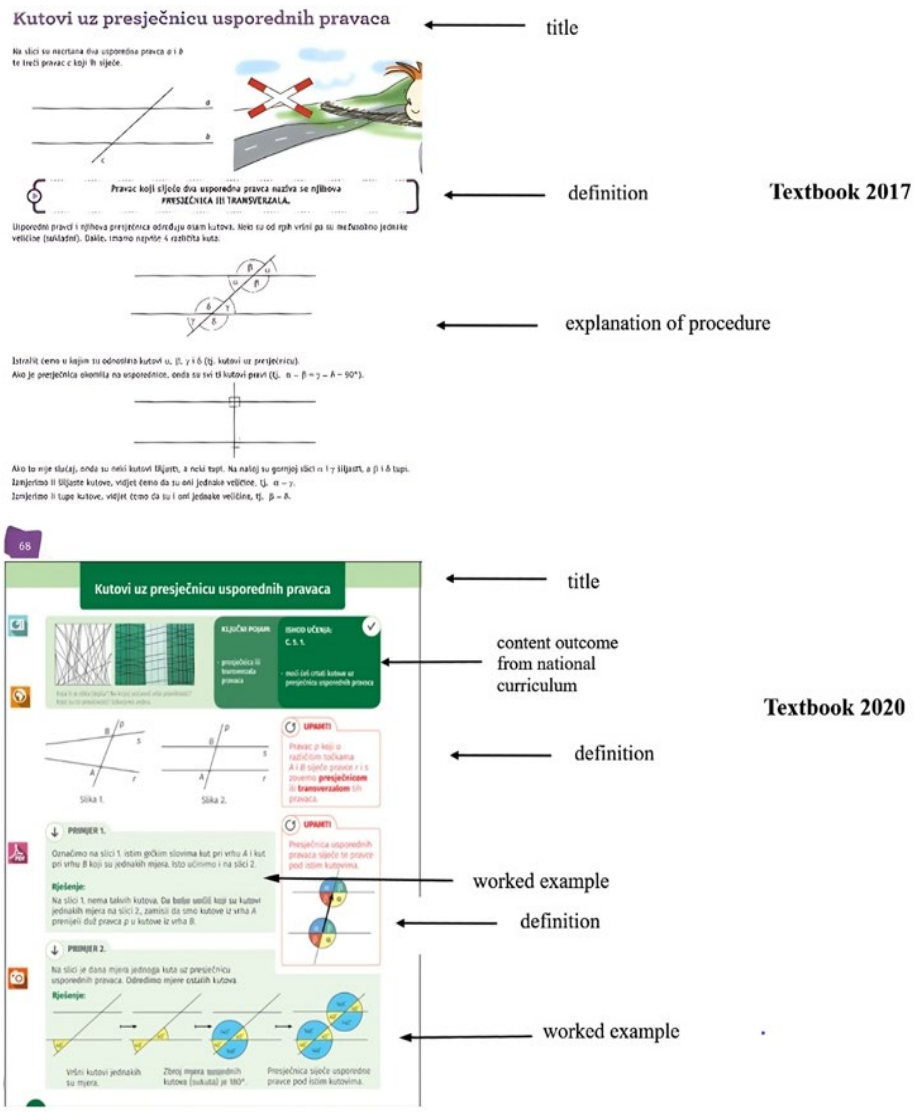

Figure 2. Examples of analyzed textbook content (Source: Author)

Table 4. Responding for the lesson parallel lines and pairs of angles

Lesson elements	2013	2017	2020	2022
Motivation	Other material	Other material	Teacher's own material	Teacher's own material
Activities for knowledge acquisition	Textbook and teacher guide	Teacher guide and other material	Teacher's own material	Teacher's own material
Definition	Textbook	Textbook	Teacher's own material	Teacher's own material
Worked examples	Textbook	/	/	/
Exercises	Textbook/other material	Other material	Textbook	Textbook/teacher's own material
Homework	Textbook	Textbook	Textbook	Textbook

curriculum materials and teaching strategies employed by the teacher evolved over time. **Table 4** summarizes key elements analyzed in the lesson plans: motivation, activities for new knowledge acquisition, definitions, worked examples, exercises for practice, and homework.

The following sections detail the teacher's curricular noticing for each year. For clarity, we have separated attending, interpreting, and responding into subsections, but this does not mean curricular noticing was linear. In reality, curricular noticing is complex and iterative. To fully capture the teacher's engagement with

the curriculum, this structured approach was used to capture what was attended to, interpreted during planning, and responded to in specific lessons.

Curricular Noticing in 2013

Attending

Sonia initiated her lesson planning by consulting the national curriculum and content in the textbook for the relevant lesson, reviewing available activities, definitions, worked examples, and exercises. Additionally, she referred to the teacher guide for the lesson structure and explored other resources, including other approved textbooks and older textbooks, to find motivational elements for her lesson. Her process also involved engaging with online resources from her colleagues, as well as reviewing the exam for the whole unit to ensure alignment with assessment requirements.

Interpreting

Sonia's lesson plan development is linked to her interpretation of the reviewed materials, particularly the official textbook, and how they align with the teaching objectives. Sonia appreciates the textbook lesson's didactic approach for its simplicity and clarity, which she believes will benefit her students and their parents. When evaluating textbook definitions, she considered their comprehensibility for students. She chose to use the textbook's worked examples while thinking about her students and learning at home.

Her use of textbook activities is dependent on whether they are aligned with her goals. She frequently finds the official textbook insufficient for motivation, such as for the upcoming lesson, prompting her to seek more engaging content in alternative sources, such as other textbooks from various publishers or older textbooks, as well as materials from colleagues. Sonia used the textbook for exercises, but she also mentioned that she occasionally uses more difficult tasks from other materials, particularly for advanced students.

The teacher guide was not followed rigidly in her lesson planning but used to validate the direction of the lesson. She also emphasizes the importance of curriculum outcomes in connecting current content with future high school material, often supplementing her lessons with additional content to bridge curricular gaps. Sonia remarked

> I provide the better achieving students with content beyond the current grade's curriculum, as it will be pertinent in high school.

Responding

In 2013, the teacher predominantly relied on the textbook for introducing new content, providing definitions/rules, exercises, and homework. The lesson plan closely mirrored the teacher guide, highlighting structured questioning as the primary teaching strategy (**Figure A1** in **Appendix A**).

Figure 3 captures the process of attending, interpreting and responding Sonia employed when working with curriculum materials in 2013.

Curricular Noticing in 2017

Attending

Sonia began her lesson preparation by consulting the monthly plan she crafted at the start of the year, outlining the goals and content aims derived from the official mathematics curriculum. For this specific lesson, she reviewed the official textbook, focusing on activities for knowledge acquisition, definitions, and worked examples. Additionally, Sonia revisited her notes and previous lesson plans for the same topic, assessing what improvements were needed. She evaluated whether to reuse, modify, or discard these old plans. She also consulted the teacher guide for lesson suggestions and explored various textbooks, including older editions and ones from different publishers, to find diverse exercises. Online resources were also scored for new activities, and she took the curriculum for higher grades into consideration to find connections with her current teaching content.

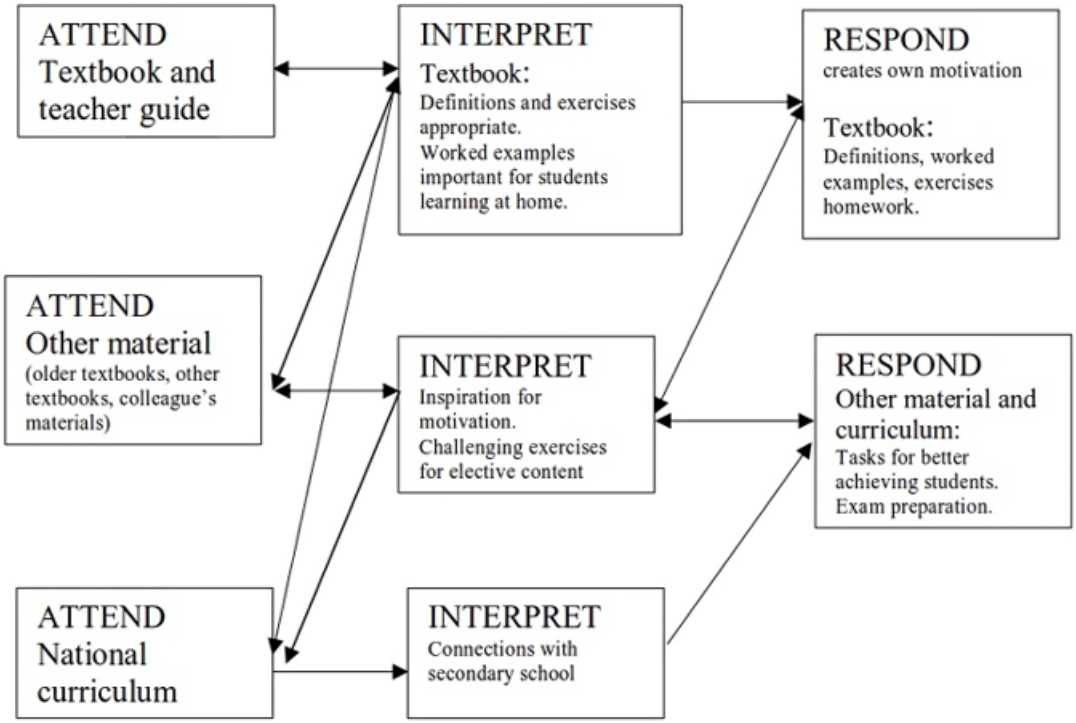

Figure 3. Curricular noticing in 2013 (Source: Author)

Interpreting

Sonia's lesson planning is guided by the goals she wants to achieve by the end of the unit. This guides her choice of activities, tasks, and teaching strategies for the particular topic within a broader unit. Sonia explained,

> Understanding the goals allows me to direct the tasks and materials.

She emphasized the importance of adapting to the unique needs of each generation of students, which necessitated customizing her plans rather than using them as is. While she prefers to use textbook elements such as definitions, worked examples, and exercises to align schoolwork with potential at-home learning, she critically evaluated their language and didactical approach, taking into account the impact on student understanding.

Sonia found the teacher guide particularly valuable for its contemporary activities and inspiration for guided discovery learning. However, she adapted the activities to suit her students' needs and her educational objectives because she found the suggested lesson plan overly ambitious.

> I take the parts of the lesson plan that I find most effective and adapt them to the content and requirements that I deem important.

She appreciated the integration of information and communication technologies (ICT) in the teacher guide, such as presentations and GeoGebra worksheets, but not for this particular lesson, so she decided to use GeoGebra activities she found on her colleague's website which promoted guided discovery learning. Moreover, Sonia decided to use exercises that differ from those in the textbook, believing that a variety of tasks is crucial for developing appropriate mathematical thinking. She noted that the latest edition of the textbook has improved with the inclusion of open-ended tasks, a change not immediately apparent from its visual presentation. Namely, the didactical approach to new concepts and procedures remained the same, but she noted a change in the exercises section.

For this particular lesson, Sonia incorporated content not explicitly included in the lower secondary school curriculum but deemed essential for upper secondary education.

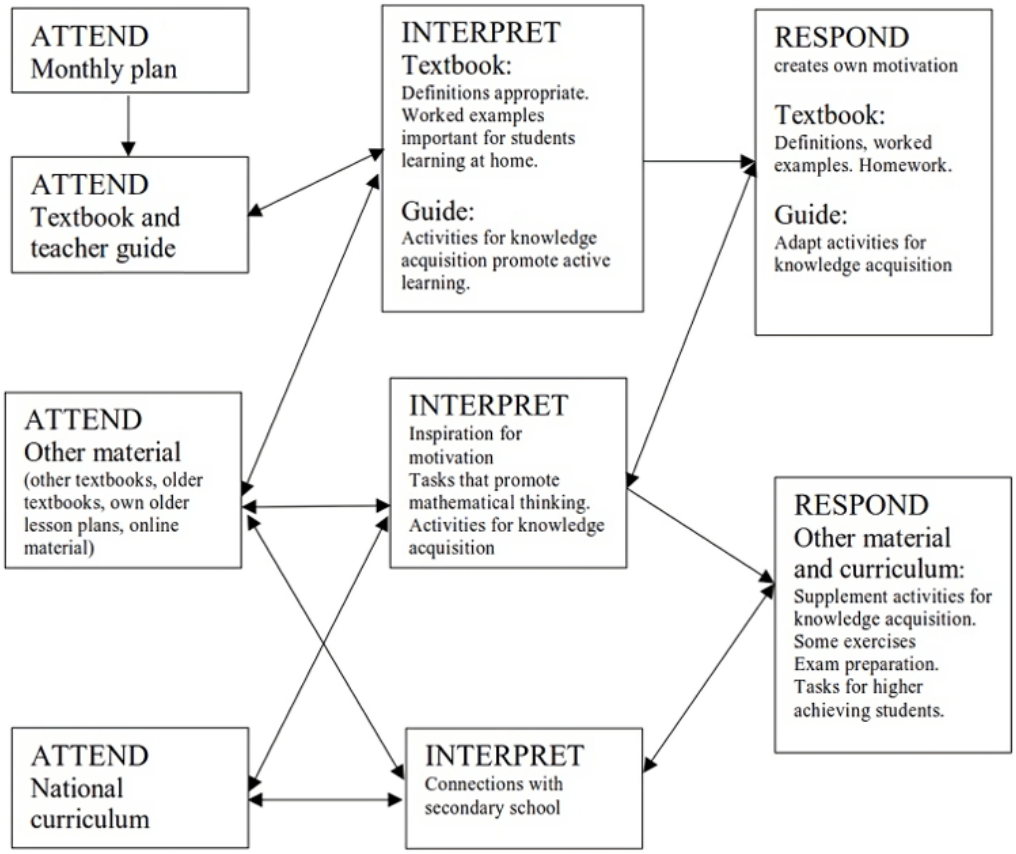

Figure 4. Curricular noticing in 2017 (Source: Author)

> I am preparing students for high school. I'm preparing them [students] to think mathematically (...) We practice the extra content, and they know they will benefit in high school because of it.

Responding

By 2017, there was a notable shift in the teacher's approach. While the central part of the lesson, particularly the acquisition of new knowledge, was adapted from the teacher guide, the teacher incorporated ICT materials from outside sources. The original worksheet focused on exploring drawn angles without requiring conclusions and the teacher modified it to include reasoning and conclusion-making, utilizing the GeoGebra for exploration. The modified worksheet used guided-discovery learning. The definition/rule section still sourced content from the textbook.

Figure 4 captures the process of attending, interpreting and responding Sonia employed when working with curriculum materials in 2017.

Curricular Noticing in 2020

Attending

Sonia began her lesson planning by consulting her annual plan, which she had created at the beginning of the year; the learning outcomes in her plan are based on the national mathematics curriculum for the specific grade. She then checked the official textbook to see if the lesson content was consistent with the outcomes from the annual plan. The teacher evaluated the lesson structure from the textbook, which included motivation, activities, definitions, and exercises. She also looked at other textbooks for more challenging activities, as well as materials from colleagues and professional development workshops with digital activities or hand manipulatives.

Interpreting

Sonia relied heavily on her annual plan to guide her lesson preparation, focusing on achieving the main content outcomes. She explained that this kind of planning helps her orchestrate longer activities like inquiry-based learning. Although she uses the official textbook, she explained that she had reduced her reliance on it over the years, favoring active learning strategies like guided discovery, small inquiries, group work, and ICT integration. Sonia has found that the textbook does not always promote these methods for all topics so, as for this lesson, she creates her own material. She noted that definitions in the textbook for this lesson are not age-appropriate.

> I closely examine the definitions in the textbook and use them if they are skillfully articulated. But these are not.

She scrutinized the textbook worked examples to ensure they aligned with her teaching goals and assessed their relevance to her lessons. She found that the exercises section aligned well with her teaching intentions, especially for at-home learning. She acknowledged challenges with some textbook topics but felt she has successfully navigated these issues. Nevertheless, Sonia believes the textbook approach mainly aligns with her teaching objectives, citing this as a key reason for her choosing this textbook. She explained:

> The introduction of open-ended tasks in the textbook is particularly appealing. Despite its appearance as a 'picture book,' the exercises effectively link reasoning and understanding.

Sonia explained she has moved away from using the teacher guide, and relied on her experience and knowledge, particularly in leveraging ICT for effective teaching and learning. She added she had learned a lot at professional development workshops and believes she can create what she wants on her own. The other existing textbooks serve also as a source for lesson planning because "each textbook has something of its own, something different". Those textbooks give Sonia inspiration for the creation of her own material or, if the intentions behind activities suit her needs, she uses them as given.

> From time to time I see more challenging tasks somewhere, so I reshape and adapt them and give them to students.

Responding

In 2020, the teacher demonstrated further innovation by creating original materials for knowledge acquisition using an inquiry-based learning approach, complemented with ICT use. The lesson was designed to engage students in using GeoGebra for inquiry. Students were expected to record their hypotheses, reasoning, and conclusions on a worksheet to deduce the rule for the angles in question (**Figure A2** in **Appendix A**). This was followed by a planned discussion of the results. Exercises and homework were assigned from the textbook. This lesson plan deviated significantly from the recommendations in the official textbook and teacher guide, which suggested exploration of angles with manipulatives and structured questioning by the teacher.

Figure 5 captures the process of attending, interpreting and responding Sonia employed when working with curriculum materials in 2020.

Curricular Noticing in 2022

Attending

Sonia started her lesson planning process by referring to the annual plan she created at the beginning of the year. This plan outlines the learning outcomes derived from the national mathematics curriculum, tailored to the grade level she teaches. Sonia then reviewed her older lesson plans for this topic, as well as the official textbook. She assessed the structure of her previous plans, which included components such as motivation, various activities, definitions, and exercises. She also reviewed the lesson plans for previous topics to understand the topics covered in class. In addition, she looked through other textbooks and digital

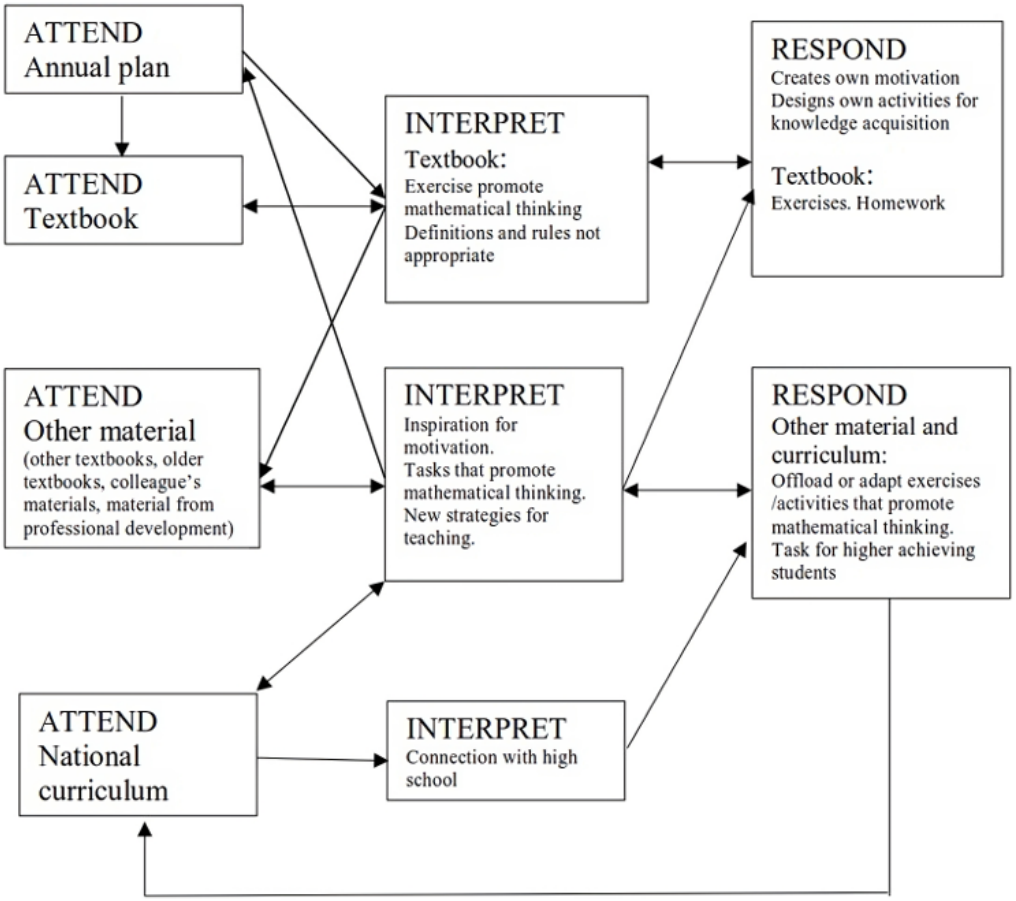

Figure 5. Curricular noticing in 2020 (Source: Author)

educational repositories, as well as professional journals and materials from professional development workshops.

Interpreting

Sonia's annual plan outlines the key learning objectives for the school year, serving as a road map. While this plan serves as a foundation, Sonia clarified that it requires adjustments throughout the year. Namely, different groups of students respond differently, necessitating more time for specific topics. Sonia also reviewed her previous lesson plans, particularly to better understand how she moved from one topic to the next. Old lesson plans serve as a reference point for Sonia, but she frequently finds herself refining them as she develops new ideas. For example, she noted aspects of previous enactments that she did not like, such as the need to connect geometry more closely to everyday life. When designing the main part of her lessons, Sonia looked not only at the official textbook but also at other textbooks and digital educational repositories, looking for different approaches to the topic, however she concluded:

> They [textbooks] have similar approaches to this topic.

Sonia has observed that her students gain a deeper understanding when they discover concepts independently, leading her to decide against using the worked examples and definitions from the textbook:

> I prioritize quality over quantity in my teaching approach. My aim is to encourage students to uncover concepts by themselves (…) Using worked examples would undermine this goal, (…) the discovery process. Students can simply look in the textbook and miss out on the experience of exploration.

Despite exploring various resources, Sonia still finds that the textbook approach generally aligns with her teaching objectives, especially tasks that encourage reasoning rather than mere procedure imitation. While

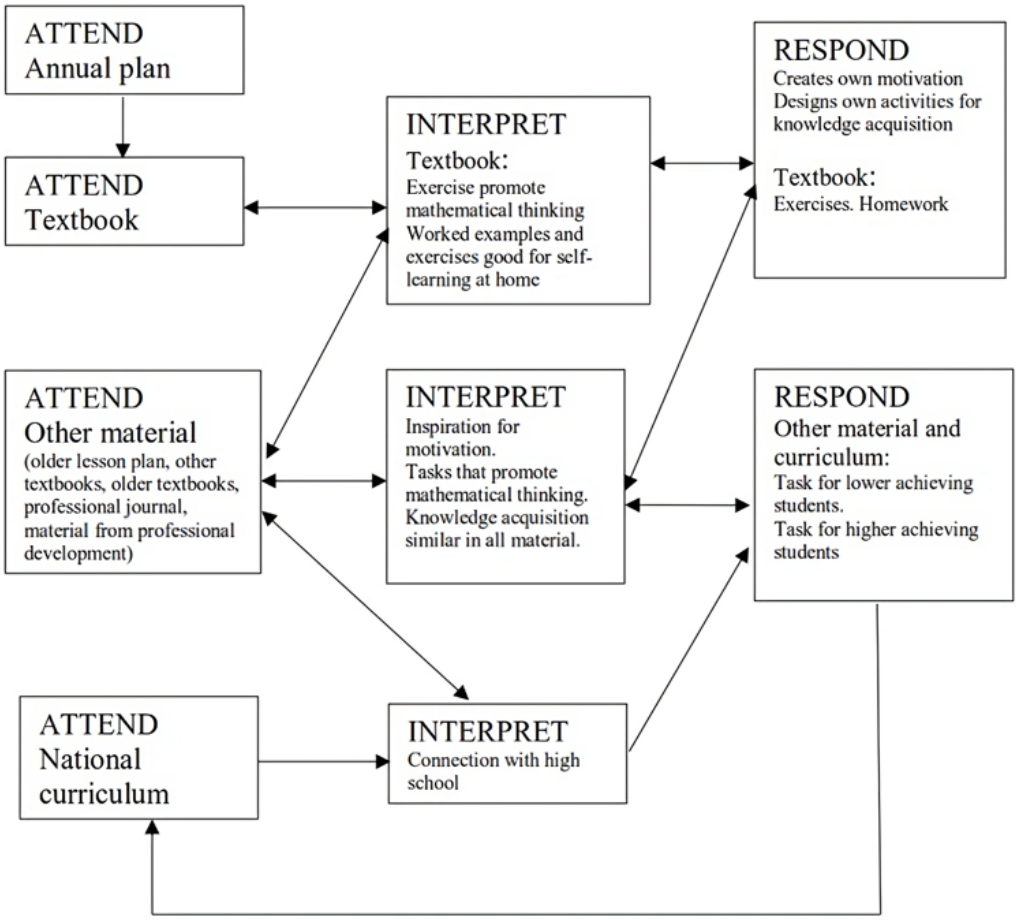

Figure 6. Curricular noticing in 2022 (Source: Author)

reviewing the textbook, Sonia identified suitable tasks for homework, ensuring that students understand the progress of the course. She planned to assign two practice exercises from the textbook, but also intended to create customized tasks in their OneNote class notebook. She explained that this approach allows her to cater to the varied needs of her students – providing more challenging tasks for higher-achieving students and simpler ones for those who require them.

> Some students will go to grammar schools, and they need more of a challenge. I try to provide that for them … I always look [at the national curriculum] what they will need at high school.

Sonia reserved the textbook exercises for self-learning at home, ensuring a balance between guided learning and independent study.

Responding

In 2022, the teacher created a real-life problem involving the concepts of parallel lines and pairs of angles. The lesson was planned as a problem-solving activity using an inquiry approach, where students worked in groups and had the option to use ICT tools like GeoGebra for exploration. The teacher prepared a worksheet for students to document their hypotheses, reasoning, and conclusions to formulate the rule for the angles in question. Although several exercises were taken from the textbook, additional exercises for practice were created in the classroom notebook on OneNote. Homework assignments continued to be based on the textbook. The planned lesson did not resemble the lesson in the official textbook or teacher guide.

Figure 6 captures Sonia's process of attending, interpreting and responding when working with curriculum materials in 2022.

DISCUSSION AND IMPLICATIONS

This longitudinal study investigated the nine-year journey of a mathematics teacher in her interaction with curriculum materials, particularly the designated curriculum in the form of textbooks. The main objective was to investigate how a teacher's curricular noticing changes over time and how this has an influence on their practice.

The findings show a shift from Sonia's reliance on textbooks to a more dynamic integration of various materials, such as digital tools. Although the teacher attended the same textbook elements, she interpreted them in a different manner at each time point (**Figure 3**, **Figure 4**, **Figure 5**, and **Figure 6**). Different interpretations resulted in lesson plans with different activities for the same topic (see **Table 4**). Sonia's interpretation was contingent on her familiarity with the textbooks she used, including an awareness of their affordances and limitations, and her ability to discern which elements (definitions, worked examples, exercises) best suit the lesson being designed. Despite using the textbook differently at the four time periods, the teacher recognized its value for student learning and continued to use it as part of her lessons. It is not an unexpected finding that Sonia attended the textbook at all four points. Many teachers continue to rely on textbooks as a source of guidance, despite the growing importance of other curriculum materials due to the fact that many other sources are more fragmented and concentrated on specific tasks, rather than the development of coherent long-term curricula (Prediger et al., 2021). Sonia was able to interpret the core ideas and suggested approaches in the curriculum materials she attended, consider their practical application, and respond by creating meaningful learning pathways. This ability is critical for teachers in order to use curriculum materials effectively in the classroom (Remillard & Kim, 2017).

Sonia's approach to lesson planning involved examining her own annual/monthly plans which were created in accordance with the national mathematics curriculum. To the teacher, this strategy ensured that the materials used in her classroom supported broader educational goals, including the development of problem-solving and reasoning skills, which were given emphasis in the national curriculum from 2019. Despite the teacher's creation of her own monthly and annual plans, the official national curriculum remained an important guide, particularly in creating a bridge to upper secondary school (**Figure 3**, **Figure 4**, **Figure 5**, and **Figure 6**). Sonia's interpretation of the content outcomes was helpful in organizing, selecting, and designing the activities necessary for the transition to upper secondary school education. Such alignment is crucial as it encompasses anticipated learning outcomes and the proper sequencing of topics, both being essential elements for effective instruction. The importance of this approach is supported by research, such as the studies conducted by Carrillo-Yañez et al. (2018) and Roth McDuffie et al. (2018b). These studies highlight the critical role that a deep understanding of mathematics learning objectives plays in engaging effectively with the curriculum.

The evolution of Sonia's curricular noticing required time, a diverse knowledge base, and a commitment to continuous professional development. The integration of technology and inquiry-based learning in Sonia's practice indicates a transition toward active learning, positioning students as autonomous learners (Choppin et al., 2020). This development is consistent with the current educational efforts that emphasize students' independent thinking and a more profound understanding of mathematical concepts (Smith et al., 2017) and the impact of professional development. Moreover, the inclusion of contextual features in her lesson in 2022 is indicative of her professional growth. Parrish et al. (2023), for instance, show that pre-service teachers often struggle with using contextual features effectively, thereby simplifying the cognitive demands of the tasks they present.

Additionally, the research revealed that Sonia's prior experiences influenced her lesson planning. Sonia made adaptations to curriculum materials, which Choppin (2011) refers to as 'learning adaptations'. These adaptations illustrate the teacher's ability to modify curriculum materials to achieve specific instructional objectives by drawing on prior experiences and provide further evidence of change in curricular noticing, i.e., interpreting and responding to that interpretation.

Implications and Further Research

It is important to acknowledge that this study, which focuses on a single experienced teacher, may not be universally generalizable. Nevertheless, it provides valuable insights into the intricacies of working with

curriculum materials. It contributes to the field by operationalizing the concept of curricular noticing within a longitudinal framework. The study offers empirical evidence that a teacher's practice and comprehension of teaching requirements can be enhanced by sustained interaction with curriculum materials. This is consistent with the concept of teacher noticing proposed by Sherin et al. (2011), which emphasizes the significance of ongoing observation and adaptation in the teaching profession. The implications of this study can be used for professional development programs for teachers. Sonia's transition from direct teaching approach to more student-centered and technology-enhanced teaching, where students explore the mathematical ideas, implies that professional development should not only introduce teachers to new strategies and methods but also provide teachers with the necessary support to navigate the transition over time Additionally, the results support a curriculum design that provides teachers with the flexibility and opportunities to customize learning experiences in accordance with their professional judgment and the needs of their students.

Future research could expand on this study by including a more diverse cohort of teachers and/or incorporating quantitative methods to provide a broader perspective of curricular noticing in different educational settings. Moreover, to deepen the understanding of curricular noticing and its development over time, future studies could focus on observations of the enactment in the classroom. By observing lessons as they happen, a more comprehensive and rounded view of curricular responding, and consequently curricular noticing, could be gained.

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Declaration of interest: The author declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the author on request.

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APPENDIX A

Here we provide example of two lesson plans.

Introduction:
- display the PowerPoint presentation with animation of parallel lines and transversal

Central part:
- define the concept of the transversal of two parallel lines
- repeat the types of angles, the concepts of acute and adjacent angles, and their properties
- emphasize that a maximum of four of the eight angles determined by the two parallel lines and their transversal are distinct
- distribute worksheets (Worksheet 1) from the teacher guide to the students (two groups with two tasks); measure four distinct angles along the transversal
- after the measurements, the students compare the solutions
- through a series of questions, guide them to the realization that acute and obtuse angles are equivalent along the transversal
- repeat the definition of supplementary angles
- state the definition from the textbook that two angles at the intersection of parallel lines of the same type are equivalent, while angles of different types (one obtuse and the other acute) are supplementary.
- use worked examples 1, 2, and 3 from the textbook to demonstrate the application of this rule in tasks.
- students orally solve tasks 1, 2, and 3 from the textbook, applying the definition
- give created worksheet (Worksheet 2)

The final part:
- independently, students complete tasks 4 and 8 from the textbook and verify the solutions by reading them aloud
- assign homework tasks

Homework:
- textbook exercises 5, 9

Worksheet 1
A.

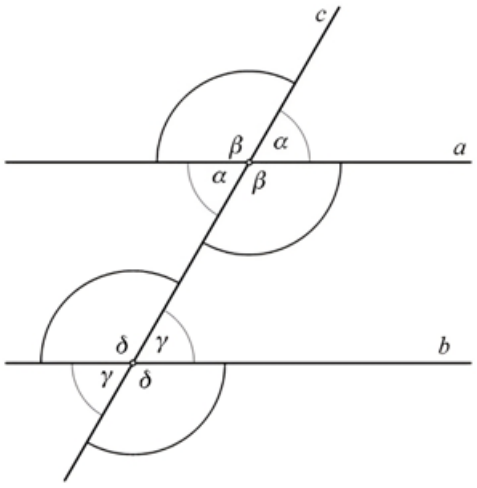
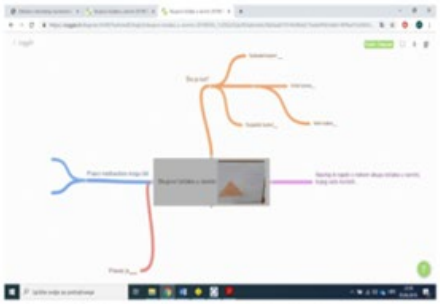


Figure A1. Part of the lesson plan from 2013 (translation by author) (Source: Sonia [anonymized name] and Author)

Introductory part: (5 min)

Activity 1.

Students, individually, fill in the conceptual map and recall some sets of points in the plane: parallel lines, angle, congruent angles, adjacent, acute angles. The concept map is displayedn the Coggle tool.



Analysis of students' solutions: students read their solutions, which other students confirm or correct or supplement. The teacher writes down some student solutions the Coggle tool.

Main part: (30 min)

I assign and explain the goal of the task: to investigate angles along the transversal, using GeoGebra. I demonstrates the use of GeoGebra, according to the tasks, and during the students' work, visits the students and help them. Based on the inquiry, the students draw conclusions and answer the questions.

Activity 2./Activity 3.

Students work individually or in pairs while learning to work in GeoGebra, and while answering the questions. Each student receives a worksheet that guides him through the tasks, and upon completing, the student will detect the concept of intersection or transversal, as well as angles adjacent to the intersection and their properties. The solutions to the tasks are checked in pairs or fours. A demonstration of the solution follows.

So, students will be working work collaboratively, i.e. first we repeat the stages of collaborative work:

1. individual

2. exchange

3. demonstration

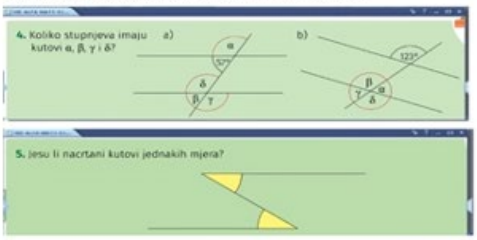
I give a signal when it is time to change a certain phase. It is also an agreement that we perform the demonstration and analysis of the solution of each task after each completed task.

To demonstrate the solution, we will use the student's oral explanation and argumentation of the solution, as well as drawing and writing on the smart board on the given sheet (The sheet is also placed in the One note -class notebook for students who were not in class or who lost the worksheet they received in class)

Final part (10 min)

Activity 4.

Students solve the tasks: 4.a), b), 5., page 75. e-textbook



After analyzing the solutions, by raising their hands, check how many students correctly solved tasks 4.a), b) and 5. Based on the received feedback, adjust the pace of work and, if necessary, provide additional explanations.

Additional Activity 5. (for students who solve the assigned tasks at a faster pace)

Students will independently come up with at least 3 different tasks related to the calculation of angles along the intersection.

Part of the worksheet for students

1. Count and answer how many angles are covered by the transversal on of two parallel lines?

   The transversal of two parallel lines encloses _____ angles.

2. Name the angles using the points that lie on the sides of the angle, and the point that is also its vertex.

   Angles along the transversal are:

   _____.

3. Now that we have established and named the angles at the transversal, think about and ask the question/questions that you could observe and investigate, related to the angles at the transversal.

Figure A2. Part of the lesson plan from 2020 (translation by author) (Source: Sonia [anonymized name] and Author)