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Research Article

Mental calculation achievement according to teaching approach: A study with eye-tracking from a neurocognitive approach

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ABSTRACT

Currently mathematics difficulties in schools are a major problem due to several factors. Some research suggest that mathematics teaching-learning methodology could be one of the causes. As a result, alternative teaching methods to the traditional approach (ciphers-based closed algorithm [CBC]) have emerged, such as numbers-based open algorithm (ABN) method. Some research about this new approach has emerged, including neuropsychological studies. The current study aims to analyze performance and potential cognitive differences in solving a computerized task linked to eye-tracking device, comparing CBC and ABN approaches. 18 5th & 6th graders participants were evaluated through a computerized mental arithmetic task. Nine participants learned mathematics with CBC, and nine with ABN approach. Participants were distributed according to his/her mathematical performance rate in three sub-groups, three students per sub-group: low, medium, and high. The ABN method group obtained a higher overall score in the computerized task (mean $[M]_{CBC}$=16.22; M_{ABN}=17.11), but the differences were not statistically significant (p=.690). However, significant differences have been found in two eye-tracking measures. ABN method group obtained a lower number of fixations average in areas of interest [AOIs]) (M_{CBC}=5.01; M_{ABN}=3.85; p=.001), and a lower pupil diameter average in AOIs (M_{CBC}=4.07; M_{ABN}=3.91; p=.001). This occurred regardless of the participants' mathematical performance. These results suggest that differences between groups were not in task performance, but in cognitive effort spent in solving the task.

Keywords: mental calculation, computerized task, ABN method, CBC method, eye-tracking

INTRODUCTION

Mathematical learning difficulties have an important prevalence, ranging from 1 to 10% depending on the research conditions (Chinn, 2014; De La Peña & Bernabéu, 2018; Lafay et al., 2019). These mathematical difficulties negatively influence classroom learning environment and daily life activities (Aguilar et al., 2015). Academic difficulties in early childhood education might have significant long-term consequences. Students who early struggling with math avoid mathematics in secondary and higher education (Luttenberger et al., 2018). This might limit their employment possibilities. Some research suggests that this problematic leads to personal, social, and even economic consequences (Loos-Sant'Ana & Brito, 2017; Passolunghi & Costa, 2016; Wilson et al., 2015).

Despite the magnitude of the problem, studies on mathematical learning difficulties are scarce compared to studies on other learning difficulties such as dyslexia. Therefore, more research about mathematical learning and cognition is needed. This could help to prevent and to intervene in different types of specific mathematical difficulties, considering multiple factors that can affect mathematical performance (Aguilar et al., 2015; Gilmore et al., 2018). In other words, low mathematical academic performance is conditioned by several factors, including cognitive, affective, cultural, and personal issues (Mello & Hernández, 2019; Szczygieł & Pieronkiewicz, 2022; Vargas, 2016). It is convenient to inquire into curricular content and teaching methodology issues that could influence these factors (Martínez & Valiente, 2020).

Numbers-Based Open Algorithm Method as an Alternative to Traditional Methods

Traditional teaching-learning mathematics methodology mostly used in Spanish classrooms, is ciphers-based closed algorithm (CBC) method. However, considering mathematical learning difficulties rates, different methodological alternatives have arisen. These new proposals argue that the traditional teaching methodology should be revised. One of these alternative approaches is numbers-based open algorithm (ABN) method, whose theoretical bases are found in Piaget's constructivist models, as well as in realistic mathematical education (RME) (Martínez-Montero, 2018). This approach fits Freudenthal (1968) principles, who conceived mathematics as a human activity for organizing or structuring reality (Zolkower et al., 2020). Consequently, ABN method seeks to move away from traditional rote learning and unrealistic mechanisms, employing a methodology more flexible and realistic. **Table 1** summarizes ABN method main characteristics (Canto et al., 2022; Martínez-Montero, 2018; Martínez-Montero & Sánchez-Cortés, 2021; Pérez et al., 2018).

Eye-Tracking and Mathematics

Neuro-learning is defined as a discipline that aims to combine pedagogy, psychology, and neuroscience to explain how the brain works in learning process. In addition, findings on this discipline can be used to optimize the brain functioning. It affects to essential cognitive processes for learning such as attention, memory, language, emotions, etc. (Rivera-Rivera, 2019). Dehaene (2019) spotted neuroscience contribution to mathematics research, by highlights the importance of analyzing brain areas and circuits that are activated during mathematical processing and execution, through neuroimaging techniques. In this regard, tools such as positron emission tomography (PET), functional magnetic resonance imaging (fMRI), electroencephalogram (EEG), and magnetoencephalography (MEG) have been used in recent decades to obtain neuropsychological data. For example, images of human beings' brain activity while reading or calculating. It can be concluded from literature on mathematics processes and neuroimaging, that no brain area deals independently with a cognitive function or task. The connection between billions of neurons, distributed in different cortical and subcortical circuits, allows humans to process information and perform cognitive tasks, including mathematical tasks.

In recent years, scientific interest in the connection between cognitive neuroscience and educational practice has increased, contributing to improve learning in specific areas such as mathematics (Araya-Pizarro & Espinoza, 2020). Consequently, studies on mathematics education using eye-tracking technique have increased, extending its notoriety in research with about twenty relevant studies published annually (Strohmaier et al., 2020). Eye-tracking device is a promising tool for research in mathematics education, as it has potential to identify visual strategies used by students in mathematical tasks. It allows to explore the underlying cognitive processes involved in these mathematical tasks (Carter et al., 2020; Schindler & Lilienthal, 2018). The eye movements registered by eye-tracking device helps to identify where the subject's interest and attention is focused (Behe et al., 2015; Van der Laan et al., 2017). This technique also contributes to analyze underlying cognitive processes and strategies used to solve a task. It requires identifying and recording the participants' eye movements, and then making inferences about their cognitive processes while performing the task (Al-Azawi, 2019; Hessels & Hooge, 2019; Lahey & Oxley, 2016; Schroeder et al., 2015).

Different eye-tracking measurements can be grouped into two main types:

1. Fixations, which refer to eye fixed states at specific points. According to the eye-mind assumption (EMA) hypothesis, formulated by Just and Carpenter (1976), what is visualized with the eyes is also processed

Table 1. ABN method main characteristics

Main character	Definitions
Based on numbers	ABN method works with numbers, while the CBC does it with ciphers*. A thorough understanding of numbering system is essential. Calculations are solved from left to right. Mental arithmetic and problem solving is especially trained.
Open calculation	Learning process must be adapted to each student individual pace. Each learner can carry out the steps needed to solve a calculation or problem. There are different ways to solve the same mathematical operation or problem. Different operations are not independent entities. It is possible to alternate the training of two of them with ease. Solving order depends on the specific decomposition into smaller numbers made by the students. There is not a certain order to solve the math problems. It is common to use strategies such as rounding to calculate quickly and easily.
Connection to reality	Calculations are not abstract operations, instead they are linked to real situations. Classroom mathematical problems are contextualized, connected to reality. Connection with reality allows solving mathematical problems through manipulation.
Self-directed learning	The teacher acts as a learning guide. Students direct their own learning process. They have an active role. Students can pose and solve their own problems. Work in small groups is encouraged, as well as reasoning exposition to the class-group. Those students who found difficult to carry out a calculation, could procced by taking additional steps or using easier calculation.
Conceptual learning	Main goal in each activity is real understanding of operations or problems to be solved. Learning always has a meaning; it is never simple memorization. ABN algorithms cannot have been previously memorized. ABN method carries out calculations all the time: never using meaningless memorized instructions or rules.
Affective-motivational factors	Students' positive attitude towards subject is an essential element in learning process. Classroom learning should generate motivation to continue with the learning process. Students should not reject mathematics, as it is an accessible subject.

Note. *Following CBC method, calculations are solved by considering numbers separately, placing them according to their place value, & using carries. This process cannot be modified. For example, 15+17 would be 5+7=12; it gives 2 and I take 1 to the tens; 1+1+1=3; 3 and 2=32. On the contrary, ABN method works with complete quantities with meaning. For example, 15+17 could be solved like this: 15+20=35; 20-17=3; 35-3=32. There are other ways to solve the same operation[1].

at a cognitive level. Therefore, a fixation on a given point would reflect the person is attending to that point or stimulus.

2. Saccadic movements, which refer to a quick and simultaneous movement of both eyes in the same direction, reflecting the shift in visual attention focusses from one stimulus to another (Alemdag & Cagiltay, 2018).

Number of fixations and number of saccadic movements are two subject's overt visual attention level markers (i.e., they are indicators of cognitive effort level and information processing depth) (Hodds et al., 2014). That is, the greater number of fixations or saccadic movements in the visual stimuli presented, the more attentional resources are being invested in them (Dewolf et al., 2015; Sillero-Rejon et al., 2019). Besides, there are other ocular measurements such as pupil size. This has been understood as a stimulus cognitive or emotional weight indicator. Thus, the higher cognitive effort or emotional reaction a stimulus involves, the greater participant's pupil dilation is (Carvajal et al., 2021; Choi, 2017; Muldner & Burleson, 2015).

Finally, studies employing eye-tracking typically define areas of interest (AOIs) (i.e., stimulus areas that are particularly relevant or informative of what is being investigated) (Holmqvist et al., 2011, cited in Hessels et al. 2016). For example, in a mathematical operation picture, AOIs would be numbers, operators, unknown quantity, etc. Following this terminology, we can refer to a specific number of fixations in each AOI per each item. So, eye-tracking data analysis can be exhaustive and specific. In short, studies using eye-tracking devices have a wide research field, considering different age groups, educational settings, teaching methods, gender, specific AOIs, etc. (Hurst & Cordes, 2016; Klein et al., 2014, 2018; Kulke et al., 2016; Michal et al., 2016).

[1] A more extensive details of characteristics of ABN method & its differences with CBC approach exceeds space available in the article. But an in-depth explanation can be found at Canto (2017), Cerda et al. (2018), & Martínez-Montero (2018).

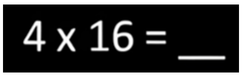

Figure 1. A multiplication item sample included in computerized task (Source: Authors)

Therefore, the research question of this study is whether eye-tracking assessment can reveal potential cognitive differences in mathematical tasks among students following different teaching-learning methodologies (ABN & CBC). This study's general goal is to compare probably differences in mental calculation between two students' groups: a group who received mathematical instruction under CBC method and a group who received ABN instruction. Specific goals are, as follows:

1. To compare students' performance who received CBC respectively ABN instruction in a computerized mental arithmetic task.
2. To contrast the differences between groups in two eye-tracking measurements, inferring potential differences in cognitive effort and emotional reaction to mental arithmetic operations.
3. To contrast potential differences (detected by eye-tracking) among the different mathematical performance levels: low, medium, and high.

METHOD

Participants

Participants were 18 5th and 6th grade (10-12 years old) primary school students. Participants assisted to two schools located in the Province of Cadiz (Andalusia), in two cities with 95.000 to 117.000 inhabitants. Nine participants learned mathematics trough CBC approach (CBC group), and nine with ABN approach (ABN group). Participants have learned mathematics with CBC or ABN methodology since their early childhood education to the present. So, they have been selected and categorized in groups incidentally. Within each group of nine participants, three had low mathematical performance rate, below 78 points (after TEDIMATH-Grands by Nöel & Grégoire, 2015), three average rate (88 to 121 points), and three high rate (over 132 points). Students sample correlates with classrooms diversity in mathematics achievement. Sample size is small because this research is a pilot study included in a larger project. This pilot study focuses on eye-tracking usefulness for studying mathematical cognition.

Instruments

Diagnostic test of basic mathematics skills

TEDIMATH-Grands instrument was used to assess mathematical performance (Nöel & Grégoire, 2015). This instrument is useful to assess general mathematical competence as well as the following specific math abilities: transcoding, numbers positional value, fractions, conceptual knowledge, measurement units use and conversion, perimeter and area calculation, math problem solving, geometry, written and mental calculations, and visuospatial reasoning.

Computer task

A computerized test (composed of 27 mathematical operations, with three ascending difficulty levels), was delivered to check the participants' mathematics performance and their eye-tracking response. The easy level was composed by 12 items, medium level by nine items and difficult level by six items. Mathematics facts included addition, subtraction, multiplication, division, combined operations, and first-degree equations tasks. Different AOIs were defined for each item, including numbers, operators, and unknown quantities. Participants first read the math-facts stimuli on the computer screen; second, they pressed the space key when they know the answer; and third, they verbalized out loud the answer so that the examiner can write it down. Finally, students again indicated their answer, but this time choosing one of the four options displayed in the computer screen. **Figure 1** shows sample of a multiplication item included in the computerized mental arithmetic task. The math task included operations whose answer space is placed to the left, to the right, and in the middle (unknown format). The computer task was designed using *Tobii Studio* software, in order to link it to eye-tracking device.

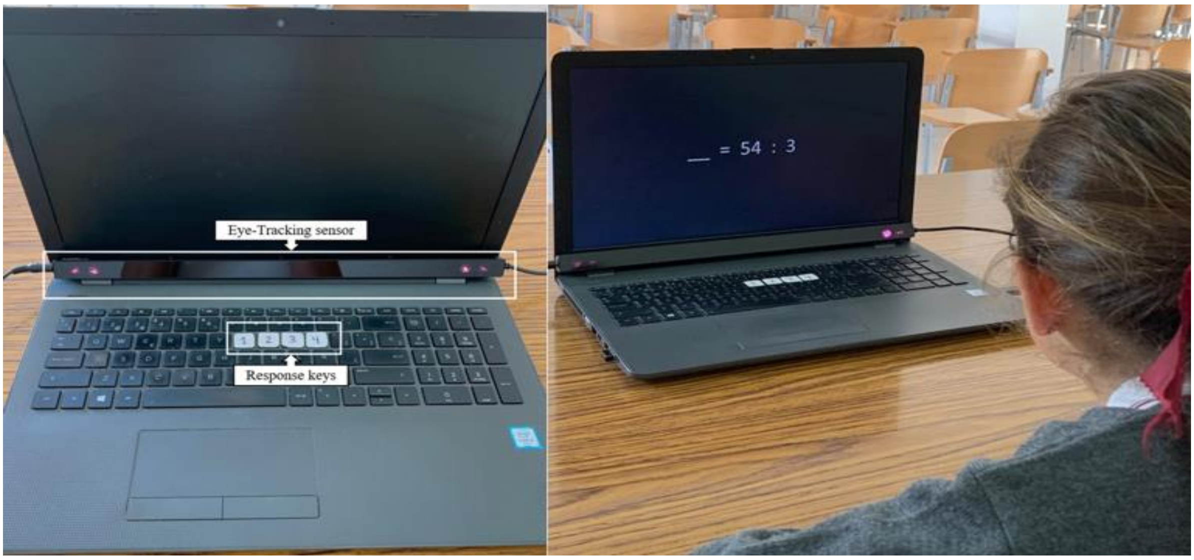

Figure 2. Eye-tracking device: Sensor & response keys (Source: Authors)

Eye tracking device

A non-invasive eye-tracking device (Eye-Tracker Tobii X2-60, Tobii AB, Stockholm) was used. This is a sensor placed on the computer screen that does not require glasses or other external tools to record the participants' eye movements (**Figure 2**). Before starting eye-tracking test, the equipment was appropriately calibrated. This is possible just with a short exercise before the real task. In the calibrating exercise participants must eye-following certain points on the screen for a few seconds. Eye tracking device allows record pupil diameter (in millimeters), and number of fixations in AOIs.

Procedure

In this study, there is no intervention, only evaluation. Participant have been selected incidentally, based on their teaching methodology. We have evaluated and compared cognitive and mathematical variables. Tasks were individually administered, in an empty and silent room, during school time. First, we administered TEDIMATH test for about 90 minutes, and then students received eye-tracking session for about 25 minutes. TEDIMATH test was administered in three separate sessions by trained psychologists. One of the authors–with specific training using eye-tracking device–administered the computer task. Participants were individually seated at a comfortable table in front of eye-tracking computer device. The first five minutes were dedicated to lowering the student behavioral activation and doing calibration. Just before starting the task, the student was warned that during this, he should remain still, not move his hands in front of his face and solve the operations by looking at the screen. Then, participants performed the task.

As for the first specific objective, measurement established to assess mental arithmetic performance was the number of correct answers in mental calculation computerized test. Average of this total score for each participants group was recorded. This score was also compared according to mathematical performance level. As for the second specific objective, two eye-tracking measurements were chosen: number of fixations and pupil diameter, in millimeters. The average number of fixations and the average pupil diameter were considered for all task AOIs and for each group. Both average measurements were compared for each mathematical performance level. According to literature, these parameters are indicators of visual attention or cognitive effort and information processing depth, as well as emotional reaction to stimuli, respectively.

Data analysis

Students' responses were recorded in a data base used by SPSS software. Statistics calculated were descriptive statistics, means comparison, and differences between means significance contrast. Because the sample size was small, data did not follow a normal distribution model, then, nonparametric statistic was used. Specifically, Mann-Whitney U test was calculated. Statistical analyses were computed to compare both groups (ABN vs CBC), and then, to analyze differences between three mathematical performance sub-groups.

Table 2. Mental arithmetic task average score according to performance level & participant group

Performance level	Math-learning group		U	p-value
	CBC	ABN		
Low	10.33	10.67	4.00	.817
Medium	15.33	17.67	.50	.072
High	23.00	23.00	4.50	1.000

Table 3. Average number of fixations according to performance level & participant group

Performance level	Math-learning group		U	p-value
	CBC	ABN		
Low	5.56	4.12	48,795.00	.001
Medium	4.99	3.46	50,182.00	.004
High	4.47	3.96	55,410.50	.415

RESULTS

Results were organized into two sections:

1. First, data related to mental arithmetic task performance (behavioral level). The main data of this analysis level was the average total score for each group and for each performance level.
2. Second, eye-tracking measures data, as indicators of underlying cognitive processes while solving the task (cognitive level).

This analysis level was focused on the average number of fixations and the average pupil diameter for each group and for each performance level.

Mental Arithmetic Tasks Data

Regarding the first section, mental arithmetic task performance data were the average total score for each group and for each mathematical performance level. Results indicates that ABN group obtained a higher overall score than CBC group (mean $[M]_{CBC}$=16.22 ; M_{ABN}=17.11), but the differences were not statistically significant after Mann-Whitney U test (U=36.00 p=.690).

Table 2 summarizes the average score according to mathematical performance level and participant group. ABN group did better than CBC group in low and medium mathematical performance levels. For high performance level, both groups scored the same.

Eye-Tracking Measures Data

Regarding the second section, the two eye-tracking measurements were:

(1) number of fixations in AOIs and

(2) pupil diameter in millimeters.

Results related to number of fixations for the whole test evidenced that ABN group obtained a lower number of fixations in AOIs than CBC group (M_{CBC}=5.01; M_{ABN}=3.85), and these differences were statistically significant after Mann-Whitney U test (U=474,918.00 p=.001).

Besides, **Table 3** shows that ABN students obtain a lower number of fixations than CBC students regardless of performance level. These differences were significant for both medium and low performance levels. However, for high performance level, the differences were not significant. When comparing students with a high mathematical performance level, according to their teaching methodology, ABN group still shows a lower number of fixations, but these differences were not significant.

Results related to pupil diameter for the whole test revealed that ABN group obtained a lower pupil size in AOIs than CBC group (M_{BC}=4.07; M_{ABN}=3.91), and these differences were statistically significant after Mann-Whitney U test (U=244,384.50 p=.001).

Besides, **Table 4** summarizes the average pupil diameter according to mathematical performance level and participant group. It shows how ABN students obtained a lower pupil size than CBC students regardless of mathematical performance level, and these differences were significant.

Table 4. Average pupil size (measured in millimeters) according to performance level & participant group

Performance level	Math-learning group		U	p-value
	CBC	ABN		
Low	4.14	3.98	29,881.00	.011
Medium	4.06	3.95	26,730.50	.016
High	3.99	3.80	27,024.00	.024

DISCUSSION

This study's main purpose was to analyze whether eye-tracking assessment was able to reveal potential cognitive differences in mathematical tasks among students' groups, as a function of teaching methodologies (ABN & CBC) and mathematical performance. This kind of study aims to evidence the benefits of using an innovative methodology for teaching mathematics in the classroom. Reasoned and connected to reality strategies help to reduce complexity and cognitive effort associated with learning mathematics (Aragón et al., 2017; Cerda et al., 2018; Díaz-López et al., 2017; Piñero-Charlo et al., 2022). Regardless of the teaching method, there are several beneficial elements that prepare students to face mathematical problems both inside and outside the classroom. Among them are connection with reality, flexibility, and practice of daily useful skills, such as mental arithmetic (Moreira, 2017).

Data suggests that differences between groups were found in cognitive resources invested in solving mathematical tasks, rather than in the final score on these tasks. Apparently, both students' groups, who follow different mathematics teaching methodologies, had a similar ability to reach mental arithmetic operations answer. In other words, ABN group scored higher on the mental arithmetic task, but the differences were not significant. Despite this, differences between groups have been found at the cognitive level, i.e., differences in cognitive processes underlying mental calculations resolution. These differences have been inferred from two eye-tracking measures, related to visual attention and cognitive effort invested in mental calculations, as well as emotional weight associated with them.

On the one hand, students who learn mathematics through ABN approach showed a significantly lower average number of fixations. This means that those students invest less visual attention and cognitive resources in solving the tasks (Dewolf et al., 2015; Sillero-Rejon et al., 2019). Specifically, cognitive effort invested in solving mental arithmetic facts seems to be lower in students who learn mathematics through ABN method. Nevertheless, although the average number of fixations is lower in ABN group for any performance level, at the high level the differences were not significant. This could be explained because students with a high mathematical performance level invest less cognitive effort, i.e., they minimize and optimize their available cognitive resources, regardless of their teaching-learning methodology (Jaime & Gutiérrez, 2021; Mato-Vázquez et al., 2017). In other words, it seems that teaching method influence could be weaker for students with high mathematical achievement. Therefore, the use of innovative and alternative teaching strategies could be mainly convenient for low or medium math achievement students.

On the other hand, smaller pupil size has been found in ABN group solving the mental arithmetic task, regardless of mathematical performance level. That is, ABN students show a smaller pupil diameter in any mathematical performance level. Pupil size reflects cognitive effort or emotional reaction to a stimulus (Muldner & Burleson, 2015). So, these results indicate that mental arithmetic operations had a lower cognitive and emotional weight for ABN students, including the high mathematical performance level participants. It could be said that students who learn mathematics using a traditional methodology showed a higher cognitive effort and emotional reaction to certain mental arithmetic facts, regardless of mathematical performance level.

This is consistent with previous literature on ABN method (Martínez-Montero, 2018), which places special emphasis on mental arithmetic, considering it an essential skill for mathematical ability and knowledge development. Thus, applying this innovative methodology in the classroom could improve mental arithmetic fluency and make students perceive mental arithmetic as an easier task. These benefits are especially relevant for students who have mathematical learning difficulties or those that consider mathematics a difficult subject. Concurrent recent studies (Alemdag & Cagiltay, 2018; Strohmaier et al., 2020) suggest that number of fixations and pupil size evaluated by eye-tracking, were appropriated visual attention or cognitive effort, and emotional weight indicators. In this regard, emotional weight and cognitive effort associated with mental arithmetic stimuli might be smaller in ABN students, because they had received more training in mental calculation. In other words, ABN participants were trained to deal with different types and formats of mental calculation operations, while CBC group were more proficient with traditional format operations.

Finally, it is worth highlighting eye-tracking device's usefulness when exploring the cognitive processes underlying mathematical mental calculations, which has already been indicated in previous studies (Hernández-Sabate et al., 2016; Schindler & Lilienthal, 2018). Eye-tracking device linked to computerized, well-structured, and visually presented tasks, is a promising tool providing valuable and unique information, such us visual attention or cognitive effort and emotional reaction to mental arithmetic operations. Therefore, future research on mathematical cognition using eye-tracking devices allows to focus on underlying cognitive processes, rather than calculation or problem outcome. In this way, we will be closer to understanding cognitive processes involved in mathematical tasks.

All scientific knowledge arising from these studies would be a useful resource for designing educational interventions. These interventions would have cognitive processes involved in mathematical learning as a central factor. In other words, it's important to systematically train general cognitive factors underlying mathematics specific skills. In addition, mathematical specific abilities training could reduce ineffectiveness in mathematics calculations and problems solving, especially for students who have or may have mathematical difficulties. Educational interventions should include reasoned and realistic strategies, instead of abstract and rote strategies. In short, innovation in research and use of eye-tracking devices should not generate a mere new knowledge accumulation: the main objective should be educational and social transformation. This transformation includes an improvement in mathematical academic performance, but also a more math-skilled society, as well as an increase in employment opportunities in the scientific-mathematical field.

CONCLUSIONS

1. This study indicates that students who learn mental computation through ABN method, showed a better performance than students who learn through CBC methodology, in challenging mathematics computerized tasks. However, these differences have not been shown to be statistically significant. There is a certain ABN group superiority over CBC group when solving mental arithmetic operations, but the superiority magnitude is not enough to be considered significant.
2. Data show that both number of fixations and pupil dilation for AOIs were lower in ABN group. That is, when considering the whole test, differences between groups were statistically significant: ABN group showed a lower number of fixations and pupil size. Considering all task items and all AOIs, average number of fixations and average pupil size was higher in students who learn mathematics trough a CBC method. Results showed that ABN group invested less cognitive effort and emotional reaction in solving mental calculations.
3. There were some differences in these eye-tracking measures considering the three mathematical performance levels. Pupil dilation remained significantly lower in ABN group for any mathematical performance level. The number of fixations was also lower in ABN group for any mathematical performance level. However, no significant differences were found for high performance level students. In other words, high-performance students invested less visual attention or cognitive effort in solving mental arithmetic calculations, regardless of teaching method.

Therefore, as a general conclusion, it can be stated that significant differences between groups have been found at the cognitive level, specifically in emotional reaction and cognitive effort associated to mental arithmetic calculations. Besides, this research has allowed us to verify the usefulness of eye-tracking devices to identify differences in cognitive processes underlying mental arithmetic calculations and to compare students' groups. This study highlights eye-tracking as an instrument with high research potential, which can be used to assess other mathematical skills in addition to mental arithmetic. Moreover, research in mathematical cognition field is still insufficient, so there are many scientific questions to be solved. Eye-tracking measurements and information inferred from them could help to resolve these scientific questions. This paper aims to encourage researchers to continue research on cognitive processes underlying mathematics learning, including studies comparing different teaching methodologies and didactic strategies.

Finally, regarding the study limitations, the sample size can be highlighted. A larger sample size would allow for greater results generalization. However, temporary, and personal resources did not allow a larger research sample to be carried out. In addition, the study has considered only two eye-tracking measures and only one mathematical skill, i.e., mental calculation. A higher number of variables, both mathematical and eye-tracking, could have been included in this kind of study. The use of innovative mathematics teaching methodologies would have a positive effect on emotional and cognitive resources invested in solving mental arithmetic calculations. For this reason, different eye-tracking measures, as well as different critical mathematical skills in addition to mental calculation, are recommended for future research. The valuable information provided by eye-tracking devices would generate a higher scientific yield.

Author contributions: MMP: contributed writing original manuscript, reviewing previous literature in mathematical cognition & eye-tracking field, administering assessment tests, analyzing data, & discussing results; **IMJ:** contributed reviewing previous literature about eye-tracking & mathematics, writing manuscript, & discussing results; **JCP-C:** contributed to this work with his experience in didactic mathematics & alternative methods, writing paper, & discussing results; & **MCC-L:** contributed reviewing previous literature about open algorithm based on numbers method, writing manuscript, & discussing results. All authors approved the final version of the article.

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Ethics declaration: The authors declared that the study was approved by the Comité de Ética de Experimentación No Biomédica y de evaluación de experimentación con Organismos Modificados Genéticamente [Ethics Committee for Non-Biomedical Experimentation and Evaluation of Experimentation with Genetically Modified Organisms] on 29 July 2022 with approval code: Ref. 003-2022. School administrators and teachers were updated about the whole current research procedure and informed consent. Timing was negotiated with the principal to guarantee that all tasks were administered during the school regular time. Then, written informed consent from participants' families was required. After finishing the study, the written report feedback was delivered to the school principal.

Declaration of interest: Authors declare no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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