European Journal of Science and Mathematics Education
Vol. 3, No. 2, 2015, 105-114

Mathematics teaching efficacy among traditional and non-traditional elementary pre-service teachers

Vivian R. Moody[1,*] and Kanita K. DuCloux[1]
[1] Department of Mathematics, Western Kentucky University, Bowling Green, KY, United States
[*]For correspondence: vivian.moody@wku.edu

Abstract:
This study examined the mathematics teaching efficacy of traditional and non-traditional elementary pre-service teachers enrolled in a three-course, three-semester mathematics sequence. Self-efficacy scales were administered at the beginning of the three-course sequence and at the end. The three-course mathematics sequence was taught from a constructivist approach and emphasized a socio-constructivist learning environment in which the pre-service teachers were challenged to construct their own meaning of mathematics. Results showed that for the non-traditional pre-service teachers, self-efficacy levels for teaching elementary school mathematics were significantly different at the end of their enrollment in the three-course mathematics sequence than at the beginning. This study also revealed that for the traditional pre-service teachers, self-efficacy levels with regards to affecting student success regardless of external factors remained the same or were not significantly different, suggesting the need for further research in this area.

Keywords: constructivism, mathematics education, self-efficacy, teacher preparation, teacher beliefs.

Introduction

Many efforts in mathematics education reform (NCTM, 1989, 1991, 1995, 2000) advocate programs with a philosophical basis consistent with constructivist theory for teaching and learning mathematics. Constructivist pedagogy provides opportunities for learning mathematics via participating in meaningful discourse about mathematics, constructing one's own knowledge and understanding of mathematics, and reflecting on processes of solving mathematics problems (Phillips, 1995; Simon, 1995; Steffe and D'Ambrosio, 1995; von Glasersfeld, 1983). In light of these reform efforts, current teacher education programs have emphasized a constructivist learning environment.

The purpose of this study was to investigate the extent to which a three-course mathematics sequence taught from a constructivist approach would impact traditional and non-traditional elementary pre-service teachers' self-efficacy beliefs for teaching mathematics. Research (Battista, 1994; Hart, 2002) has shown that teacher education programs have the propensity to shape teachers' perceptions of what mathematics is important and appropriate to teach and how that mathematics should be taught. Further, several studies (De Mesquita and Drake, 1994; Swars, Hart, Smith, S., Smith, M., and Tolar, 2007; Hart, 2002) have found that pre-service teachers' self-efficacy beliefs affect how they perceive and think about the many ideologies with which they are faced in their teacher preparation programs. Some scholars have argued that teachers' self-efficacy beliefs develop over time (Lortie, 1975) and are resistant to change (Pajares, 1992; Schoenfeld, 1989; Thompson, 1992). Therefore, in order to effect change with regards to self-efficacy beliefs, teacher education programs must provide opportunities for pre-service teachers to examine and reflect on their beliefs.

According to Hart (2002) and Swars et al. (2007), research studies on teachers' beliefs are usually examined in a single methods course. Unfortunately, "if mathematics content is taught by lecture and

the methods courses use a constructivist environment, the experience is diluted, and the chance for change is significantly decreased" (Hart, 2002, p. 5). By contrast, this research examined pre-service teachers' self-efficacy and change in self-efficacy beliefs over a period of three semesters in which the pre-service teachers participated in a sequence of three required content mathematics courses, each taught using a socio-constructivist learning environment.

Theoretical Perspectives

Self-efficacy beliefs. The influence of pre-service teachers' self-efficacy beliefs on their conceptions of mathematics teaching and learning is well-documented in the literature. Although several research studies in mathematics education have been conducted on teachers' beliefs, the authors specifically used Bandura's (1977, 1986) work on self-efficacy to frame this study. Bandura (1977) defined self-efficacy as individuals' viewpoints, perceptions, or judgments of their capabilities to succeed. Self-efficacy beliefs influence how persons think, motivate themselves, and ultimately, behave (Bandura, 1993). With regards to pre-service teachers, self-efficacy beliefs influence how they think about mathematics, how they think about teaching mathematics, and how they respond to the process of *learning to teach* mathematics.

Constructivist pedagogy. Teacher education programs consistent with past reform efforts (NCTM 1989, 1991, 1995, 2000) in mathematics education often emphasize preparation programs analogous to the constructivist paradigm. Although the idea of *constructivism* as it relates to mathematics education has evolved over the years, the present study is built upon one major premise of constructivist pedagogy: *students (in this case, pre-service teachers) are active participants in creating their own knowledge of mathematics rather than passive recipients of information from an authority, such as a teacher or a textbook.* Specifically, in constructivist classrooms, teachers are *facilitators* of knowledge construction, rather than *givers* of information. von Glasersfeld (1990, p. 37) expounds on this notion as follows: *Knowledge is the result of an individual subject's constructive activity, not a commodity that somehow resides outside the knower and can be conveyed or instilled by diligent perception or linguistic communication.*

Central to the ideology of constructivist pedagogy is the notion that mathematical knowledge emerges from social interactions within the learning environment (Vygotsky, 1986). Hence, a socio-constructivist learning environment is predicated on social interactions between *student and teacher* or *student and student*. Notably, these social interactions are influenced by the prior knowledge and learning experiences of the students, as well as the teacher, that participate in the learning environment (Vygotsky, 1986).

Because of the significant philosophical contributions of several scholars such as Dewey (1933), Piaget (1970, 1972), von Glasersfeld (1983, 1990), and Vygotsky (1986), many mathematics teacher education programs focus on creating socially-constructed learning opportunities for pre-service teachers. However, pre-service teachers usually enter their teacher preparation programs with ideologies and self-efficacy beliefs that are inconsistent with this philosophy for teaching and learning. Further, pre-service teachers' self-efficacy beliefs determine how much effort they will initiate, expend, and sustain in the face of aversive experiences (Bandura, 1977), such as participating in a mathematics learning environment that is paradoxical in relation to their prior schooling experiences with mathematics.

Research Questions

The authors used Bandura's work (1977, 1986, 1993) on self-efficacy and behavioral change as a basis for the present study and sought to answer the following research questions:

(1) To what extent does the participation in a three-course mathematics sequence emphasizing constructivist pedagogy influence the self-efficacy beliefs of elementary pre-service teachers (PSTs)?

   a) To what extent does this participation influence the self-efficacy beliefs of *traditional* elementary PSTs?
   b) To what extent does this participation influence the self-efficacy beliefs of *non-traditional* elementary PSTs?

(2) How do the self-efficacy beliefs of *traditional* elementary PSTs differ from the self-efficacy beliefs of *non-traditional* elementary PSTs after participating in a three-course mathematics sequence that emphasized constructivist pedagogy?

Methodology

Participants. Fifty-one elementary pre-service teachers participated in this study. The authors categorized the pre-service teachers as *traditional* and *non-traditional*. The non-traditional group was defined as those pre-service teachers who had received a degree in another field or who had training in another field other than teaching; who had delayed collegiate study at least five years after completing high school in order to attend to raising children; or who could not attend courses prior to 5:00 PM due to working a full-time job or attending to a family full-time. The traditional group was defined as those pre-service teachers who were able to attend classes prior to 5:00 PM and who did not have training or a degree in another field.

The authors categorized 27 pre-service teachers as traditional. The ages in this group ranged from 19 to 24 years old. There were 25 females in this group. Of these 25, twenty-one were Caucasian, one was Bosnian, one was Asian American, one was African American, and one was Native American. The remaining two pre-service teachers in this group were Caucasian males.

Twenty-four of the pre-service teachers were categorized as non-traditional. The ages in this group ranged from 33 to 58 years old. This group consisted of 20 Caucasian females, one African American female, and three Caucasian males.

Mathematics sequence. The elementary pre-service teachers in this study participated in a three-course mathematics sequence as part of the requirements necessary to complete their teacher certification. The three-course mathematics sequence was taught over the course of three independent semesters, and each class met twice each week for one hour and twenty minutes. Each course in the sequence focused on a particular set of mathematics topics. The focus of the first course was number sense, whole number and integer operations, and algebraic thinking. The second course emphasized the study of geometric concepts with a focus on both two- and three-dimensional shapes, and the third course emphasized the study of rational numbers, data analysis, and probability.

Although both authors developed course materials for each of the three courses in the mathematics sequence, to maintain consistent instructional practices, only the first author of this paper was the instructor of record for all three courses. Creating a socio-constructivist learning environment was the focus in each of the three courses, and as such, each course emphasized teaching and learning consistent with the philosophy of reform in mathematics education (i.e., NCTM 1989, 1991, 1995, 2000). Hence, during every class meeting the instructor of record implemented consistent pedagogical strategies that included challenging the pre-service teachers to investigate mathematics problems, find their own solutions, and to discuss and justify those solutions. Opportunities were provided for the pre-service teachers to engage in problem solving and to engage in discourse and social interactions with their peers, as they completed tasks similar to one shown in Figure 1.

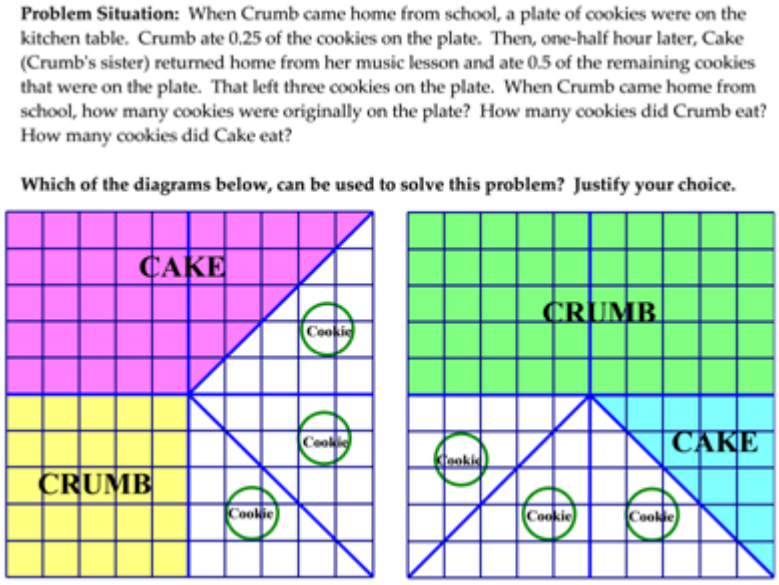

Figure 1. Problem-solving involving rational numbers.

Typically, during each class meeting the pre-service teachers completed these tasks in three stages. During the first stage, the pre-service teachers were asked to consider the tasks individually, during which time they were not allowed to talk to or ask questions of their peers or the instructor. During the second stage, the pre-service teachers were assigned to a small group and were asked to discuss, justify, and negotiate meanings of the mathematics embedded in the tasks. In the small groups, they were challenged to justify the solutions they found during their individual considerations of the tasks. Further, each small group was challenged to reach consensus. Finally, in the third stage, each small group was asked to present their solutions and justifications. During this final stage, the pre-service teachers were allowed to ask questions of the instructor. Also, during this stage, the instructor asked questions of the pre-service teachers as they presented their solutions, and the instructor facilitated the dialogue that ensued during the whole-group discussions. Tasks that did not involve a particular problem situation, similar to the one shown in Figure 2, were also completed using this three-stage instructional strategy.

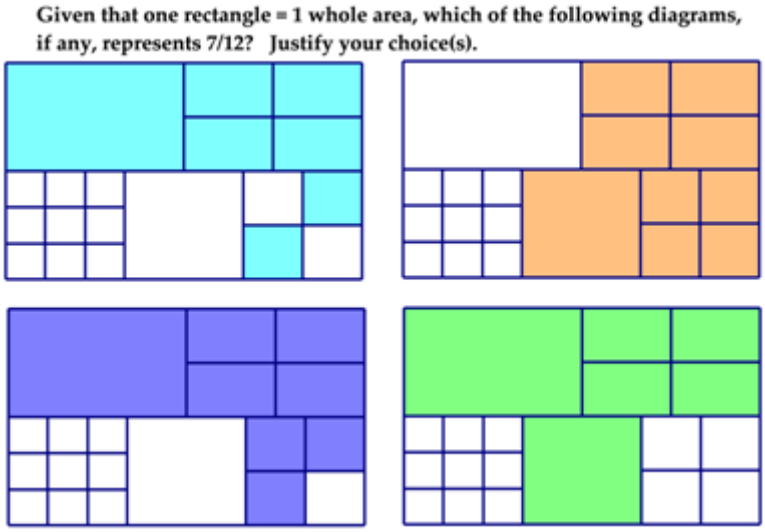

Figure 2. Units and fractional parts (adapted from Lamon, 2012).

Mathematics instruction in all three courses emphasized conceptual understanding, and as a consequence, the pre-service teachers were challenged to determine "why" specific algorithms and

procedures worked for solving particular mathematics problems. Also, the pre-service teachers connected concrete and theoretical mathematical models and used manipulatives, problem solving, and reflection to make sense of concepts and to construct their own meaning of mathematics. As shown in Figure 3, rather than give rules for operations with numbers as is usually done in traditional mathematics classes, the pre-service teachers in this study worked with models and diagrams to determine why particular rules worked.

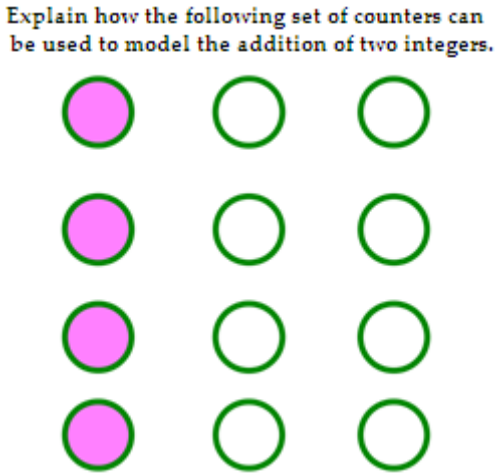

Figure 3. Operations with integers.

Data instrument and measures. At the beginning of the first course in the three-course sequence, pre-service teachers completed the *Mathematics Teaching Efficacy Beliefs Instrument* (MTEBI) (Enochs, Smith, and Huinker, 2000) and then again at the end of the third course in the mathematics sequence. The development of the MTEBI was based on the Science Teaching Efficacy Beliefs Instrument (STEBI-B) developed by Enochs and Riggs (1990). The MTEBI is aligned with Bandura's (1977, 1986) work on self-efficacy and emphasizes two constructs: *personal teaching efficacy* and *teaching outcome expectancy*. The personal teaching efficacy can be defined as a teacher's belief in their own knowledge, skills, and abilities to be an effective teacher. The teaching outcome expectancy can be defined as a teacher's belief in the notion that student learning hinges on effective teaching, and moreover, effective teaching can elicit student achievement and success regardless of external factors affecting the student.

The MTEBI consists of 21 items and is a Likert-scale instrument that has five response categories: strongly agree, agree, uncertain, disagree, and strongly disagree. Higher scores on the MTEBI indicate a greater teaching efficacy with lower scores indicating a lower teaching efficacy. Thirteen of the MTEBI items are classified as the Personal Mathematics Teaching Efficacy (PMTE) subscale, and eight are classified as the Mathematics Teaching Outcome Expectancy (MTOE) subscale (Enochs, Smith, and Huinker, 2000). The PMTE subscale addresses pre-service teachers' self-efficacy about their capabilities—specifically, their own knowledge and skills—to become effective mathematics teachers. The MTOE subscale addresses pre-service teachers' self-efficacy about effective teaching augmenting students' mathematics achievement regardless of external factors that may influence student learning.

The MTEBI has been regarded as a measurement tool that accurately measures the concept of self-efficacy and that yields the same results on repeated measures, suggesting that it is both valid and reliable. Specifically, Enochs et al. (2000) found that a reliability analysis produced the Cronbach alpha coefficient of 0.88 for the PMTE subscale and an alpha coefficient of 0.75 for the MTOE subscale (N = 324). Further, confirmatory factor analysis indicated that the two scales (PMTE and MTOE) are independent, adding to the construct validity of the MTEBI (Enochs et al., 2000).

Data analysis. Particular *t*-tests were used to analyze the distribution of the pre-service teachers' responses on the MTEBI, to determine significant differences between mean scores. Comparing mean

score differences were used as a way to examine propensities within and across the group of pre-service teachers with respect to the two constructs – personal mathematics teaching efficacy and mathematics teaching outcome expectancy. The authors specifically investigated the pre-service teachers' general mathematics teaching efficacy beliefs and then used the PMTE and the MTOE subscales to independently examine two separate categories of these teaching efficacy beliefs.

Paired-samples *t*-tests were used to analyze the pre-service teachers' mean score differences from pre-test to post-test and to compare mean score differences *within* the group of pre-service teachers. Specifically, the authors tested the null hypothesis that there were no differences in the pre-service teachers' levels of self-efficacy at the end of the three-course mathematics sequence. Since the authors were testing the null hypothesis that the two mean scores from pre-test to post-test were equal, a two-tailed test was used.

Independent-samples *t*-tests were used to analyze the mean score differences between the traditional and non-traditional pre-service teachers. The authors particularly sought the likelihood that the mean scores from pre-test to post-test obtained from the two groups were significantly different. It was assumed that the sample of pre-service teachers represented a random sample and that the traditional and non-traditional groups were independent of each other. Further, it was assumed that the variances within each group were equal. A two-tailed test was used to test the null hypothesis that the mean scores from pre-test to post-test between the traditional and non-traditional pre-service teachers were equal.

Results and Discussion

The authors used the work of Bandura (1977, 1986, 1993) and examined the two constructs from the MTEBI—personal teaching efficacy and teaching outcome expectancy—to answer the first research question. As described previously in this article, the Personal Mathematics Teaching Efficacy (PMTE) belief subscale of the MTEBI focuses on pre-service teachers' self-efficacy about their capabilities to teach mathematics effectively and their self-efficacy about their knowing mathematics well enough to teach it effectively. Further, the Mathematics Teaching Outcome Expectancy (MTOE) belief subscale of the MTEBI specifically focuses on pre-service teachers' beliefs that their teaching can affect students' achievement in mathematics regardless of external factors that may hinder students' achievement and mathematics performance. Paired-samples *t*-tests were used to determine what differences existed in the self-efficacy of the elementary pre-service teachers. As shown in Table 1, the authors found statistically significant differences in the self-efficacy of the pre-service teachers, analyzed collectively as one group. The mean scores were significantly different at the 0.01 confidence level for both the PMTE and the MTOE subscales. Thus, from pre-test to post-test, the t-values show that the pre-service teachers' self-efficacy levels were more positive at the end of the three-course mathematics sequence than at the beginning.

Table 1. Paired-samples *t*-test efficacy results

Assessment	*Mean*	*SD*	*t-value*
PMTE			
Pre – Test	4.006	0.467	
Post – Test	4.271	0.454	**5.487****
MTOE			
Pre – Test	3.667	0.481	
Post – Test	3.982	0.498	**4.083****
N = 51	***df = 50***	***Two – tailed***	****p < 0.01***

Paired-samples *t*-tests were used to examine what differences existed in the self-efficacy of traditional and non-traditional elementary pre-service teachers in order to answer research questions 1(a) and

1(b). The authors analyzed the mean score differences in self-efficacy of the traditional (N = 27) pre-service teachers to answer research question 1(a) and tested the null hypothesis that there were no differences in the mean scores from pre-test to post-test. As shown in Table 2, the authors found that for the PMTE subscale self-efficacy levels were significantly different at the end of the three-course mathematics sequence than at the beginning.

Table 2. Paired-samples *t*-test efficacy results (traditional pre-service teachers)

Assessment	*Mean*	*SD*	*t-value*
PMTE			
Pre – Test	3.915	0.486	
Post – Test	4.051	0.477	**2.606****
MTOE			
Pre – Test	3.708	0.370	
Post – Test	3.741	0.417	0.462
N = 27	***df = 26***	***Two – tailed***	****p < 0.01***

However, for the MTOE subscale, the authors did not find a significant difference in the self-efficacy levels from pre-test to post-test. Recall that the MTOE subscale measures pre-service teachers' beliefs that effective teaching can positively impact students' success in mathematics regardless of external factors that may hinder students' achievement or performance in mathematics. While enrolled in the third mathematics content course, many of the pre-service teachers in this study were simultaneously enrolled in a "block" of methods courses and field placements. Some research studies (Swars et al., 2007) have found that pre-service teachers' mathematics teaching outcome expectancy (MTOE) beliefs tend to remain the same during student teaching, while other studies (Hoy and Woolfolk, 1990) have found that pre-service teachers' self-efficacy beliefs with regards to their MTOE decline significantly during student teaching. It is the authors' contention that during student teaching or even during participation in multiple field placements, pre-service teachers may become overwhelmed with being immersed in teaching and learning environments that may be inconsistent with the constructivist teaching and learning environments emphasized in their teacher preparation programs. Consequently, pre-service teachers may not feel empowered or may not feel as though they are able to control whether students perform well or achieve in mathematics.

The authors used paired-samples *t*-tests to answer research question 1(b) and analyzed the mean score differences from pre-test to post-test of the group of non-traditional (N = 24) pre-service teachers. As shown in Table 3, mean scores were significantly different for both the PMTE and the MTOE subscales, suggesting that the self-efficacy levels of the non-traditional pre-service teachers were more positive at the end of the three-course mathematics sequence.

Table 3. Paired-samples *t*-test efficacy results (non-traditional pre-service teachers)

Assessment	*Mean*	*SD*	*t-value*
PMTE			
Pre – Test	4.109	0.431	
Post – Test	4.519	0.264	**5.488****
MTOE			
Pre – Test	3.620	0.587	
Post – Test	4.255	0.443	**5.557****
N = 24	***df = 23***	***Two – tailed***	****p < 0.01***

The authors used independent-samples *t*-tests to answer the second research question and sought to determine if significant differences existed in the self-efficacy *between* traditional and non-traditional

elementary pre-service teachers. As shown in Table 4, there was no significant difference in the mean scores between the traditional and non-traditional pre-service teachers at the beginning of the three-course mathematics sequence. This was the case for both the PMTE and the MTOE subscales, suggesting that upon entering the three-course sequence, both the traditional and the non-traditional pre-service teachers held similar beliefs with regards to their teaching mathematics. However, the authors found significant differences in mean scores between the traditional and non-traditional pre-service teachers at the end of their participation in the three-course mathematics sequence. Significant differences were found for both the PMTE and the MTOE subscales.

Table 4. Independent-samples *t*-test efficacy results

Assessment	*Mean*	*SD*	*t-value*
PMTE Pre-Test			
Traditional	3.915	0.486	
Non-Traditional	4.109	0.431	1.504
PMTE Post-Test			
Traditional	4.051	0.477	
Non-Traditional	4.519	0.264	**4.258****
MTOE Pre-Test			
Traditional	3.708	0.370	
Non-Traditional	3.620	0.587	0.652
MTOE Post-Test			
Traditional	3.741	0.417	
Non-Traditional	4.255	0.443	**4.269****
N = 51	***df = 49***	***Two – tailed***	****p < 0.01***

Conclusions and Implications

This research investigated elementary pre-service teachers' mathematics teaching efficacy with regards to two constructs—personal mathematics teaching efficacy beliefs (PMTE) and teaching outcome expectancy efficacy beliefs (MTOE). Specifically, the authors sought to determine whether the self-efficacy levels of traditional and non-traditional elementary pre-service teachers were significantly different at the end of a three-course mathematics sequence than at the beginning. Data were analyzed using null hypothesis statistical testing, in which mean scores were compared from pre-test to post-test.

Results of this study revealed that for *non-traditional* elementary pre-service teachers, learning mathematics from a constructivist approach and participating in a socio-constructivist mathematics learning environment significantly influenced their self-efficacy beliefs with regards to both efficacy constructs—PMTE and MTOE. However, for *traditional* pre-service teachers, there was no significant difference in self-efficacy levels with regards to their mathematics teaching outcome expectancy beliefs (MTOE). It is the authors' contention that since the traditional pre-service teachers in this study entered their teacher education programs almost immediately after completing their K-12 schooling experiences, their encounter with processes of teaching and learning contrary to these K-12 experiences were met with resistance. By contrast, for non-traditional pre-service teachers in this study, the gap between their K-12 schooling experiences and their participation in a teacher education program was quite considerable. Scholars (Brown, Cooney, and Jones, 1990; Lortie, 1975; Pajares, 1992; Tatto, 1998) have posited that pre-service teachers' beliefs are influenced by prior experiences as students themselves. Thus, it is important to conduct further research to determine how and to what extent K-12 schooling experiences help to formulate and sustain pre-service teachers' self-efficacy beliefs.

Although there was no significant difference in the self-efficacy levels with regards to mathematics teaching outcome expectancy beliefs (MTOE) for traditional pre-service teachers, findings revealed that for this group, self-efficacy levels with regards to personal mathematics teaching efficacy beliefs (PMTE) were significantly different at the end of the three-course mathematics sequence. According to Bandura (1993), self-efficacy beliefs that fall under the umbrella of the PMTE construct are those beliefs or internal feelings that teachers believe they can control. However, those self-efficacy beliefs that are categorized as MTOE beliefs are ones that hinge on controlling the behaviors of others, in this case, students' mathematics performance. Perhaps the collection of qualitative data such as participant interviews could help determine why self-efficacy beliefs in this area (MTOE) are more resistant to change for traditional pre-service teachers than their perceived confidence in their skills and abilities to effectively teach mathematics.

As discussed previously in this article, several scholars (Brown, Cooney, and Jones, 1990; Lortie, 1975; Pajares, 1992; Tatto, 1998) have cautioned that pre-teachers' beliefs develop over time and are resistant to change. Since this study examined pre-service teachers' beliefs over the course of only three semesters, it is unclear whether the significant change in the pre-service teachers' self-efficacy beliefs at the end of the three-course mathematics sequence is *lasting* change. In other words, it cannot be predicted whether this change in beliefs will continue into practice as the pre-service teachers become in-service teachers. Thus, it is important that further research investigate whether pre-service teachers who are highly efficacious during their teacher education programs are also highly efficacious during their first years of in-service teaching. Further, since it has been well established in the literature that many mathematics teacher education programs emphasize teaching and learning consistent with reform efforts in mathematics (NCTM 1989, 1991, 1995, 2000), it is important for research studies to examine how self-efficacy beliefs influence teaching practices and subsequent student performance and student achievement in mathematics. Thus, additional research studies that determine causal and correlational relationships between teachers' self-efficacy beliefs and teaching practices and between teachers' self-efficacy beliefs and student achievement are needed.

This study has provided insight into the essential components needed in teacher education programs to effect change in pre-service teachers' self-efficacy beliefs. Moreover, unlike most studies on teachers' beliefs that examine the impact on beliefs of pre-service teachers' participating in methods courses, this study specifically investigated the impact on beliefs of pre-service teachers' participating in three content mathematics courses. In terms of teacher education programs' changing pre-service teachers' beliefs, this study has shown that content mathematics courses emphasizing constructivist pedagogy hold promise in contributing to that change.

References

Bandura, A., (1993). Perceived self-efficacy in cognitive development and functioning. *Educational Psychologist*, ***28***(2), 117-148.

Bandura, A., (1986). *Social foundations of thought and action: A social cognitive theory*. Englewood Cliffs, NJ: Prentice Hall.

Bandura, A., (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review*, ***84***, 191-215.

Battista, M. T., (1994). Teacher beliefs and the reform movement in mathematics education. *PhiDelta Kappan*, ***75***(2), 462-470.

Borko, H., and Putnam, R. T., (1995). Expanding a teachers' knowledge base: A cognitive psychological perspective on professional development. In T.R. Guskey and M. Huberman (Eds.), Professional development in education: New paradigms and practices, (pp 35-65). New York: Teachers College Press.

De Mesquita, P. B., and Drake, J. C., (1994). Educational reform and self-efficacy beliefs of teachers implementing nongraded primary school programs. *Teaching and Teacher Education*, ***10***(3), 291-302.

Dewey, J., (1933). *How we think. A restatement of the relation of reflective thinking to educative process*. Boston, MA: D.C. Heath.

Enochs, L. G., and Riggs, I. M., (1990). Further development of an elementary science teaching efficacy belief instrument: A preservice elementary scale. *School Science and Mathematics*, ***90***(8), 695-706.

Enochs, L. G., Smith, P. L., and Huinker, D., (2000). Establishing factorial validity of the mathematics teaching efficacy instrument, *School Science and Mathematics*, ***100***(4), 194-202.

Hart, L., (2002). Preservice teachers' beliefs and practice after participating in an integrated content/methods course, *School Science and Mathematics,* ***102***(1), 4-14.

Hoy, A. W., and Woolfolk, A. E., (1990). Socialization of student teachers. *American Educational Research Journal*, ***27***, 279-300.

Lamon, S. J., (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*, New York: Routledge.

Lortie, D., (1975). *Schoolteacher: A sociological study*. Chicago: University of Chicago Press.

National Council of Teachers of Mathematics (NCTM)., (1989). *Curriculum and evaluation standards for school mathematics*, Reston, VA: Authors.

National Council of Teachers of Mathematics (NCTM)., (1991*). Professional standards for teaching mathematics*, Reston, VA: Authors.

National Council of Teachers of Mathematics (NCTM)., (1995). *Assessment standards for school mathematics*, Reston, VA: Authors.

National Council of Teachers of Mathematics (NCTM)., (2000). *Principles and standards for school mathematics*, Reston, VA: Authors.

Pajares, M. F., (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, ***62***(1), 307-332.

Patton, M. Q., (1990). *Qualitative evaluation and research methods*. Newbury Park, CA: Sage Publications.

Phillips, D. C., (1995). The good, the bad, and the ugly: The many faces of constructivism. *Educational Researcher*, **24**(7), 5-12.

Piaget, J., (1972). *Psychology and epistemology: Towards a theory of knowledge*. Harmondsworth, UK: Penguin.

Piaget, J., (1970). *Science of education and the psychology of the child*. New York: Orion Press.

Schoenfeld, A., (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education*, ***20***(4), 338-355.

Simon, M., (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, ***26***(2), 114-145.

Steffe, L., and D'Ambrosio, B. S., (1995). Toward a working model of constructivist teaching: A reaction to Simon. *Journal for Research in Mathematics Education*, ***26***(2), 146-159.

Swars, S., Hart, L., Smith, S., Smith, M., and Tolar, T., (2007). A longitudinal study of pre-service teachers' mathematics beliefs and content knowledge, *School Science and Mathematics*, ***107***(9), 325-335.

Tatto, M. T., (1998). The influence of teacher education on teachers' beliefs about purposes of education, roles, and practice. *Journal of Teacher Education*, ***49***(1), 66-77.

Thompson, A. G., (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning, (pp. 127-146). New York: MacMillan.

Tirosh, D., and Graeber, A. O., (1989). Preservice teachers' explicit beliefs about multiplication and division. *Educational Studies in Mathematics*, ***20***, 79-96.

von Glasersfeld, E., (1990). Environment and education. In L.P. Steffe and T. Wood (Eds.), Transforming children's mathematics education: International perspectives, (pp. 200-215). Hillsdale, NJ: Lawrence Erlbaum.

von Glasersfeld, E., (1983). Learning as a constructive activity. Proceedings of the Fifth Annual Conference of the International Group for the Psychology of Mathematics Education, 41-69.

Vygotsky, L. S., (1986). *Thought and knowledge*. Cambridge, MA: The MIT Press.

Wilson, S. M., Floden, R. E., and Ferrini-Mundy, J., (2001). *Teacher preparation research: Current knowledge, gaps, and recommendations*. University of Washington: Center for the Study of Teaching and Policy.