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Research Article

Investigation of the relationship between learning concepts and mathematical thinking of secondary school students with canonical correlation analysis

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ABSTRACT

In this study, the relationship between secondary school students' conceptions of learning and their mathematical thinking was examined. The study was conducted with 311 secondary school students using relational survey model. The data were collected with the learning conceptions scale and mathematical thinking scale, and the relationship between the variables was evaluated by canonical correlation analysis. The results revealed that there was a high level of relationship between students' learning conceptions and mathematical thinking skills. It was determined that the sub-dimensions of learning conceptions, knowledge acquisition and use, and personal changes had a moderate inverse effect on mathematical thinking. In addition, the tendency for higher-order thinking sub-dimension made the highest contribution to the model but had an inverse relationship with learning conceptions. These findings provide important clues to understand the effect of learning conceptions on mathematical thinking.

Keywords: sustainable education, learning conceptions, mathematical thinking, canonical correlation analysis, secondary school students

INTRODUCTION

The concept of learning has been a fundamental issue that has attracted the attention of many researchers for many years. While behaviorists explain learning as an observable behavioral change that occurs as a result of the connection between stimulus and behavior, cognitive learning theorists focus on how the individual learns and accept that learning is a mental process in which the human brain is similar to a computer. Learning from the perspective of constructivist approach has been defined by the researchers with the criticisms that human cannot be a machine and that creating meaning is a different process than processing information; as an event that is unique to the individual, cannot be transferred from one individual to another, and occurs in the mind of the individual, as a result of each individual attributing a meaning to their own experiences (Olkun & Toluk Uçar, 2014). In order to understand how students' approach and learn information and how these processes affect learning outcomes, two concepts come to the fore: Deep and surface learning conceptions. Deep learning is an approach that requires discovery and meaning making in order for knowledge to be permanent (Biggs et al., 2001). When students adopt this approach, since they focus on making sense of information, analyzing it and associating it with different contexts; learning processes become more meaningful and information is better retained in long-term memory (Case, 2004). Surface learning, on the other hand, is an approach that requires short-term recall of an unconnected body of knowledge through memorization (Biggs et al., 2001; Marton & Säljö, 1976). In short, surface learning can be characterized as rote learning and deep learning as learning by understanding (Beattie et al., 1997).

Different learning conceptions emerge as a result of individuals' experiences (Tsai, 2004). In this context, learning conceptions are specific to each individual and field. In order to understand the conceptions of learning mathematics, it is necessary to understand the nature of learning mathematics. Learning mathematics is the ability to master and apply mathematical concepts effectively. It involves explaining the relationships between concepts and applying algorithms correctly and efficiently in problem-solving (Schaathun, 2022). In this context, mathematical learning requires a deep learning approach that necessitates the understanding of the relationships between concepts and the meanings underlying the procedures instead of surface learning that causes memorization of calculation skills (Case, 2004). Another fundamental element that enables deep learning in mathematics is mathematical thinking. Having mathematical thinking increases students' capacity to use mathematics actively (Stacey, 2006). However, in the literature, it is seen that the vast majority of studies on mathematical thinking focus on problem situations and problem-solving. Yet, all of the components and skills related to students' mathematical thinking process require a deep understanding of learning that involves meaningful and permanent learning. In this direction, the effect of deep learning conceptions on students' mathematical thinking skills, especially in the context of its relationship with skills such as problem-solving, reasoning, generalization, and higher-order thinking, constitutes an important research area in education. This is because supporting students' deep learning conceptions is considered a critical factor that will contribute to their development of analytical thinking, problem-solving, and high-level cognitive skills, thereby supporting long-term academic success and lifelong learning. Based on this, the present study investigates the relationship between students' learning conceptions and their mathematical thinking. Since studies on this subject have mostly been limited to univariate analyses, this study aims to contribute a more holistic perspective to the literature by examining the relationship between secondary school students' learning conceptions (acquiring and using knowledge, personal change, social skills) and their mathematical thinking (tendency for higher order thinking, reasoning, mathematical thinking skill, problem-solving) with canonical correlation analysis, a multivariate method. In this regard, answers to the following questions were sought:

1. Is there a significant relationship between secondary school students' learning conceptions and mathematical thinking?
    a. How do the sub-dimensions of learning conceptions (acquiring and using knowledge, personal change, social skills) relate to mathematical thinking?
    b. How do the sub-dimensions of mathematical thinking (tendency for higher order thinking, reasoning, mathematical thinking skill, problem-solving) relate to conceptions of learning?

LITERATURE REVIEW

Learning Conceptions

Contemporary educational design and practices are undergoing a significant transformation from traditional teaching-learning approaches based on the direct transmission of knowledge to modern approaches where knowledge is actively constructed by the student (Gray, 1997). In this context, teaching is generally considered along two main approaches. The first approach views the teaching process as content delivery and is therefore based on a teacher-centered conception. The other approach is based on the student's active participation in the process of accessing, questioning, and constructing meaning; it is therefore defined as a learner-centered approach. The fundamental distinction between these two approaches lies in the nature of the roles assigned to the teacher and the student (Kember, 1997). Students' learning approaches, however, can be shaped not only by the teaching conception but also by the quality of the learning environment and assessment methods. Indeed, although students may at times gravitate towards a surface learning approach, it is observed that they prefer a deep learning approach when the course content is meaningful and engaging, or when assessment methods requiring deeper thought, such as essay writing or fill-in-the-blanks, are used (İlhan Beyaztaş & Senemoğlu, 2015). This situation indicates that students' learning approaches are not fixed; on the contrary, they can orient towards different forms of learning depending on the conditions they are in (Yılmaz & Orhan, 2011). Therefore, the fact that learning approaches can change contextually necessitates a closer examination of the factors affecting students' learning conceptions. In this context, studies conducted in the literature also focus on investigating the relationships between students' learning conceptions and various individual, environmental, and cognitive variables. The researches demonstrate that students' learning conceptions significantly impact key cognitive outcomes, influencing both their critical thinking skills (Beşoluk & Önder, 2010) and their academic achievement (Alamdarloo et al., 2013; Bahadır et al., 2025). These conceptions and the resulting learning approaches are not formed in a vacuum; they are also closely related to a range of contextual variables, including university, grade level, subject area, the teaching-learning environment, and perceptions of academic success (Ekinci, 2010). Delving deeper, these conceptions are often rooted in students' underlying epistemological beliefs. For instance, the belief that ability is innate and fixed, or that knowledge is absolute and unchanging, is significantly and positively correlated with a traditional approach to teaching and learning (Chan & Elliott, 2004). Indeed, the importance of a deep learning conception is highlighted in specific domains. The study of Bahçivan and Kapucu (2014), for example, reveals that science requires an approach centered on understanding knowledge coherently and applying scientific laws to improve daily life, rather than one focused on merely memorizing formulas for success on tests.

Mathematical Thinking

Research on mathematical thinking reveals it to be a complex and multi-component skill. The literature consistently shows that students' mathematical thinking skills develop as their grade level increases (Cai, 2003; Ferdianto et al., 2022). With advancing grade levels, students increasingly employ appropriate solution strategies with sound mathematical reasoning, leading to a higher rate of correct answers (Cai, 2003). This progression naturally affects not only the overall problem-solving process but also its sub-components. Ferdianto et al. (2022) determined that students with high mathematical thinking skills tend to execute all sub-components of the process. Conversely, as skill levels decrease, difficulties emerge, particularly with persuasion and generalization. The process of generalization stands out as a prominent area of difficulty across various mathematical domains and grade levels. For instance, research in geometry has shown that while students can successfully express a general rule verbally or geometrically, they struggle significantly to articulate it algebraically (Yıldırım, 2015). This challenge persists in other domains, such as with exponential expressions, where students find the generalization stage markedly more difficult than making assumptions or working with special cases (Düzgün & İpek, 2023). This suggests students' difficulties are not merely about finding a pattern but about abstracting it into a formal mathematical structure, a challenge that becomes more pronounced when they encounter unfamiliar problems (Yıldırım & Yavuzsoy, 2017). Nonetheless, students are often able to perform other components of mathematical thinking, such as making conjectures by using equation setting and estimation strategies during problem-solving (Kükey et al., 2019).

Beyond these cognitive processes, the literature indicates that students' mathematical thinking performance is linked to a range of individual and contextual factors. For example, significant differences are observed between gifted and non-gifted students in the stages of specialization, generalization, and making assumptions (Aygün et al., 2021). It is not only giftedness; research also reveals that demographic variables such as gender and reading habits are influential. Benli and Gürtaş (2021) found that female middle school students exhibited higher mathematical thinking skills and that reading for at least one hour daily also enhanced these skills. In conjunction with these factors, affective dimensions like mathematics self-efficacy are also seen to be related to students' mathematical thinking across different stages (Tüzün & Cihangir, 2020). Moreover, studies demonstrate that these skills can be developed through targeted instructional methods, such as material-supported learning environments (Kılıç et al., 2013), highlighting the importance of the learning context.

While these studies collectively offer a multifaceted perspective on mathematical thinking, they have predominantly focused on examining single sub-skills in isolation, often using limited or specific sample groups (e.g., gifted students, a single grade level). Furthermore, while the literature investigates what students can do and the factors that influence them, it largely overlooks the question of how students' fundamental learning conceptions–their core beliefs about what it means to learn and do mathematics–shape their approach to mathematical thinking. This situation points to a significant gap that constitutes the core rationale for the present study.

Current Study

In the literature, it is seen that most of the studies on mathematical thinking focus on problem situations and problem-solving. However, all of the components and skills related to the students' mathematical thinking process require a deep understanding of learning. How deep learning conceptions in mathematics education affect students' mathematical thinking, especially in relation to skills such as problem-solving, reasoning, generalization, and higher-order thinking, is an important research topic in education. This is because supporting students' deep learning conceptions can contribute to their development of analytical thinking, problem-solving, and high-level cognitive skills. This, in turn, is a critical requirement for long-term success and lifelong learning. However, studies in the literature that systematically investigate the relationship between students' learning conceptions and mathematical thinking using a multivariate method like canonical correlation analysis are limited. While existing studies generally focus on univariate analyses, the present study aims to provide a more comprehensive understanding by addressing the relationship between multiple sets of variables with a multivariate approach. In this context, the main problem of the study is to reveal the nature of the relationship between middle school students' learning conceptions (acquiring and using knowledge, personal change, and social skills) and their mathematical thinking (tendency for higher order thinking, reasoning, mathematical thinking skill, and problem-solving), and to determine how this relationship interacts across different dimensions. The investigation of this relationship will enable the more effective development of mathematical thinking by contributing to sustainable development goals through the promotion of deep learning conceptions in education, thus filling a significant gap in literature.

METHOD

Research Design

The aim of this study is to examine the relationship between the main variables and sub-variable groups of the variable data set of 6th, 7th, and 8th grade secondary school students' learning conceptions and mathematical thinking with canonical correlation. For this purpose, relational survey model, one of the quantitative research methods, was used. The relational survey model is an approach used to determine the existence and degree of change between more than one variable. In this context, correlational analysis can be carried out in two different ways: determining the relationship through correlation analysis and revealing the relationship through comparison method. This model allows the determination of attitudes and tendencies (Creswell, 2017; Karasar, 2005). Canonical correlation analysis is a multivariate statistical method used to evaluate the relationship between more than one set of variables and to determine the strength of this relationship (Tabachnick & Fidell, 2007). In other words, this analysis method aims to understand the relationship between X and Y variable groups by representing them with fewer variables (Çankaya, 2005). Furthermore, it is a flexible and comprehensive statistical method used to examine the complex relationships between multiple sets of independent and dependent variables. It is seen as a special case of many dependency analysis methods and is considered the most general relational analysis. This is because it comprehensively evaluates the linear relationships between two sets of variables (Pugh & Hu, 1991). Canonical correlation analysis is a suitable method for examining the multivariate relationships between learning conceptions and mathematical thinking in this study, as it evaluates the complex relationships between two sets of variables in a single analysis, reduces the risk of Type I error, and clearly reveals the contributions of the sub-dimensions (Hasbek, 2020; Stangor, 2010; Thompson, 2000). In the study, some assumptions necessary for canonical correlation analysis were tested. For this purpose, Skewness and Kurtosis values were examined for the normality testing of the data.

Participants

The study group consisted of 311 students studying in the 6th, 7th, and 8th grades in a public school on the European side of Istanbul. Of the participants, 164 were female and 147 were male. 93 of the students were sixth grade, 90 were seventh grade and 128 were eighth grade students.

Research Variables

In the study, a model suitable for canonical correlation analysis was designed; in this direction, learning conception was considered as one independent variable group and mathematical thinking as the other independent variable group. The main and sub-variables used in the study are given in **Table 1**.

Table 1. Main variable and sub-variables

Set no	Main variables	Sub-variables	Variable code
Set 1	Learning conceptions	Acquiring and using knowledge	L1
		Personal change	L2
		Social skills	L3
Set 2	Mathematical thinking	Tendency for higher order thinking	M1
		Reasoning	M2
		Mathematical thinking skills	M3
		Problem-solving	M4

Data Collection Tools

In the study, learning conceptions scale and mathematics thinking scale were used as data collection tools. The characteristics of the data collection tools are given below.

Learning conceptions scale

The scale named 'learning conceptions' developed by Baş (2013) was developed to determine the learning conceptions of primary school students. The scale is a 5-point Likert-type, with responses ranging from 1 (strongly disagree) to 5 (strongly agree). The suitability of the scale for middle school students (6^{th}–8^{th} grades) was tested in the present study. The knowledge acquisition and utilization dimension of the scale consists of 7 items, personal change dimension consists of 5 items and social skills dimension consists of 3 items. Cronbach's alpha coefficient for the first factor of the scale, 'learning as acquiring and using knowledge', was 0.79, for the second factor, 'learning as personal change', was 0.80, for the third factor, 'learning as social skills', was 0.94, and for the whole scale, Cronbach's alpha coefficient was 0.87. In the present study, the Cronbach's alpha coefficient for the scale was found to be 0.814, which indicates that the scale is highly reliable.

Mathematical thinking scale

The "mathematical thinking" scale, developed by Ersoy and Başer (2013), is a 5-point Likert-type scale designed to measure mathematical thinking levels, and responses are rated on a scale from 1 (strongly disagree) to 5 (strongly agree). The tendency for higher order thinking dimension of the scale consists of 6 items, reasoning dimension consists of 4 items, mathematical thinking skill consists of 8 items and problem-solving dimension consists of 7 items. Cronbach's alpha coefficient for the whole scale was found to be 0.78. In this study, the Cronbach's alpha coefficient of the scale was calculated as 0.759. This value indicates that the scale is reliable.

Data Analysis

The relationship between learning conceptions and mathematical thinking was analyzed by canonical correlation analysis. Canonical correlation analysis is a method used to examine the relationships between two sets of variables ($X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$, $n \geq 2$ and $m \geq 2$) with at least two variables in each (Bordens & Abbott, 2011; Huo & Budescu, 2009; Varmuza & Filzmoser, 2009). This analysis offers the advantage of revealing the links between two data sets in a single process. At the same time, by reducing the risk of Type I error that may be involved in the measurement process (Stangor, 2010), it reduces the possibility of non-significant relationships being mistakenly accepted as significant. In canonical correlation analysis, it is not necessary to classify two sets of variables as dependent or independent variables (Albayrak, 2010). Therefore, the sets of variables are usually named as set 1 and set 2 and the analysis focuses on determining the relationship between these two sets (Pedhazur, 1997; Stevens, 2009). The first step in canonical correlation analysis is to examine the relationship between two sets of variables.

In canonical correlation analysis, linear components that will maximize the relationship between two sets of variables are first obtained (Everitt & Hothorn, 2011; Fan, 1997; Henson, 2000; Leech et al., 2005). The new variables derived from these linear components are called canonical variables (Afifi & Clark, 1996). The canonical variables on both sides of the canonical correlation equation are together called canonical variable pairs (Tabachnick & Fidell, 2007). The relationship between pairs of canonical variables is defined as canonical function or canonical root (Sherry & Henson, 2005). Each canonical function is expressed by a pair of two canonical variables (Hair et al., 2010). The maximum number of canonical variable pairs that can be formed is limited to the number of variables in whichever of the variable sets has fewer variables (Cohen et al., 2003; Fan, 1996). The first canonical variable pair is calculated to maximize the relationship between the two sets of variables (Afifi & Clark, 1996; Härdle & Simar, 2012; Rencher, 2002). Then, the analysis is continued by creating a second canonical variable pair. This second pair is uncorrelated with the first pair and represents a new maximum relationship between the two sets of variables that has not been taken into account (Stevens, 2009). As the canonical functions progress, the correlation values obtained gradually decrease (Hair et al., 2010). In practice, only statistically significant canonical functions are interpreted (Tabachnick & Fidell, 2007). The general structure of the canonical correlation analysis created in the study is shown in **Figure 1**.

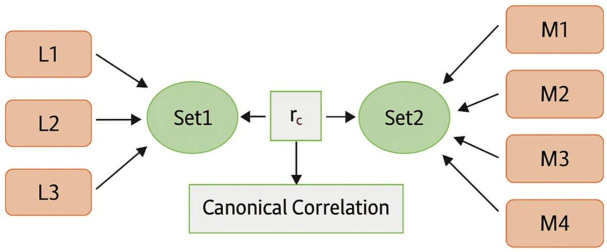

Figure 1. Canonical correlation analysis scheme (Source: Authors' own work)

In the study, the relationship between two different data sets was analyzed using canonical correlation analysis method. The first data set includes the learning conceptions variable consisting of weighted combinations of knowledge acquisition and use, personal change and social skills variables, while the second data set includes the mathematical thinking variable consisting of weighted combinations of tendency for higher order thinking, reasoning, mathematical thinking skills and problem-solving variables. In the analysis, since there are 3 variables in the learning comprehension (set 1) data set and 4 variables in the mathematical thinking (set 2) data set, the maximum number of canonical variable pairs that can be created is equal to the number of variables in the set with fewer variables. In this case, maximum 3 canonical variable pairs can be created. These pairs are used to make sense of and measure the relationship between the two data sets.

Table 2 shows the Skewness and Kurtosis values of the variables in the research model. Whether the variables show normal distribution or not was evaluated by analyzing skewness and kurtosis values.

Table 2. Skewness and Kurtosis values

Variable	Minimum	Maximum	Skewness	Kurtosis
Problem-solving	1.000	5.000	.008	.688
Mathematical thinking skills	1.500	5.000	–.085	.627
Reasoning	1.000	5.000	–.632	.885
Acquiring and using knowledge	1.000	5.000	–.654	.541
Tendency for higher order thinking	1.000	5.000	–.656	.774
Social skills	1.000	5.000	–.645	–.018
Personal change	1.400	5.000	–.681	.292

According to **Table 2**, it is seen that Skewness values vary between –0,681 and 0,008 and Kurtosis values vary between –0.018 and 0.774. In other words, Skewness and Kurtosis values are between +3 and –3. These results show that the variables meet the univariate normality assumption (Kline, 2011).

In canonical correlation analysis, not only the overall significance of the canonical model obtained, but also the significance of each canonical function in the model should be evaluated separately. In determining the significance of the canonical model, the cumulative values of the canonical functions obtained from the analysis are taken as a basis. Therefore, in a canonical model where the cumulative values are statistically significant, some canonical functions may be found significant, while in others the relationship between canonical variables may be quite weak and this relationship may not show statistical significance. For this reason, when interpreting the results of canonical correlation analysis, it is important to consider the significance of both the overall canonical model and each canonical function separately. To assess the significance of canonical functions, the eigenvalues and canonical correlation coefficients of the related functions are taken into consideration (Sherry & Henson, 2005).

FINDINGS

Canonical correlation analysis was conducted to determine whether there is a significant relationship between the learning conceptions and mathematical thinking of 6th, 7th, and 8th grade secondary school students. Canonical correlation analysis was performed using a data set that meets the assumptions that variable sets consist of more than one variable and that these sets are in linear relationship with each other. Within the scope of this analysis, 3 sub variables of the learning conceptions scale and 4 sub variables of the mathematical thinking scale were evaluated.

In this study, three canonical functions emerged as a result of the canonical correlation analysis conducted to evaluate the relationship between the learning conceptions and mathematical thinking data sets. The eigenvalues and canonical correlation coefficients of these functions are presented in **Table 3**.

Table 3. Multivariate significance tests

	Correlation	Eigenvalue	Wilks statistics	F	Num df	Denom df	p
1	.755	.325	.415	26.437	12.000	804.600	0.000
2	.172	.031	.965	1.833	6.000	610.000	0.090
3	.075	.006	.994				

* $p < 0.05$

According to **Table 3**, three canonical functions were calculated as a result of canonical correlation analysis. Among these functions, only one of the canonical variable pairs tested using Wilks statistics was found to be statistically significant. The fact that the eigenvalue of this significant variable is quite high compared to the other variables and the Wilks statistics have a value close to .05 supports this finding.

As a result of the analysis, the canonical correlation coefficient between the sub variables of the learning conceptions scale and the sub variables of the mathematical thinking scale was calculated as 75.5%. According to this result, a one-unit change in the learning conceptions levels of 6th, 7th, and 8th grade secondary school students leads to a 0.755-unit change in their mathematical thinking. Büyüköztürk (2012) classifies correlation coefficients as low level between 0.00–0.30, medium level between 0.30–0.70 and high level between 0.70–1.00. Based on this classification, it was concluded that the relationship between the scales in question was at a high level and strong.

The three functions obtained in the canonical correlation analysis represent the relationships between the variable sets in different dimensions. Based on the analysis results and the contributions of the variables, these functions have been named as follows:

1. Relationship between deep learning and higher-order mathematical thinking: This function expresses the high-level inverse relationship between the "acquiring and using knowledge" (–0.934) and "personal change" (–0.906) sub-dimensions of learning conceptions, and the "tendency for higher order thinking" (–0.871) and "reasoning" (–0.771) sub-dimensions of mathematical thinking skills. The canonical correlation coefficient (0.755, $p < 0.05$) shows that this function strongly explains the primary relationship.
2. Interaction of social skills and problem-solving: The second function represents that the "social skills" sub-dimension of learning conceptions shows a weaker relationship with the "problem-solving" sub-dimension of mathematical thinking skills. Although the Wilks' statistics (0.965, $p = 0.090$) indicate that this function is not statistically significant, it is seen that social skills have a limited interaction with problem-solving processes.

3. Weak and residual relationships: The third function covers the minimal variance not explained by the previous functions and does not provide a statistically significant contribution with its low correlation coefficient (0.075). This function represents the weak and residual relationships between learning conceptions and mathematical thinking skills.

The findings of the standardized canonical correlation coefficients of the scales are given in **Table 4**.

Table 4. Correlation coefficients for the canonical functions

		Standardized canonical coefficients
Sub-variables of learning conceptions	Acquiring and using knowledge	–.543
	Personal change	–.471
	Social skills	–.098
Sub-variables of mathematical thinking	Tendency for higher order thinking	–.515
	Reasoning	–.331
	Mathematical thinking skills	–.317
	Problem-solving	–.106

Among the variables that constitute the sub-dimensions of learning conception, the variable with the most weight is the 'acquiring and using knowledge' dimension, with a coefficient of –.543. This negative loading indicates that the ways students acquire and apply knowledge may have an inverse relationship with some of the mathematical thinking skills in the opposing variable set. This finding suggests that students may struggle with cognitively higher-order skills such as problem-solving, reasoning, or generalization, especially when the knowledge acquisition process is limited to a memorization-based or surface approach.

According to **Table 4**, the canonical function obtained from the sub-variables of the learning conception is (–0.543 × acquiring and using knowledge) + (–0.471 × personal change) + (–0.098 × social skills).

Among the variables related to mathematical thinking, the highest canonical loading, with a coefficient of –.515, belongs to the 'tendency for higher order thinking' dimension. This finding shows that the most determinant variable within the mathematical thinking set for the first canonical function is the 'tendency for higher order thinking'. This result suggests that when students' approaches to acquiring and using knowledge remain at a surface level, it may negatively affect their higher-order thinking tendencies.

The canonical function obtained from the sub-variables of mathematical thinking is (–0.515 × tendency for higher order thinking) + (–0.331 × reasoning) + (–0.317 × mathematical thinking skills) + (–0.106 × problem-solving).

The findings of the canonical loadings of the scales are given in **Table 5** and **Table 6**.

Table 5. Canonical loadings of the learning conception scale

Sub-variables	1
Acquiring and using knowledge	–.934
Personal change	–.906
Social skills	–.679

According to **Table 5**, among the sub-variables of the learning conception scale, the sub-variable with the highest explanatory power in terms of the first canonical value was determined to be "acquiring and using knowledge" with a value of –0.934. This indicates that how students' approach and apply knowledge plays a central role in the relationship established with mathematical thinking skills. The fact that all three sub-dimensions carry high negative canonical loadings demonstrates that all dimensions of the learning conception contribute significantly to the canonical structure. However, the difference between "acquiring and using knowledge" and "personal change" shows that students are concerned not only with acquiring information but also with how this knowledge transforms them. In contrast, the lower loading of the "social skills" dimension may suggest that the social aspect remains weaker in the relationship established with mathematical thinking in this study.

Table 6. Canonical loadings of the mathematical thinking scale

Sub-variables	1
Tendency for higher order thinking	–.871
Reasoning	–.771
Mathematical thinking skills	–.728
Problem solving	–.610

According to **Table 6**, among the sub-variables of the mathematical thinking scale, the sub-variable with the highest explanatory power in terms of the first canonical value was determined to be 'tendency higher order thinking' with –0.871. This finding reveals that students' orientation towards higher-order cognitive skills such as abstraction, analysis, synthesis, and evaluation is the strongest indicator of the overall structure related to mathematical thinking. The fact that all sub-dimensions carry significant canonical loadings (ranging from –.610 to –.871) indicates that mathematical thinking skill is a holistic structure and cannot be reduced to a single dimension. This, in turn, suggests that to develop students' mathematical thinking, their skills in higher-order thinking, reasoning, problem-solving, and general thinking must be addressed collectively.

Table 7. Cross correlation results of the learning conception scale

Sub-variables	1
Acquiring and using knowledge	–.705
Personal change	–.684
Social skills	–.512

According to **Table 7**, the high cross-loading value of –.705 for the "acquiring and using knowledge" dimension reveals that this dimension demonstrates the strongest, yet inverse, relationship with mathematical thinking. This finding suggests that as students' learning conceptions based on knowledge acquisition increase, the higher-order components of their mathematical thinking skills may decrease. This situation may point to a surface form of learning in which the student merely receives and uses information without questioning or processing it in depth.

Table 8. Mathematical thinking scale cross correlation results

Sub-variables	1
Tendency for higher order thinking	–.658
Reasoning	–.582
Mathematical thinking skills	–.550
Problem-solving	–.460

According to **Table 8**, the cross-loading value of –.658 for the "tendency higher order thinking" dimension shows that this dimension has the strongest, yet inverse, relationship with learning conceptions. This finding suggests that students prone to higher-order thinking may have less developed or differently oriented approaches to learning. Specifically, traditional or surface learning conceptions may be insufficient to support high-level thinking.

Table 9. Mutually explained effect rate

Canonical variable	Set 1 by self	Set 1 by set 2	Set 2 by self	Set 2 by set 1
1	.718	.409	.564	.321

According to **Table 9**, the rate of the learning conception scale measuring learning conception was determined as 71.8%. The rate of the learning conception scale sub variables affecting the mathematical thinking scale was 40.9%, and the rate of the mathematical thinking scale measuring mathematical thinking was 56.4%. In addition, the rate of the mathematical thinking scale sub variables affecting the learning conception scale was found to be 32.1%. According to these data, it was determined that the levels of the scales affecting each other were close and medium level according to Büyüköztürk's (2012) classification. This situation reveals that supporting a deep learning conception in educational settings can directly contribute to the development of students' mathematical reasoning, problem-solving, and higher-order thinking skills.

CONCLUSION

In this study, the relationship between the learning conceptions and mathematical thinking of 6^{th}, 7^{th}, and 8^{th} grade middle school students was investigated using canonical correlation analysis. The results of the analysis indicate a significant and strong relationship between these two sets of variables. Specifically, the first canonical function explains approximately 75.5% of the total variance between the variable sets. This situation demonstrates that learning conceptions are linked to mathematical thinking skills at a statistically significant level.

According to the canonical loadings, the "acquiring and using knowledge" sub-dimension has the highest weight in the learning conception variable set, with a coefficient of –0.543. This reveals that this dimension's effect on the model is pronounced, but it also shows that this effect is in a negative direction. In other words, as the learning conception geared towards acquiring and using knowledge increases, students' mathematical thinking tendencies may decrease. This suggests that when students focus on acquiring knowledge in a superficial manner, their processes of meaning-making and higher-order thinking may not develop sufficiently. Therefore, for the development of mathematical thinking, it is important to adopt a deep learning approach aimed at discovering cause-and-effect relationships, rather than surface learning that focuses only on exam-oriented or rote knowledge acquisition. Similarly, the "personal change" sub-dimension also shows an inverse relationship with mathematical thinking. Although it provides a lower contribution compared to the 'acquiring and using knowledge' dimension, it shows that learning approaches focused on personal development may affect mathematical thinking skills through indirect, rather than direct, means. This result suggests that the development of individual awareness and self-regulation skills may not be sufficient on its own to enhance mathematical thinking but could play a supportive role. The effect of the 'Social skills' dimension on mathematical thinking is limited compared to the other dimensions. The fact that mathematical thinking is generally based on individual skills such as problem-solving, analysis, and reasoning suggests it is more influenced by cognitive dimensions, while social skills have a limited role in directly shaping these processes. However, this result does not mean that social skills are entirely ineffective. Environments for developing arguments and discussing mathematical ideas can also support mathematical thinking as they allow students to question the causality of mathematical knowledge. Therefore, this limited effect of social skills is indirect rather than direct.

In the mathematical thinking variable set, the "tendency for higher order thinking" sub-dimension stands out with the highest canonical loading of –0.515 and shows an inverse relationship with the learning conception. This situation suggests that even if students have developed higher-order thinking skills, they might be using them merely to complete given tasks or take notes, rather than to develop an in-depth understanding. Particularly in learning environments focused on outcome assessment, if students use higher-order thinking only to solve a complex problem presented to them, their learning conceptions may transform into surface learning. For this reason, it is important to evaluate the process and the metacognitive skills employed by students in problem-solving, rather than just the results they achieve. The sub-dimensions of 'reasoning' and 'mathematical thinking skills', which have a lower effect compared to the 'tendency for higher-order thinking', contribute at a moderate level and show an inverse relationship. This inverse relationship indicates that the cognitive approaches students exhibit in their mathematical thinking processes do not fully align with their learning conceptions. This finding points to the fact that problem-solving does not directly contribute to deep learning, as students solve frequently encountered routine problems with memorized solution paths. A deep learning conception is based on the ability to understand concepts and the relationships between them, as well as to apply this knowledge in different contexts. However, students can arrive at a solution directly without questioning the underlying meaning of a concept while solving a problem. Indeed, the existence of an inverse relationship between higher-order thinking tendencies and learning conceptions supports this situation.

The cross-relational and explained variance analyses in the study also support the canonical function results. Learning conceptions show a relationship with the components of mathematical thinking, particularly between the "acquiring and using knowledge" dimension and the "tendency for higher order thinking" dimension. However, the negative direction of these relationships reveals that approaches based solely on knowledge acquisition or task-oriented thinking in learning processes may provide a limited contribution to the development of mathematical thinking. In this context, designing learning environments that support students' conceptual understanding, creative thinking, and analytical solution-generation skills may be one of the keys to developing mathematical thinking. Nevertheless, since this study does not offer a causal analysis, the relationships remain only at a correlational level and do not provide an in-depth explanation.

DISCUSSION

The secondary school period is a developmental stage where students develop beliefs about whether they like mathematics and often conclude that they are not good at it. During this period, students' perceptions of mathematical thinking are generally limited to performing mathematical calculations and following formulas (Bransford et al., 2012). Placing too much emphasis on remembering formulas and rules can lead to a surface learning approach and a low level of mathematical understanding (Murphy, 2016). The fact that mathematical thinking requires deep mathematical knowledge, reasoning ability, and heuristic strategies for problem-solving (Stacey, 2006) necessitates a deep understanding of mathematical learning. Indeed, individuals with a deep learning conception use the metacognitive processes necessary for mathematical thinking more effectively (Chin & Brown, 2000; Lim & Hwa, 2006). In this context, the need for a deep understanding of mathematical learning comes to the fore for the development of mathematical thinking skills. As a matter of fact, the results of the present study have revealed a significant and inverse relationship between middle school students' learning conceptions and mathematical thinking. Due to the nature of mathematics, a deep mathematical learning conception is necessary, which requires the comprehension of the underlying meanings of inter-conceptual relationships and procedures (Case, 2004; Skemp, 1976, 2006). Notably, in the learning conception variable set, the highest canonical loading value belonged to the "Acquiring and using knowledge" sub-dimension (–0.934), and a negative relationship was observed between this dimension and the sub-dimensions of mathematical thinking. This situation shows that when students carry out their processes of acquiring and applying knowledge in a superficial manner, their mathematical thinking skills may be weakened.

Another concept related to mathematical thinking is personal change. There is an inverse relationship (–.906) between mathematical thinking and personal differences. These differences generally include cognitive style, learning approaches, and motivation levels (Entwistle & Peterson, 2004). In this context, while individuals who adopt a more analytical and systematic thinking style can use their mathematical thinking skills more effectively, individuals who exhibit an intuitive or emotional approach may encounter difficulties in mathematical problem-solving processes (Holmes, 2012). Furthermore, personal changes can also be explained by emotional factors such as mathematics anxiety and self-efficacy perception. Mathematics anxiety, which is related to individual differences, plays a limiting role in individuals' mathematical thinking processes (Ashcraft & Kirk, 2001; Dowker, 2005). Finally, the inverse relationship of mathematical thinking with individual differences may be influenced by contextual factors such as the education system, family attitudes, and socio-economic status (Boaler, 2002). This situation reveals that a multidimensional approach should be adopted to understand to what extent and in what ways individual differences affect mathematical thinking.

In the study, the effect of social skills on mathematical thinking was found to be limited and inverse (–.679). Although an inverse relationship was observed between social skills and mathematical thinking in the study, this situation should not be generalized to mean that mathematical thinking develops solely through individual processes. When appropriately structured, social interaction can also be a supportive element for mathematical thinking. Indeed, when evaluated in the context of individuals' learning styles, socially oriented learning styles generally focus on more practical and applied skills rather than the abstract and logical thinking required by mathematical processes (Entwistle & McCune, 2004b). This situation suggests that social skills may indirectly create negative effects in areas that require individual focus, such as mathematical thinking.

Surface learning restricts individuals' cognitive deepening processes (Bigss, 1987; Entwistle & Mccune, 2004a). While higher-order thinking requires individuals to effectively use complex cognitive processes such as analysis, evaluation, and creation (Anderson & Krathwohl, 2001), individuals with a surface learning conception approach information superficially and ignore the underlying conceptual structures; this causes them to tend towards processing information disconnected from context and based on memorization

(Entwistle, 2009; Marton & Säljö, 1976). While individuals with a deep learning conception are more successful in the process of solving complex problems, those with a surface learning conception use information temporarily and do not make an effort to understand deeply in the process; they act in a more result-oriented manner (Marton & Säljö, 1976; Trigwell & Prosser, 1991). The inverse relationship found in the study between tendency higher-order thinking (–.871), reasoning (–.771), mathematical thinking (–.728), and problem-solving (–.610) on one hand, and the learning conception on the other, may be due to the participants having a surface learning conception. Therefore, ensuring that assessment processes, in particular, are not solely result-oriented may contribute to students integrating these cognitive processes with more meaningful learning. Research supporting this position reveals that assessment practices focusing on understanding, reasoning, and self-assessment support the development of students' intellectual maturity as well as their cognitive capacities, and that the importance of such assessment approaches becomes even more significant as students grow and their metacognitive awareness increases (Demetriou et al., 2019; Demetriou et al., 2020; Saltanat et al., 2023).

Limitations and Future Research

Although this study provides well-founded conclusions about the relationship between secondary school students' learning conceptions and mathematical thinking through a canonical correlation analysis in different variables, it has certain limitations. The present study is a quantitative study. In future studies, it is recommended to supplement the results with qualitative data in order to gain more in-depth knowledge and to examine the reasons for the obtained results in the context of the components of mathematical thinking. In addition, the direct and indirect effects between learning conceptions and mathematical thinking can be examined with structural equation modelling. With this method, for example, problem-solving strategies can be selected as a mediating variable and its effect can be evaluated. The interpretation of mathematical thinking in the problem-solving process in the context of different learning conceptions (surface/in-depth) can provide a clearer picture of the related situation. In future studies, examining this relationship in different age groups and learning contexts may allow for more comprehensive inferences about mathematics education.

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