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Research Article

Investigating Pre-Service Teachers' Beliefs Towards Mathematics: A Case Study

Bhesh Mainali [1*]
0000-0002-4158-2399

[1] Rider University, Lawrenceville, NJ, USA
[*] Corresponding author: bmainali@rider.edu

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ABSTRACT

This case study investigated two pre-service elementary teachers' changes in beliefs towards mathematics, learning mathematics, and teaching mathematics from the beginning to the end of mathematics methods courses. One of the participants had mathematics learning disability (MLD). The analysis of data gathered using concept maps and interviews at the beginning and end of the method courses revealed mixed findings between the two participants. The findings indicated that both pre-service teachers held negative beliefs before they took methods courses because they believed mathematics was boring, confusing, time-consuming, and a difficult subject. The data also suggested that learning mathematics was difficult and teaching mathematics was challenging. Their negative beliefs were associated primarily with their past school experiences as well as instructional strategies employed at school. One of the participants positively changed her beliefs at the end of the method courses since she described mathematics as a dynamic, creative, and useful subject. However, the other participant with MLD still held negative beliefs primarily about mathematics, as well as for learning and teaching mathematics at the end of the method courses. Data also revealed that utilizing appropriate instructional strategies based on individuals learning styles helped to make learning and teaching mathematics easy and interesting.

Keywords: belief, concept map, pre-service teachers, mathematics learning disability, methods course

INTRODUCTION

The meaning and definition of beliefs about mathematics are not uniform and vary from scholar to scholar (Ernest, 1989; Francis et al., 2015; Pajares, 1992; Phillip, 2007; Torner, 2002). Generally, belief is a psychologically held-standings, premises, or proposition about the world (Philipp et al., 2007), whereas Eynde et al. (2002) state that "beliefs refer to what "I" believe to be true, even though others "know" it to be true or not" (p. 23). In the domain of mathematics education, beliefs define as a cognitive dimension that develops based on one's mathematical world view (Schoenffeld, 1985), which develops based on an individual's comprehension, ideology, judgments, and experiences about mathematics as they interact with the mathematical world (Ernest 1989; Raymond,1997). Thompson (1992) further states that belief is an understanding that an individual has about the interaction of mathematics and the world that one believes to be true. Thus, belief refers to an individual's perception, thinking, and understanding about the nature of mathematics (can be true or false) along with the associated ideology and philosophy about learning and teaching mathematics.

Teachers' beliefs towards teaching and learning mathematics can have an important role in their instructional strategies (Phillip, 2007; Xie & Cai, 2021; Chan & Wong, 2014). The fact is that beliefs refer to teachers' integrated system of personalized assumptions about the learning and teaching mathematics, and of students (Artzt, 1999). Generally, teachers' beliefs towards mathematics teaching refer to the notions of

understanding mathematics and principles of effective classroom teaching, whereas beliefs towards learning refer to understanding mathematical ideas in the classroom in constructive ways (Xie & Cai, 2021).

Understanding teachers' beliefs is an important factor for educational reform. What teachers believe is a significant determiner of what gets taught, how it is taught, and what gets learned in the classroom (Wilson & Cooney, 2002). Thus, it is important to have a positive belief towards mathematics as early as possible when an individual begins a teacher education program at higher education. The fact is that negative beliefs likely contribute to negative classroom instructional strategies, which tend to develop negative beliefs in pupils as early as in elementary school (White et al., 2005). They further state that these students likely carry negative beliefs when they become teachers. In fact, pre-service teachers enter teacher education programs with pre-existing beliefs about mathematics based on various factors, including their experience of learning mathematics at school (Kane et al., 2002). The pre-existing beliefs could be positive or negative. The specific types of beliefs, particularly negative beliefs, that pre-service teachers bring with them in the teacher education program are considered to be the obstacles in the reform of classroom instruction (Richardson, 2003). The positive beliefs towards mathematics include various positive items such as creativity in mathematics, daily life application, interesting facts, the logic behind mathematics, etc. In contrast, negative beliefs towards mathematics include various items such as mathematics is all about rules, formulas, memorization, learning mathematics is difficult and boring, mathematics is only for mathematicians and scientists, and so forth (Collier, 1972). In general, positive belief refers to a positive disposition, and negative belief refers to a negative disposition towards mathematics. Some people consider mathematics as a static body of disconnected knowledge and skill that should be memorized without understanding is an example of negative beliefs, whereas others perceive mathematics as a set of connected–logical ideas and tools for solving problems in real life is an example of positive beliefs towards mathematics (Berk & Cai, 2019).

Several studies reported that students develop specific types of beliefs (positive or negative) towards mathematics primarily during mathematics learning experiences at school (Kane et al., 2002; Lappen & Theule-Lubienski, 1994; Maasepp & Bobis, 2014). Despite forming specific types of beliefs about mathematics at an early grade, teacher education programs at the college/university level need to foster positive beliefs towards mathematics for the pre-service teachers in addition to focusing on content knowledge and pedagogical content knowledge (Lappen & Theule-Lubienski, 1994). In fact, the teacher education program needs to change pre-service teachers' negative beliefs to positive beliefs towards mathematics (Looney et al., 2017. Focusing on pedagogical content knowledge–how to teach mathematics in effective ways–increase the positive belief towards mathematics in pre-service teachers as a result, they tend to be effective mathematics teachers in the future (Chang, 2015). Therefore, one of the aims of the teacher education program, particularly mathematics education, strive to develop positive beliefs towards mathematics for pre-service teachers.

THEORETICAL BACKGROUND

Several studies conducted in different fields, including mathematics education, suggest that belief is an important factor for teachers (Phillip, 2007; Richardson, 2003). Furthermore, teachers' instructional strategies and mathematical understanding are influenced by their existing beliefs, which affect the quality of their classroom teaching practices (Maasepp & Bobis, 2014; Xie & Cai, 2021). Furthermore, teachers' various types of beliefs shape and influence the actions taken during mathematics instruction (Bray, 2011). Ernest (2016) further stated that belief in mathematics has an important role in planning and theorizing mathematics teaching. In fact, the relationship between teachers' beliefs about mathematics and their classroom practices is a dynamic process, with each influencing the other (Nisbet &Warren, 2000). Thus, related studies suggested that teachers' beliefs towards mathematics influence their instructional decisions, classroom–action, and various lesson activities in the classroom (Beswick et al., 2011; Hart, 2002a; Lloyd, 2002; Stuart & Thurlow, 2000; Vacc & Bright, 1999).

The formation of belief about mathematics, teaching, and learning mathematics primarily are shaped by teachers' personal and professional learning experiences. The formation process likely goes for the entire life in conjunction with an individual mathematical experience (Clark et al., 2014). In fact, the belief system is a complex concept, and likely consists of different clusters (Chan & Wong, 2014), and various researchers have attempted to explain the concept of belief system.

Green (1971) proposed that there are three dimensions of the beliefs system:

(1) quasi-logicalness (primary and derivative beliefs),

(2) psychological centrality (central and peripheral beliefs), and

(3) cluster structure.

An individual who has beliefs about certain things is primarily based on his own logic. However, forming an individual's beliefs based on his own logic is not necessarily logical for somebody else. Similarly, Rokeach (1968) states that belief systems are organized psychologically but not necessarily logically. Quasi-logical has some primary beliefs and some derivative beliefs. Primary beliefs are generally developed from direct experiences over time and considered more effective than derivative beliefs. Some beliefs are more important than others for an individual. The more important belief is called central beliefs, which have a greater impact than peripheral beliefs. Thus, it is not easy to change the deeply-rooted central beliefs of an individual (Andrews, 2000), whereas peripheral beliefs can be changed easily (Furinghetti & Pehkonen, 2002). Cluster structure is the third dimension of beliefs, where beliefs are held in clusters. The formation of cluster structure beliefs is dependent on various other related beliefs, which occur in a set or groups.

Ernest (1989) classified belief towards mathematics as instrumental, Platonist, and problem-solving. Platonist view refers to mathematics as a structured and unchanged body of knowledge. The instrumental view alludes to mathematics as a collection of procedures, facts, and skills, whereas the problem-solving view considers mathematics as an expanding field of human discovery and dynamic in nature. Ernest (1989) further describes mathematics teachers' beliefs in three categories: beliefs towards the nature of mathematics, beliefs about teaching mathematics, and beliefs towards learning of mathematics. Schoenfeld (1992) describes typical students' beliefs about mathematics. Some of the beliefs described by Schoenfeld (1992) include, but are not limited to, mathematics problems that have only one answer, mathematics learned in school has little or nothing to do with real life, mathematics is all about memorization, and ordinary students cannot understand mathematics. Whereas, Goldin (2002) explains about different types of mathematics beliefs. The different types of beliefs are

(1) mathematical facts, rules, equation theorem, etc.,

(2) beliefs about mathematical validity or how mathematical truths are established, and

(3) beliefs about mathematical reasoning and strategies, beliefs about mathematical ability, beliefs about nature of mathematics and beliefs about teaching and learning mathematics, beliefs about the connection of mathematics with the physical world, individual's motivation and emotions towards mathematics.

Furthermore, learners' beliefs towards mathematics, indeed, included how they perceive and think about mathematics, such as mathematics is a boring (or interesting) subject as well as whether they love (hate) mathematics or not. The fact is that once students are exposed to the mathematical world, then they start to form some kind of mathematical beliefs.

The formation of beliefs about mathematics is a long process and intertwines with various factors. For example, Furinghetti and Pehkonen (2002) suggested that teachers' beliefs towards good mathematics teaching are deeply rooted in various other factors such as curriculum, teaching materials, and other resources. Therefore, it is not easy to change pre-service teachers' deeply rooted beliefs about mathematics regardless of providing appropriate intervention. A conceptual framework, proposed by Anderson et al. (1997, 2005), suggested that beliefs about mathematics are shaped by past school experiences, teacher education programs, mathematics teaching practices, and mathematical knowledge. Despite reluctant to change in negative beliefs towards mathematics held by pre-service teachers (Kane et al., 2002), teacher education programs strive to change pre-service teachers' beliefs towards mathematics (Aldridge & Bobis, 2001; Beswick & Dole, 2001, Karatas et al., 2017). More importantly, at the university/college level, mathematics educators can play an important role in fostering positive beliefs about mathematics for pre-service teachers. The fact is that the pre-service teachers' beliefs can be changed to be more cognitively aligned to support their future students' effective learning of mathematics (Hart & Memnun, 2015). Particularly, it is important to change pre-service teachers' negative beliefs about mathematics, such as mathematics is abstract, dry, boring, math-hate, difficult, etc., for them to be more effective in their future math-teaching career.

Some frameworks have been suggested, and research has been done on pre-service teachers' beliefs towards mathematics. However, little has been done to examine pre-service teachers' beliefs who have mathematics learning disability (MLD). The fact is that research on MLD is still in the emerging stage (Kaufmann et al., 2013). Learning disabilities are generally defined in terms of deficits in various psychological processes that affect certain areas of academic achievement including mathematics (Penny, 2018). National Center for Learning Disabilities (2007) states that learning disability (LD) is a neurological disorder, which affects the brain's ability to receive, process, store, and respond to information. The disorder can be in one or more of the basic psychological processes involved in understanding or using spoken language, manifesting itself in an imperfect ability on various factors, including mathematical calculation (USDOE, 2004). The disabilities might exist together with mental or physical disabilities (Pullen et al., 2011).

The LD that pertains particularly to mathematics such as computation, numeration, etc., is considered to be MLD. MLD refers to an inherent weakness in mathematical cognition not attributable to sociocultural or environmental causes (Mazzocco, 2007). There is, however, a general consensus among researchers that MLD is a biological disorder in the brain (Geary, 2007). MLD includes different types of disability such as processing mathematical information slowly, a disorder in biological and neurological development, or any other types of issues that are (in)directly linked with the ability to do mathematical tasks. A child with a disability is not simply a child less developed than his peers but is a child who has developed differently because of problems in biological and sociocultural lines of development (Vygotsky, 1993). Because of the neurological disorder, students with MLD experience pervasive difficulties in doing mathematical tasks. Mazzocco et al. (2008) reported that students with MLD made more errors even for the easiest task, unlike their low-achieving peers. As a result, students with MLD need more practice and ample amounts of time to solve even simple mathematics problems, and certainly they would have a lot of struggles for moderate difficulty problems.

As aforementioned, psychological centrality beliefs that are developed from direct experiences over the time have a greater impact than peripheral beliefs. We can argue that students with MLD are likely to develop more primary beliefs towards mathematics when they are exposed to mathematics in early grades. Thus, the deeply-rooted primary beliefs towards mathematics are not easy to change for MLD students compared to students without MLD. However, a number of studies reported that with appropriate instructional strategies and interventions were beneficial for students with MLD and had positive outcomes for learning mathematics (Marita & Hord, 2017; Shalev et al., 2005).

Belief Change Towards Mathematics

Changing the beliefs of pre-service teachers towards mathematics is an important domain in the field of mathematics education (Liljedahl et al., 2021). McCleary et al. (2013) reported fewer research studies conducted focusing on MLD. The number of studies conducted in this field also revealed inconsistent findings. Some studies reported a positive effect of mathematics methods courses on pre-service teachers' beliefs towards mathematics, while others reported no effect. Massepp and Bobis (2014) reported that pre-service teachers' negative beliefs about the nature of mathematics and teaching of mathematics were mostly influenced by their past school experiences. They further reported that pre-service teachers changed their negative beliefs about mathematics and teaching mathematics after completing the mathematics content/method courses. Beswick (2006) and Vacc and Bright (1999) also reported that pre-service teachers changed their beliefs after taking a mathematics methods course.

Similarly, Loone et al. (2017) and Markovits (2011) reported that most pre-service teachers hold negative beliefs towards mathematics; however, after taking a mathematics method course, participants had positive beliefs about mathematics. They further reported that the method course had a huge impact on changing in pre-service teachers' beliefs, but one method course might not be enough. In fact, mathematics method courses not only help to create positive beliefs but also help to improve the pre-service teachers' overall quality of teaching mathematics (Karatas et al., 2017). Because of the various pedagogical components and conceptually oriented instructional strategies in the mathematics methods courses, they can help to change in pre-service teacher belief towards mathematics (Lau, 2021). Whereas White et al. (2005) reported conclusive and inconclusive findings regarding pre-service teachers' beliefs towards mathematics. Their study further revealed positive beliefs towards constructivist approaches of teaching mathematics but contradictory

findings regarding the nature of mathematics after taking mathematics methods courses. Similarly, Yang et al. (2020) also reported that pre-service teachers tend to hold mixed beliefs towards mathematics.

Wilkins and Brand (2004) reported that pre-service teachers changed their beliefs about mathematics positively that was more consistent with current mathematics education reform after taking a mathematics methods course. Similarly, Hart (2002b) and Rolka et al. (2006) also reported that pre-service teachers changed their beliefs positively towards mathematics after completing an integrated content/mathematics methods course. In a different study, Philipp et al. (2007) reported that pre-service teachers developed more sophisticated beliefs towards mathematics when participants were given opportunities to learn about children's mathematical thinking processes. Pre-service teachers' beliefs about mathematics changed from mathematics as a system aspect and a utility to mathematics as a process aspect (Rolka et al., 2006). The fact is that method courses help develop a positive belief by providing opportunities for new experiences of teaching and learning mathematics (Francis et al., 2015).

Some studies reported that pre-service teachers' beliefs about mathematics are mainly rooted in their past school experiences and are difficult to change. For example, Holt-Reynolds (1992) suggested that teaching and learning mathematics beliefs are shaped by their own experiences as students. In fact, learning new theories in mathematics methods courses have little effect in changing pre-service teachers' beliefs about mathematics and teaching practices (Calderhead & Robson, 1991; Kagan, 1992). Vacc and Bright (1999) contended that to bring a positive belief about mathematics for pre-service teachers, the framework for mathematics content and methods courses needs to be consistent with each other. The fact is that mathematics methods and content courses that are housed in different departments at university likely cause inconsistency in the content frameworks, as well as there is a disconnect between the content and method courses.

Statement of the Problem

LD is a challenging problem in the teaching and learning field. The problem is even more widespread, particularly in the domain of mathematics. Various studies reported that (Badian, 1983; Gross-Tsur et al., 1974, Satsangi et al., 2018) approximately 5% to 8% of the school-age population have MLD. The recent statistics revealed that 14% of all public-school students have some sort of learning disabilities (NCES, 2020). Some of the population with learning disabilities will likely choose to pursue teaching as their future career. While some research has been done on pre-service teachers' beliefs towards mathematics, little has been done to examine the beliefs of pre-service teachers with MLD. It is the aim of this study to investigate some of the facets of pre-service teachers' beliefs towards mathematics, in particular, a pre-service teacher with MLD along with a pre-service teacher without MLD.

The Objectives of the Study

The related research studies regarding beliefs towards mathematics suggest that beliefs play an important role in instructional decisions and the quality of classroom teaching practices for mathematics teachers. It is important to develop positive beliefs towards mathematics for pre-service teachers because teachers with positive beliefs are more likely to be enthusiastic and interested in teaching and learning mathematics. As a result, they could be more effective in teaching mathematics lessons. In this context, this study is sought to investigate two pre-service elementary teachers' beliefs about mathematics, to learn mathematics, and to teach mathematics after taking two mathematics methods courses. However, the aim of this study was not to compare the two pre-service teachers in regard to their beliefs. The researcher posed the following research questions:

1. What is the belief of two pre-service elementary teachers, one with MLD, towards mathematics, learning mathematics, and teaching mathematics? What are the causes for the specific beliefs towards mathematics?
2. Do the two pre-service elementary teachers change their beliefs towards mathematics, learning mathematics, and teaching mathematics after taking two prescribed mathematics method courses?

RESEARCH DESIGN AND METHODOLOGY

Setting and Participants

The case study took place at a small liberal arts public university in a northern state of the United States during the spring semester of 2017. Two undergraduate elementary pre-service female white teachers in the teacher education program took part in the study during a mathematics methods course. As a part of the teacher education program, students were required to take general education courses, teacher education core courses, and courses in their minor track. Students were required to take three mathematics methods courses under the education core course. However, this study was conducted during the second mathematics methods course. There were 15 students in the course with three male and 12 female students.

This study was primarily sought to examine the belief in mathematics of a pre-service teacher with MLD and a regular student. As stated earlier, the purpose of this study is not to compare the two participants rather to present the two different cases. At the beginning of the course, participants were debriefed about the study and were requested to volunteer for the study. However, out of the two participants, the researcher asked a student who had MLD to participate in the study. The researcher received a letter from the Academic Success Center, a wing of the university, where accommodation was requested for the student because of the MLD. Before conducting the study, the researcher received consent forms from both participants. There was no compensation for the participants.

The first methods course primarily aimed to cover various basic mathematics skills and some pedagogical content knowledge required for elementary school teachers. The various content areas were problem-solving, basic concepts of set theory, the concept of problem-solving, number theory, operation of whole numbers, rational and irrational numbers, and fundamental geometry concepts and a short history of mathematics and mathematics education in the USA with conjunction common core state standards of mathematics (CCSSM). The second course was geared more for pedagogical content knowledge and teaching and learning theories. One of the main objectives of the course was to focus on conceptual understanding of mathematics, where students were encouraged not just to find the answers based on standard algorithms. Rather they were required to justify their mathematical solution process as to how they solved the given problems.

Furthermore, the emphasis was given to a learner-centered approach that promotes an understanding of mathematics that combines theory with pedagogical practices. Similarly, one of the main objectives of the history of mathematics was to provide insights into how mathematics has evolved as part of human activities. For example, a brief history of the development of number systems (Babylonian, Greek, and Mayan System) was discussed as to how number systems originated from the primitive stage to the place value system. To provide bigger pictures to the pre-service teachers about teaching and learning mathematics, a brief history of mathematics education was also discussed as what were the different eras in including CCSSM in the USA.

Various hands-on activities were utilized throughout the course. The various types of manipulatives were constantly utilized during the course to enhance mathematical reasoning and justification. The various hands-on activities included, but not limited were, Base Ten block, Fraction Kit, AngLegs, Geoboard, Cuisenaire Rod, construction paper, and so forth. Additionally, some virtual manipulatives, such as the National Council of Teachers of Mathematics: Illumination, National Library Virtual Manipulative, etc., were also utilized during the course. One of the main benefits of using various manipulatives in class was promoting a learning environment where students can explore, experiment, and discover mathematical ideas (Mainali, 2021b). Some of the video clips about the history of mathematics and teaching and learning about mathematics were also utilized.

The various content areas were learning and teaching mathematics in conjunction with learning theories and CCSSM, developing mathematical thinking and problem-solving ability, developing number concepts, understanding of numeration and place value concepts, developing whole number operations, alternative strategies (non-standard algorithm) for operations of the whole number system, understanding the meaning of fraction and its operation, and developing about basic geometric thinking. Additionally, analyzing mathematical errors and misconceptions based on sample works done by elementary school students was part of the course. The 75-minutes classes were taught every Tuesday and Thursday during the semester.

One of the main expectations during the course was that students should be able to explain the mathematical reasoning and solution process; not just to get the correct answer to the given problem. In order to provide some insights, consider the example of fraction addition. When students were asked to add two fractions with unequal denominators and if they made a common denominator before adding the fractions, they were required to justify why they needed to find the common denominator. In order to explain their reasoning, for example, students were required to utilize the fractions model (drawings, manipulatives, etc.) and utilized real-life examples such as cutting and slicing pizza. A sample work done by a student is provided in **Appendix A**, where students were required to write a story problem and complete the problem and explain their solution process by using a non-standard algorithm approach. A brief description of the two participants are as follows.

Participant one: Nichole

In her junior year, Nichole (pseudonym) was a full-time undergraduate (BA) doing an elementary education major with a minor in early childhood. She had no formal teaching experiences other than some field experiences required by the Teacher Education Program. She was normally shy and did not ask questions, and was hesitant to participate in the class's group discussion.

Nichole had MLD. She stated that her brain processes mathematical information very slowly, requiring additional time to complete tests and exams. The MRI report, which was taken right after she was born, showed that she had some neurological disorder in her brain, according to her doctor. As a result, Nichole had mathematical learning disabilities. Nichole informed that her doctor said that she would have difficulty even going through high school.

Nichole did not know that she had MLD until she was in the 6^{th} grade. As she advanced from elementary to middle school, she struggled more in doing mathematics tasks. As a result, she was placed in an intervention group at middle school, where she used to get additional support for mathematics. She also had a hard time doing mathematics problems when she was in elementary grades, but she spent lots of time getting them done. During elementary school, she thought everything was okay; she only needed additional time to complete mathematics tasks. When she asked her parents why she had so much struggles in doing mathematics, her mother explained that Nichole needed additional help and support in doing mathematics. Thus, Nichole somehow knew indirectly from her parents that she had some MLD when she was in 6^{th} grade. Later on, her mother explained the entire story about the MLD they were informed by the doctor when she was born.

Participant two: Megan

Megan (pseudonym) was a full-time undergraduate (BA) in her junior year doing an elementary education major with an early childhood minor. Mathematics was always a tough subject for her during school. She had no formal teaching experiences other than some field experiences during the teacher education program. She was very candid and active in the class and the teacher education program.

Research Instruments and Data Collection

Two different instruments were used to gather data: semi-structured interviews and concept maps. The data were collected before and after the intervention, viz. pre- and post-data collection. The concept map and a semi-structured interview were used to collect the data. In order to explore multiple aspects of pre-service teachers' beliefs about mathematics, learning mathematics, and teaching mathematics, the data was collected at the beginning and end of the second mathematics method course. One of the instruments utilized in this study was concept maps, which have been useful diagnostic and research tools (Buhmann & Kingsbury, 2015). Concept maps are essentially visual tools for organizing and representing knowledge (Kinchin et al., 2010). Various ideas can be easily connected with links in a concept map is a diagrammatic representation. In fact, a concept map helps to examine the relationship and pattern within the data (Streatfield & Hay, 2010).

In the visual representation form, participants have opportunities to express their ideas in a free-flow manner in the concept map. In fact, different modes of representation help students express their ideas more freely and clearly (Mainali, 2021b). Moreover, a concept map can be reviewed quickly, allowing participants to complete more passes, ideas, and thoughts through given information (Nesbit & Adesope, 2006). Because of

the visual nature of concept maps, it offers a multidimensional aspect of the ideas expressed in the map, in contrast to verbal representation (Barbro, 2008; Grevholm, 2008).

Participants were given three different concept maps to draw on a blank sheet of paper, with different questions on each concept map, at the beginning and end of the second mathematics method course. The three questions were, as follows:

(a) When you think about mathematics, what comes to your mind? Include as many factors as you can in your concept map.
(b) Draw a concept map about your beliefs towards mathematics.
(c) Draw a concept map regarding your belief about learning and teaching mathematics.

A semi-structured interview was the other instrument utilized to collect data at the beginning and end of the second method course. Interview questions were aimed to explore participants' beliefs about mathematics, learning, and teaching mathematics and the reasons as to why they held either positive or negative beliefs towards mathematics. The interview questions were the same for both participants; however, some additional questions were geared only for the MLD participant. The additional questions were sought to explore when the participant was notified about MLD during the school education system.

The researcher explained the basic idea of concept maps demonstrating some examples before they were given the maps to complete. 1st and 2nd concept maps were completed in the class. Once they completed the 1st concept map, they were given the 2nd concept map. In the same week, a semi-structured interview was conducted individually for each participant. Prior to the interview, participants were asked to draw a 3rd concept map.

At the end of the semester, participants were given the same set of instruments to complete. Participants did not have access to the pre-concept maps. However, rather than drawing a 3rd concept map, participants were given the 3rd pre-concept map back that they drew in the beginning and were asked if they wanted to make any changes in the concept map. And then post-interviews were administered to each participant. During the post-interview, participants were also asked why they did(not) change the third concept map.

Data Analysis

Choosing a case study for a small population is a challenging task. Seawright and Gerring (2008) suggested choosing a case study representing a sample and use variations on the dimension of theoretical interest. There are always limitations in time and access in case studies. Regarding this, researchers needed to select a case that should provide as much information as it can, and also, the selected participants should represent different cases. Thus, one of the participants was an average case, while the other served as a student differently than an average case. The analysis, particularly the audiotape interview, was carried out in several steps. The audiotape interviews were carefully transcribed word by word. Glesne (2011) recommends that the researcher starts a codebook soon after the data collection starts.

The pre- and post-concept map were analyzed based on key nodes, node links, crosslinks, and different mathematical domains used in the concept map. A key node represents a main point or idea that leads to constructing a concept map. A node represents ideas, phenomena, or associated terms in a concept map that links to key nodes. Links are the lines that connect key nodes and nodes. Crosslinks refer to the lines used to connect nodes in the concept map. The actual area and level of the concept map were not taken into consideration in the analysis.

RESULTS AND FINDINGS

Participant One: Nichole

Interviews

A semi-structured interview was conducted to examine Nichole's beliefs towards mathematics in conjunction with her MLD. The analysis of pre-interview data indicated that Nichole held negative beliefs towards mathematics because she believed mathematics was a boring and difficult subject. However, she also believed that the difficulty level depends on the way teachers present materials while teaching

mathematics lessons. One of the main reasons that created negative beliefs towards mathematics was her mathematics learning at school, where teachers did not utilize instructional strategies that addressed Nichole's learning needs. The data suggested that the development of negative belief was not only because of the MLD; rather, it was due to the instructional strategies in school mathematics. Nichole stated that

> "I think teachers are great people, but they may need to work on how they present materials, particularly in primary grades, and many teachers at school did not have time to understand my learning needs."

She further stated,

> "having a lot of ways to show materials and just being very interactive, teachers make learning mathematics easier."

Below, a part of the interview transcript between the researcher and Nichole is given:

> Researcher: Why did you have negative beliefs towards mathematics?
>
> Nichole: I was not good at mathematics. I was totally an average student until 6th grade.
>
> Researcher: Because you were average in the class, do you think that was a reason, which fostered negative beliefs towards mathematics?
>
> Nichole: No, I never liked mathematics for some reason. I went back and forth between liking science and mathematics. I liked science. It is my favorite subject, and I did great in science; math is not my subject. I didn't like mathematical concepts.
>
> Researcher: Why didn't you like mathematical concepts?
>
> Nichole: I did not find any interest working in number systems.
>
> Researcher: Why?
>
> Nichole: I do not know. I liked the concept better in science. For some reason, science concepts are interesting.
>
> Researcher: Why do you think mathematics is not interesting?
>
> Nichole: I think lots of time the way it was taught. If you sit there (class) for a lecture by someone in a middle school classroom on how to do a specific algebra problem, you are not going to be able to hold my attention for 75 minutes.

The examination of the post-interview also revealed that Nichole held a tendency towards negative beliefs towards mathematics at the end of methods courses. She stated that

> "mathematics is difficult, frustrating, and a boring subject."

Data analysis reflected Nichole's concerns about instructional strategies, which was one of the main reasons that developed negative beliefs towards mathematics. Nichole's statement further attested this,

> "mathematics can be interesting based on the way mathematical materials are presented."

The negative beliefs towards mathematics were further revealed in the post-interview, as follows:

> Researcher: Do you still have negative beliefs after going through the two math methods courses?
>
> Nichole: Yes, I still have the same belief.
>
> Researcher: Why do you still have the same belief?

> Nichole: It could be just… I had that thought process from the beginning, and it is really hard to change that because I had that for such a long time in elementary school, and I did not get any good changes until I was in middle school. So, for 5-6 years, I did not like it, and then all the sudden, it was now only a matter of (like) certain teachers that was making the difference, and like my mathematics and how I learned it was not consistent from 6 to 12 grades.

The data further suggested that Nichole did not change her belief despite taking mathematics methods courses because her negative belief towards mathematics was deeply rooted for a long time at school. The data further contended that Nichole's negative belief was also associated with the difficulty of doing mathematics. She had much struggle in doing mathematics, which fostered negative beliefs. However, she also stated that teachers could make learning mathematics fun and interesting. She further stated:

> "Freshman year in my high school, my teacher sat there and readout of the book for the entire class, whereas my other teacher switched to the second half did not utilize the same strategies. But he made everything connected to the students rather than books, so it was a lot more energetic, so I went from C grade to A grade."

Nichole stated that she would do her best to generate positive beliefs towards learning mathematics for her future students despite her negative beliefs. She hopes that the MLD could be beneficial rather than deleterious for her teaching profession.

> Researcher: How do you think your negative beliefs would impact your future students?
>
> Nichole: The way I looked at it is, I think it will be hard for me to change my natural beliefs in mathematics in general. But, if I take the way that I was taught and make the way I am teaching the way I wish I had been taught, it can totally make the difference in kids choosing to go more in mathematics.
>
> Researcher: As you know, you have MLD, does this somehow inspire you to pursue a teaching career, or it have nothing to do with mathematics?
>
> Nichole: Actually, that was part of the reason that why I was going back and forth about my teaching career because I thought I was going to get some sort of retaliation from some education departments. I think some teachers in school would say it is not a great path for me because of MLD. But also look the other way. I have advantages in the sense that I do have this difference so that I can better understand students who struggle in learning mathematics. I would have more empathy for struggling learners. And I know how to make changes for those kids because I know what was for me.

The pre- and post-interview data analysis revealed that Nichole held negative beliefs towards mathematics. She did not change her beliefs after taking the mathematics methods courses. Despite having negative beliefs, Nichole wanted to develop a positive belief towards mathematics for her students in the future. Further, she stated that you shouldn't get discouraged if you want to pursue a teaching profession, even if you have an LD. She stated:

> "I would say make mathematics fun for the kids because kids are going to enjoy mathematics, and you will be successful teacher regardless of your disability."

Concept map

Displaying mathematics in the central node, Nichole drew six links and several crosslinks to connect different nodes in the 1st pre-concept map, as shown in **Figure 1**. Three out of the six nodes indicated that Nichole held negative beliefs about mathematics because she believed mathematics is *confusing*, *difficult*, and *time-consuming*. Apparently, other nodes in the concept map are linked to mathematics content. The analysis also suggested that she believed mathematics was simple initially and eventually gets difficult, which also implied a tendency towards negative beliefs towards mathematics.

Figure 1. 1st pre-concept map

Figure 2. 2nd post-concept map

Nichole added several new nodes in her 1st post-concept map. Examination of the concept map revealed that Nichole held negative beliefs about mathematics because she still believed that mathematics was difficult and confusing. She, however, did not include "time-consuming" in her post-concept map, which was part of the 1st pre-concept map. Furthermore, various math topics were added in the post-concept map, which was not part of the pre-concept map. It seemed that the method courses she took helped to add these new math topics, which were discussed during the methods course. The data also suggested that people learn mathematics differently and, if instructional strategies don't address an individual's learning needs, the instructional strategies can confuse learning and understanding mathematical concepts.

Nichole used five links with several crosslinks to connect different nodes in the 2nd pre-concept map. The analysis of the 2nd pre-concept map did not provide a clear finding regarding Nichole's beliefs about teaching and learning mathematics. However, different terms she used in the concept map were associated with the importance of learning and teaching mathematics. Data further suggested that Nichole believed learning and teaching mathematics should be explorative styles in which students could explore and learn conceptual understanding rather than just computational skills. Also, it seemed that mathematics teachers should always be ready to help students by opening their office doors regardless of learning differences in students.

Nichole generated six links with various crosslinks to connect different nodes in her 2nd post-concept map, as shown in **Figure 2**. The 2nd post-concept map examination revealed that Nichole still had negative beliefs about learning and teaching mathematics at the end of the methods course since she described learning and teaching as "hard, confusing, and time-consuming."

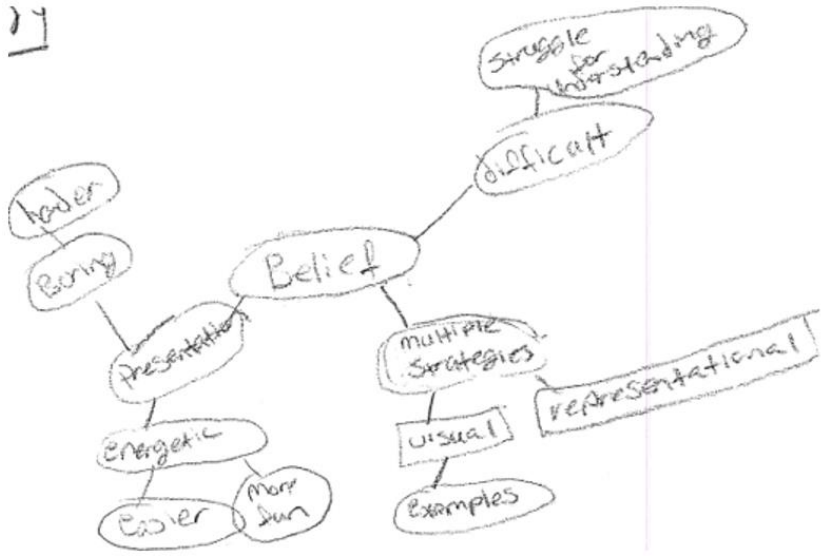

Figure 3. Pre-concept map

However, she also used various nodes that demonstrated the importance of teaching and learning mathematics. For example, visual representation and connecting mathematics with real-life used in the concept map expressed the importance of learning and teaching mathematics. The concept map also reflected various aspects of the methods course, such as using visual representation in math instructions, connecting math with real-life, and focusing on word (story) problems.

Nichole used three main links with several crosslinks to connect different nodes to express her belief about mathematics, as shown in **Figure 3**. The pre-concept map revealed that Nichole held negative beliefs towards mathematics because she described mathematics as a boring and difficult subject. She also believed mathematics was hard and difficult to understand, which further attested to Nichole's negative beliefs. She, however, also believed that mathematics could be fun to learn and easier to understand depending on the way materials are presented in mathematics lesson activities. Thus, data suggested that Nichole's beliefs about mathematics seemed to rely more on the way teachers present/teach materials and how energetic they were in the class.

Right before the post-interview, the 3rd pre-concept map was given back to Nichole, and asked if she wanted to make any changes. She simply replied that she did not want any changes, which suggested that her beliefs towards mathematics at the end of the method courses did not alter. Further, she was asked why she included words such as difficult and boring in her concept map. The analysis indicated that it was associated with her mathematics learning experiences from elementary to high school. She stated that

> "I think a lot of time the way it was taught. If you sit there and are lectured by someone in a middle school classroom on how to do a specific algebra problem, for example, you would not be able to hold my attention for 75 minutes."

She further stated:

> "If teachers had utilized more manipulatives stuff and had made learning mathematics fun early on, I might have a different point of view than now I do. But I don't know my belief is going to change now that I am way past elementary school, where that belief could have changed probably."

The concept map revealed that the negative belief generated towards mathematics was as early as elementary school. The concept map also included words such as energetic and presentations, which seem to have positive beliefs towards mathematics. However, these two words were linked with instructional strategies and teachers' way of presenting mathematical contents. She states that

> "some teachers were really energetic about mathematics because they really liked mathematics; probably why they are mathematics teachers. Then it's going be a lot easier for me because the teacher makes learning mathematics fun in the class."

The cross-analysis of three different data sets further suggested that Nichole had negative beliefs towards mathematics and did not change after the method course. In fact, cross-analysis of data in qualitative research could increase confirmation and credibility of findings when data gathered through different instruments are found to be consistent (Knafl & Breitmayer, 1991). Some of the common themes that were revealed in both data sets are: math is boring, hatred of mathematics (concept map), learning mathematics is frustrating, mathematics is difficult and time-consuming (interview). Thus, these common themes suggested that Nichole had negative beliefs towards mathematics. Nichole's mathematics learning experiences are primarily associated with her past school experiences, which helped to foster negative beliefs. One of the main factors for developing negative beliefs was the instructional strategy utilized in school that did not address Nichole's learning needs. As a result, she never really understood mathematical concepts from her early grades to the end of high school. Therefore, she did not like mathematics. The data set also confirmed that learning mathematics can be interesting and fun based on the way materials are presented in mathematics lessons and the role of teachers in the classroom.

The fact is that Nichole had negative beliefs towards mathematics for a long time when she was in school. Despite believing that mathematics was a confusing and difficult subject, she was able to find the application of mathematics in real life during the mathematics methods courses. The data analysis indicated that the instructional strategies need to address students' needs and learning preferences. Furthermore, the analysis also indicated that appropriate instructional strategies and carefully selected lesson materials likely make math learning fun and interesting for students.

The cross-comparison further confirmed that Nichole emphasized the importance of multiple strategies, the role of visual representation, the effect of hands-on learning, and making mathematics more meaningful in conjunction with real-life application. Thus, it seemed that method courses helped her understand the importance of various strategies in teaching and learning mathematics; however, it did not help to change her beliefs towards mathematics. Nichole believed that her MLD would be beneficial instead of harmful for her teaching profession in the future. She thought that she would be more effective in teaching MLD students in her future teaching. She also advised that anyone can be a successful teacher despite having MLD. She also believed that we need to take MLD as a strength instead of a weakness for college students who want to be in teaching.

Second Participant: Megan

Interviews

The analysis of the pre-interview revealed that Megan held negative beliefs towards mathematics. She described mathematics as a confusing, boring, and difficult subject because she never understood mathematics, and she always hated the subject. She stated that "stupid peoples' mathematics course" refers to a college developmental mathematics course she recently took. One of the main reasons she hated mathematics was how her mathematics teachers taught lessons in her school mathematics courses.

> Researcher: Why did you have negative beliefs towards mathematics?
>
> Megan: I never really understood mathematics, so I hated it.
>
> Researcher: Why did you hate mathematics?
>
> Megan: Because I did not understand mathematics. So, I got frustrated and I would give up. So, I think because I did not understand, I just let that overtake my opinion and formed negative beliefs towards mathematics.
>
> Researcher: Do you believe that you developed negative beliefs towards mathematics because of that reason?

> Megan: I think when I did not understand mathematics, I just had a negative outlook on it.
>
> Researcher: Why do you think that you did not understand mathematics?
>
> Megan: I think because I remembered my 7th grade, I did not like to turn in my homework, for instance, and I fell behind and finally got on track, but I missed like fundamental skills I believe. And then algebra I was fine, but the geometry was really hard.
>
> Researcher: Did the negative belief emerge out at elementary school?
>
> Megan: No, elementary school was fine, like it was always hard for me, but I did not hate mathematics. Negative beliefs developed more as I progressed to middle school and kept going in high school because I did not understand mathematics. Because probably did not put enough effort into mathematics and began to hate it.
>
> Researcher: Did you understand math at elementary school?
>
> Megan: Yes, I did understand it at elementary school, so I did not hate it.
>
> Researcher: Why do you think that you did not hate math at elementary school?
>
> Megan: Because I did understand the concepts of math at elementary school.

The data analysis suggested that Megan developed negative beliefs, which were associated mainly with the way she was taught mathematics lessons, primarily when she was in middle school. Because of the different instructional strategies utilized by Megan's teachers, she liked algebra more than geometry. She did not understand mathematics and hated it, generating negative beliefs towards mathematics. Megan also believed that learning and teaching mathematics is challenging but can be made teaching and learning easy by incorporating various hands-on activities in mathematics lessons.

The analysis of the post-interview revealed that Megan changed her beliefs towards mathematics. She did not hate mathematics anymore; however, she still believed mathematics was difficult. She stated that

> "now, I know how mathematics works and applies in our daily life. For example, with volume, I did not realize that literally what fits in the cylinder is the area of a circle and multiplied by its height. I did not get it until I took mathematics method courses."

She also changed her beliefs towards learning and teaching mathematics at the end of the method courses. She stated that mathematics is not just about getting correct answers; instead, it is about understanding mathematical concepts. The change in beliefs towards teaching mathematics was further illustrated by the fact that she strongly advocated using manipulative and hands-on activities in learning and teaching mathematics. She still believed learning mathematics is difficult because she believed that the language of mathematics is different; however, she further stated,

> "if we start with a good base and focus on conceptual understanding, then learning mathematics can be fun and interesting."

She also believed that mathematics is more than just doing standard algorithms. She further stated that, after taking the method courses,

> "I feel more confident teaching it, and I understand math and now have positive beliefs. I think I hated it because I did not understand it".

Thus, the examination of data suggested that the methods courses helped Megan change her beliefs towards mathematics.

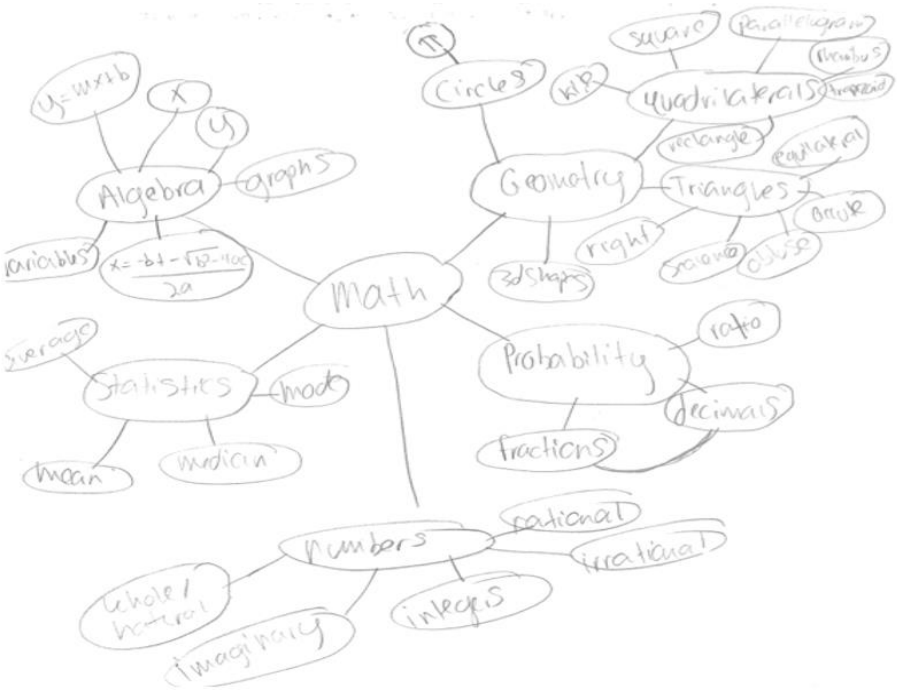

Figure 4. 1st pre-concept map

Concept map

The question for the 1st concept map was what comes to your mind when you think about mathematics. Megan included many mathematical topics linked to each other in the 1st pre-concept map, as shown in **Figure 4**. With mathematics as a key node, she drew several nodes connected by links and crosslinks in the 1st concept map. Many nodes used in the concept map were primarily connected with different mathematics content areas. The analysis of the 1st pre-concept map analysis did not explicitly indicate any positive or negative beliefs towards mathematics since it did not include any words associated with beliefs about mathematics. The concept map also suggested that Megan viewed mathematics in a holistic way, connecting various mathematical domains to each other.

Megan added three new nodes: everyday life mathematics, money, and fractions in her 1st post-concept map. She clearly indicated the importance of mathematics by connecting it to daily life, such as in banking, measurement, and shopping. The method courses were focused on meaningful mathematics learning in conjunction with real-life application and conceptual understanding, which can be seen in her post-concept map. It seemed that Megan developed positive beliefs to some extent based on the concept map.

Analysis of the 2nd pre-concept map suggested that Megan did not have clear positive or negative beliefs towards learning and teaching mathematics. Repetition was one of the nodes in the concept map, which seemed to indicate negative beliefs to some extent; however, no conclusion can be drawn about beliefs towards learning and teaching mathematics.

Megan used four main links and several crosslinks to connect several nodes in her 3rd post-concept map, as shown in **Figure 5**. The analysis revealed that Megan clearly shifted her beliefs about learning and teaching mathematics at the end of the methods course because she used several words such as be positive, make mathematics fun, focus on process not on the outcome etc., in the 2nd post-concept map. One of the nodes in the concept map states, "teach kids never to give up," which further conceded a strong positive belief towards learning and teaching mathematics.

Emphasizing the role of manipulatives and connecting teaching and learning mathematics with real-life application were indicative of a constructivist approach, which further demonstrated positive beliefs towards learning and teaching mathematics. The examination of the 2nd post-concept map also exhibited a strong tendency towards differentiated instruction, multiple instructional strategies, and individual needs in learning and teaching mathematics, which also showed Megan changed her beliefs at the end of method courses.

Figure 5. 2nd post-concept map

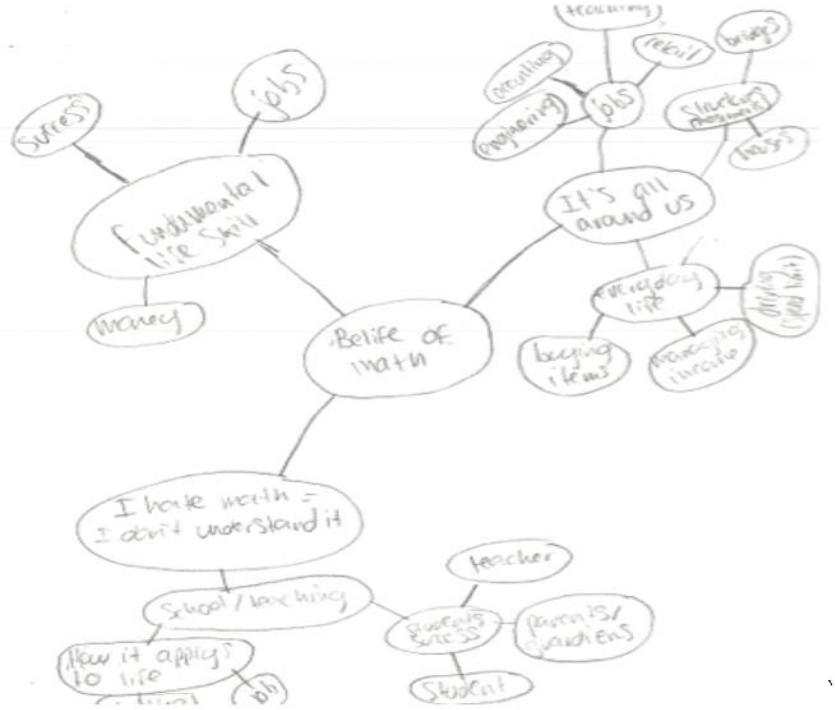

Figure 6. 3rd pre-concept map

Megan used three links and several crosslinks to connect various nodes to express her belief about mathematics in the 3rd pre-concept map, as shown in **Figure 6**. One of the nodes in the 3rd concept map, "I hate mathematics," clearly indicated negative beliefs towards mathematics. Though Megan did hate mathematics, in the meantime, she also emphasized the importance of mathematics since she believed that mathematics was all around us. The data further suggested that Megan's negative belief was primarily associated with her past school experiences, particularly in middle school.

Right before the post-interview, Megan was given the 3rd pre-concept map back and asked if she wanted to make any changes. She added one crosslink with three nodes under a new node "process." The addition of the new node in the 3rd concept map indicated positive beliefs towards mathematics since she emphasized more in the mathematical process than getting a correct answer. She was also asked why she added one node in her post-concept map. She stated, "I added the process because one thing I learned during methods courses was that process was more important than product." The positive beliefs towards mathematics were also revealed in the post-concept map since Megan further stated that

> "teachers have a big impact on mathematics, as my story kind of show, and the approaches they gave and the different manipulatives used. Like, I remember you (researcher/instructor) brought manipulatives in these mathematics methods classes, which was fun since we can actually use them in our classroom to help our students concretely see it."

Thus, analysis of the post-concept maps revealed that Megan changed her beliefs towards mathematics, teaching and learning mathematics at the end of the method course.

The cross-analysis of the pre-data set revealed that Megan held negative beliefs towards mathematics. Some of the common themes derived in each data set prior to the intervention were "hate mathematics, mathematics is all about computation, and mathematics is boring," which indicated negative belief towards mathematics. Megan did not understand mathematics. Thus, she hated mathematics, which fostered negative beliefs about mathematics. The way her school teachers taught mathematics, particularly in middle school, also cultivated negative beliefs towards mathematics. The date further confirmed that mathematics teachers did not make her understand the mathematical concept; as a result, she lost her interest and began to hate mathematics.

The cross-analysis of the post-data set revealed a change in belief towards mathematics. She understood the mathematical concept, which helped change her belief in mathematics. Megan did not hate mathematics anymore (interview); she agreed that mathematics is creative, beautiful, and useful to human endeavor, and she stated that the mathematical process is more important than that of the product. The common theme derived from each data set further suggested positive beliefs towards mathematics. The cross-analysis also indicated that Megan believed mathematics was an important subject since it is all around us and useful in everyday life. Analysis of data also confirmed that utilizing various strategies such as hands-on activities, real-life mathematics application, etc., helped her to promote positive beliefs towards teaching and learning mathematics at the end of the method course.

DISCUSSION

As stated earlier, the purpose of this study is not to compare the two participants rather to present the two different cases. Both participants held negative beliefs towards mathematics preceding the method course; however, only one of the participants changed her beliefs towards mathematics after completing the method courses. The findings of Megan support several studies (Beswick, 2006; Francis et al., 2015; Hart, 2002a; Karatas et al., 2017; Looney et al., 2017; Markovits, 2011; Massepp & Bobis, 2014; Stuart & Thurlow, 2000; Vacc & Bright, 1999) in which pre-service teachers changed their beliefs about mathematics after taking methods courses. Results revealed that pre-service teachers' past school experiences, which also aligns with similar studies, mostly influenced the beliefs about mathematics. This study further suggested that mathematics teachers fostered negative beliefs towards mathematics because instructional strategies they employed might not align with participants' learning preferences. Similar to the findings of Beswick (2006) and Rolka et al. (2006), this study also contended that the role of hands-on activities in the lesson, real-world mathematical activities, and instructional strategies focusing on conceptual understanding could play an important role in developing positive beliefs towards mathematics at school. The fact is that mathematics teaching and learning can be enhanced by incorporating various hands-on activities (Mainali, 2021a).

Nichole's negative beliefs about mathematics began as early as in elementary school, which still prevail. Nichole's primary beliefs were developed from direct experiences over the time during elementary school, which was not easy to change. This is consistent with the findings of Forgasz and Leader (2008), Green (1971), Kim et al. (2013), and Yang et al. (2020). It seemed that because of MLD, Nichole held strong negative beliefs

towards mathematics in early elementary grades. The fact is that students with MLD experience pervasive difficulties in doing mathematics and make more errors even for the easiest task, struggling in doing mathematics. As she had lots of struggles in mathematics, she developed primary negative beliefs towards mathematics. Thus, Nichole's primary beliefs towards mathematics did not change, further supported by Andrews's (2000) study.

The findings of Nichole did not support the result reported by Marita and Hord (2017) and Shalev et al. (2005), where appropriate instructional strategies and interventions were beneficial for students with MLD and had positive outcomes for learning mathematics. However, it seemed that method courses helped her understand the importance of various strategies regarding teaching and learning mathematics. Despite not changing her beliefs, Nichole tends to develop a positive belief towards mathematics for her future students in her future teaching career. Furthermore, Nichole constantly stated that if the appropriate instructional strategies had been utilized by mathematics teachers when she was in school, she would have had positive beliefs towards mathematics. Normally, MLD affects an individual's ability to receive, process, store, and respond to mathematical information, consistent with the National Center for Learning Disabilities (2007). As a result, she was likely frustrated spending lots of time-solving even easier mathematics problems and could not solve the problems. Thus, she developed primary negative beliefs so strongly that she was not ready to change her beliefs despite after taking the methods course, which contradicts the findings of Marita and Hord (2017) and Shalev et al. (2005).

Nichole's result was also consistent with the findings of various studies (Calderhead & Robson, 1991; Holt-Reynolds, 1992; Kagan, 1992), where learning new theories in mathematics methods courses or elsewhere have little effect in changing pre-service teachers' beliefs about mathematics because beliefs are mostly shaped by their own experiences as students at school mathematics. Similar to Vacc and Bright's (1999) suggestions, the lack of change in Nichole's beliefs likely has been caused by inconsistent mathematics content and methods courses based on different frameworks employed in method and content courses.

Megan held negative beliefs towards mathematics prior to the intervention. She did not understand mathematics while she was in school. Thus, she hated mathematics, which fostered negative beliefs about mathematics. One of the main reasons that cultivated negative beliefs towards mathematics was the instructional strategy utilized to teach math lessons in school. Similar to Massepp and Bobis (2014) findings, Megan's negative beliefs about mathematics and teaching mathematics were mostly influenced by her past school experiences. The finding of this study also supports the findings reported by Holt-Reynolds (1992), where beliefs about teaching and learning mathematics are shaped by students' own experiences as students in the school.

This study found that Megan changed her beliefs about mathematics at the end of the method course, which was consistent with other studies (Beswick, 2006; Rolka et al., 2006; Vacc & Bright,1999; Wilkins & Brand, 2004) as well as contradicted other studies (Calderhead & Robson, 1991; Kagan, 1992). One of the main focuses of the method courses was conceptual understanding, doing mathematics tasks rather than just looking for the final answer, learning some best teaching practices, and analyzing students' misconceptions. As a result of the focus on different aspects of mathematics, Megan changed her beliefs since Megan emphasized understanding the problem-solving process and dynamic nature of mathematics, which is likely derived from the problem-solving aspect of mathematics beliefs proposed by Ernest (1989).

Limitation, Implication, and Significance

This study revealed mixed findings. This study is limited to only two pre-service teachers. Therefore, the findings cannot be generalized for all pre-service teachers. The method courses were not explicitly aimed at changing pre-service teachers' beliefs towards mathematics, learning mathematics, and teaching mathematics; rather, method courses were focused on exploring pedagogical content knowledge suitable for elementary school pre-service teachers. A different mathematics method courses oriented particularly more to change pre-service teachers' beliefs about mathematics may yield a different result. As explained in the data analysis, the 3rd pre-concept map was provided back, and participants were asked if they wanted to make any changes. Different instruments (questions) would probably lead to a different result.

Regardless of utilizing some differentiation instructions, instructional strategies in method courses were primarily geared to meet the expectations of the majority of students in the class. A different methods course designed to address Nichole's needs might yield a different result. Additionally, the intervention of multiple mathematical methods courses could yield a different result. This study also reported that pre-service teachers' negative beliefs towards mathematics could be changed despite their resistance. Fullan (1991) stated that

> "the relationship between prior beliefs and program experiences is crucial, complex, and not straightforward" (p. 296).

However, teacher education programs strive to change pre-service teachers' pre-existing negative beliefs towards mathematics. Once pre-service teachers start their teaching career after completion of the teacher education program, hopefully, they will likely employ instructional strategies that help to instill positive beliefs about mathematics for their future students.

In both the pre- and post-data set, Nichole strongly suggested taking students' learning preferences and needs into account. As she stated, her teachers did not address her learning needs, which eventually fostered negative beliefs towards mathematics. This is an important implication for researchers and teachers to consider students' learning needs to develop positive beliefs about mathematics as early as elementary school. Data also indicated that pre-service teachers did not always hold negative beliefs. The finding suggested that teacher education programs, particularly mathematics methods courses, endeavor to develop positive beliefs for future mathematics teachers. This study further suggested that it is important to design curricula that address students' preferences for learning in school mathematics. The fact is that it is instructional strategies should be focused on incorporating both preferences visual and nonvisual solution methods in mathematics lesson activities (Mainali, 2019). Also, from the teaching standpoint, this study suggested utilizing instructional strategies that address students' MLD as early as possible so that students likely have positive beliefs towards mathematics.

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REFERENCES

Aldridge, S., & Bobis, J. (2001). Multiple learning contexts: A vehicle for changing preservice teachers' mathematical beliefs, knowledge and practices. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond: Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 43-49). Australia.

Andrews, P. (2000, September 23). *The influence of context on teachers' conceptions of mathematics and its teaching* [Paper presentation]. The European Conference on Educational Research, Edinburgh, UK.

Artzt, A. F. (1999). A structure to enable preservice teachers of mathematics to reflect on their teaching. *Journal of Mathematics Teachers Education, 2*(2), 143-166. https://doi.org/10.1023/A:1009999004407

Badian, N. A. (1983). Arithmetic and nonverbal learning. In H. R. Myklebust (Ed.), *Progress in learning disabilities* (pp. 235-264). Grune and Stratton.

Barbro, G. (2008). Concept maps as research tool in mathematics education. In *Proceedings of the 3rd International Conference on Concept Mapping*. Finland.

Berk, D., & Cai, J. (2019). Mathematics teacher beliefs. In M. A. Peters (Ed.), *Encyclopedia of teacher education*. Springer. https://doi.org/10.1007/978-981-13-1179-6_236-1

Beswick, K. (2006). Change in preservice attitudes and beliefs: The net impact of two mathematics education units and intervening experiences. *School Science and Mathematics, 106*(1), 36-47. https://doi.org/10.1111/j.1949-8594.2006.tb18069.x

Beswick, K., & Dole, S. (2001). Dispelling the myths: Influencing the beliefs of pre-service primary teachers. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond: Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 90-97). Australia.

Beswick, K., Callingham, R., & Watson, J. (2011). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education, 14*(1), 1-27.

Bray, S. W. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2-38. https://doi.org/10.5951/jresematheduc.42.1.0002

Buhmann, S. Y., & Kingsbury, M. (2015). A standardized, holistic framework for concept-map analysis combining topological attributes and global morphologies. *Knowledge Management & E-Learning, 7*(1), 20-35. https://doi.org/10.34105/j.kmel.2015.07.003

Calderhead, J., & Robson, M. (1991). Images of teaching: Students' teachers' early conceptions of classroom practice. *Teaching and Teacher Education, 7*, 1-8. https://doi.org/10.1016/0742-051X(91)90053-R

Chan, Y.-C., & Wong, N.-Y. (2014). Worldviews, religions, and beliefs about teaching and learning: Perception of mathematics teachers with different religious backgrounds. *Educational Studies in Mathematics, 87*(3), 251-277. https://doi.org/10.1007/s10649-014-9555-1

Chang, Y. (2015). Examining relationships among elementary mathematics teachers' efficacy and their students' mathematics self-efficacy and achievement. *EURASIA Journal of Mathematics, Science, & Technology Education, 11*, 1307-1320. https://doi.org/10.12973/eurasia.2015.1387a

Clark, M. L., DePiper, J. N., Frank, T. J., Nishio, M., Campbell, P. F., Smith, T. M., Griffin, M. J., Rust, A. H., Conant, D. L., & Choi, Y. (2014). Teacher characteristics associated with mathematics teachers' beliefs and awareness of their students' mathematical dispositions. *Journal for Research in Mathematics Education, 45*(2), 246-284. https://doi.org/10.5951/jresematheduc.45.2.0246

Collier, P. (1972). Prospective elementary teachers' intensity and ambivalence of beliefs about mathematics and mathematics Instruction. *Journal for Research in Mathematics Education, 3*(3), 155-163. https://doi.org/10.5951/jresematheduc.3.3.0155

Ernest, P. (1989). The knowledge, beliefs, and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*, 13-33. https://doi.org/10.1080/0260747890150102

Ernest, P. (2016). The problem of certainty in mathematics. *Educational Studies in Mathematics, 92*(3), 379 -393. https://doi.org/10.1007/s10649-015-9651-x

Forgasz, H., & Leder, G. (2008). Beliefs about mathematics and mathematics teaching. In P. Sullivan, & T. Wood (Eds.), *The international handbook of mathematics teacher education: Knowledge and beliefs in mathematics teaching and teaching development* (pp. 173-192). Sense Publisher. https://doi.org/10.1163/9789087905439_010

Francis, D., Rapacki, L., & Eker, A. (2015). The individual, the context, and practice: A review of the research on teachers' beliefs related to mathematics. In H. Fives, & M. G. Gill (Eds.), *International handbook of research on teachers' beliefs* (pp. 336-352). Routledge.

Fullan, M. (1991). *The new meaning of educational change*. Teachers College Press.

Glesne, C. (2011). *Becoming qualitative researchers: An introduction*. Pearson.

Goldin, G. A. (2002). Affect, meta affect, and mathematical belief structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 59-72). Kluwer Academic Publisher. https://doi.org/10.1007/0-306-47958-3_4

Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 38*, 25-33. https://doi.org/10.1111/j.1469-8749.1996.tb15029.x

Hart, L. (2002a). A four-year follow-up study of teachers' beliefs after participating in teacher enhancement project. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp 161-176). Kluwer Academic Publisher. https://doi.org/10.1007/0-306-47958-3_10

Hart, L. (2002b). Preservice teachers' belief and practice after participating in an integrated content/methods course. *School Science and Mathematics, 102*(1), 4-14. https://doi.org/10.1111/j.1949-8594.2002.tb18191.x

Hart, L., & Memnun, D. S. (2015). The relationship between preservice elementary mathematics teachers' beliefs and metacognitive awareness. *Journal of Education and Training Studies, 3*(5), 70-77. https://doi.org/10.11114/jets.v3i5.840

Holt-Reynolds, D. (1992). Personal history-based beliefs as relevant prior knowledge in course work. *American Educational Research Journal, 29*, 325-349. https://doi.org/10.3102/00028312029002325

Kagan, D. M. (1992). Professional growth among preservice and beginning teachers. *Review of Educational Research, 62,* 129-169. https://doi.org/10.3102/00346543062002129

Kane, R., Sandretto, S., & Heath, C. (2002). Telling half the story: A critical review of research on the teaching beliefs of university academics. *Review of Educational Research, 32*(1), 177-228. https://doi.org/10.3102/00346543072002177

Karatas, I., Guven, B., Ozturk, Y., Arslan, S., Gursoy, K. (2017). Investigation of pre-school teachers' beliefs about mathematics education in terms of their experience and structure of their education. *EURASIA Journal of Mathematics, Science & Technology Education*, *13*(3), 673-689. https://doi.org/10.12973/eurasia.2017.00638a

Kaufmann, L., Mazzocco, M. M., Dowker, A., von Aster, M., Göbel, S. M., Grabner, R. H., Henik, A., Jordan, N. C., Karmiloff-Smith, A. D., Kucian, K., Rubinsten, O., Szucs, D., Shalev, R., & Nuerk, H.-C. (2013). Dyscalculia from a developmental and differential perspective. *Frontiers in Psychology, 4*, 516. https://doi.org/10.3389/fpsyg.2013.00516

Kim, C., Kim, M. K., Lee, C., Spector, M., & DeMeester, K. (2013). Teacher beliefs and technology integration. *Teaching and Teacher Education, 29*, 76-85. https://doi.org/10.1016/j.tate.2012.08.005

Kinchin, I. M., Streatfield, D., & Hay, D.B. (2010). Using concept mapping to enhance the research interview. *International Journal of Qualitative Methods, 9*(1), 52-68. https://doi.org/10.1177/160940691000900106

Knafl, K. A., & Breitmayer B. J. (1991). Triangulation in qualitative research: Issues of conceptual clarity and purpose. In J. M. Morse (Ed.), *Qualitative nursing research: A contemporary dialogue* (pp. 226-239). SAGE. https://doi.org/10.4135/9781483349015.n26

Lappen, G., & Theule- Lubienski, S. (1994). Training teachers or educating professionals? What are the issues and how they are resolved? In D. Robitaille, D. Wheeler, & C. Kieran (Eds.), *Selected lectures from the 7th International Congress on Mathematical Education* (pp. 249-262). de L'Universite Laval.

Lau, W. W. F. (2021). Pre-service mathematics teachers' professional learning in a pedagogy course: Examining changes in beliefs and confidence in teaching algebra. *Mathematics Education Research Journal, 33,* 223-239. https://doi.org/10.1007/s13394-019-00285-y

Liljedahl, P., Rosken, B., & Rolka, K. (2021). Changes to preservice elementary teachers' beliefs about mathematics and the teaching and learning of mathematics: How and why? *Journal of Adult Learning, Knowledge and Innovation, 4*(21), 20-30.

Lloyd, G. (2002). Mathematics teachers' beliefs and experiences with innovative curriculum materials: The role of curriculum in teacher development. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 149-159). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47958-3_9

Looney, L., Perry, D. R., & Steck, A. K. (2017). Turning negatives into positives: The role of an instructional math course on preservice teachers' math beliefs. *Education, 138*(1), 27-40.

Maasepp, B., & Bobis, J. (2014). Prospective primary teachers' belief about mathematics. *Mathematics Teachers Education and Development, 16(2*), 89-107.

Mainali, B. (2019). 9955 difficulty, and high school students' geometry performance geometry investigating the relationships between preferences, gender, and task. *International Journal of Research in Education and Science, 5*(1), 224-236.

Mainali, B. (2021a). Making online mathematics method course interactive and effective with OER. In J. P. Howard, & J. F. Beyers (Eds.), *Teaching and learning mathematics online*. Chapman and Hall/CRC. https://doi.org/10.1201/9781351245586-19

Mainali, B. (2021b). Representation in teaching and learning mathematics. *International Journal of Education in Mathematics, Science, and Technology, 9*(1), 1-21. https://doi.org/10.46328/ijemst.1111

Marita, S., & Hord, C. (2017). Review of mathematics interventions for secondary students with learning disabilities. *Learning Disability Quarterly, 40*(1), 29-40. https://doi.org/10.1177/0731948716657495

Markovits, Z. (2011). Beliefs hold by pre-school prospective teachers toward mathematics and its teaching. *Procedia-Social and Behavioral Sciences, 11*, 117-121. https://doi.org/10.1016/j.sbspro.2011.01.045

Mazzocco, M. M. M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. B. Berch, & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 29-47). Paul H Brookes Publishing. https://doi.org/10.1097/DBP.0b013e31817aefe8

Mazzocco, M. M. M., Devlin, K. T., & McKenney, S. J. (2008). Is it a fact? Timed arithmetic performance of children with mathematical learning disabilities (MLD) varies as a function of how MLD is defined. *Developmental Neuropsychology*, *33*, 318-344. https://doi.org/10.1080/87565640801982403

McCleary, D. F., Rowlette, E. F., Pelchar, T. K., & Bain, S. K. (2013). Interventions for learning disabilities: Does a journal-based change in focus and article type reflect or influence legal mandates? *Review of Educational Research, 83*(2), 196-210. https://doi.org/10.3102/0034654313476143

National Center for Learning Disabilities. (2007). *SLD identification overview: General information and tools to get started.* https://files.eric.ed.gov/fulltext/ED543737.pdf

NCES. (2019). Children and youth disabilities. *National Center for Educational Statistics*. https://nces.ed.gov/programs/coe/indicator_cgg.asp

Nesbit, J. C., & Adesope, O. O. (2006). Learning with concept and knowledge maps: A meta-analysis. *Review of Educational Research, 76*(3), 413-448. https://doi.org/10.3102/00346543076003413

Nisbet, S., & Warren, E. (2000). Primary teacher's belief relating to mathematics, teaching and assessing mathematics factors that influence these beliefs. *Mathematics Teachers Education and Development, 2*, 34-47.

Penny, G. C. (2018). Rethinking the concept of learning disability. *Canadian Psychology, 59*(2), 197-202. https://doi.org/10.1037/cap0000128

Philipp, R. A., Ambrose, R., Lamb, L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., Thanheiser, E., & Chauvot, J. (2007). Effects of early field experiences on the mathematical knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38*(5), 438-476.

Pullen, P. C., Lane, H. B., Ashworth, K. E., & Lovelace, S. P. (2011). Learning disabilities. In J. M. Kauffeman, & D. P. Hallahan (Eds.), *Handbook of special education (*pp. 187-197). Taylor & Francis.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*(5), 550-576. https://doi.org/10.2307/749691

Richardson, V. (2003). Constructivist pedagogy. *Teachers College Record, 105*(9), 1623-1640. https://doi.org/10.1046/j.1467-9620.2003.00303.x

Rokeach, M. (1968). *Beliefs, attitudes, and values: A theory of organization and change.* Jossey-Bass.

Rolka, K., Rosken, B., & Liljedahl, P. (2006). Challenging the mathematical belief of preservice elementary school teachers. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education* (pp. 441-448). PME.

Satsangi, R., Hammer, R., & Hogan, C. D. (2019). Video modeling and explicit instruction: A comparison of strategies for teaching mathematics to students with learning disabilities. *Learning Disabilities Research & Practice, 34*, 35-46. https://doi.org/10.1111/ldrp.12189

Schoenfeld, A. H. (1992). *Mathematical problem solving*. Academic Press.

Seawright, J., & Gerring, J. (2008). Case selection techniques in case study research: A menu of qualitative and quantitative options. *Political Research Quarterly, 61,* 294-308. https://doi.org/10.1177/1065912907313077

Shalev, R., Manor, O., & Gross-Tsur, V. (2005). Developmental dyscalculia: A prospective six-year follow-up. *Developmental Medicine and Child Neurology, 47*(2), 121-125. https://doi.org/10.1111/j.1469-8749.2005.tb01100.x

Stuart, C., & Thurlow, S. (2000). Making it their own: Preservice teachers' experiences, beliefs, and classroom practices. *Journal of Teacher Education, 51*(2),13-21. https://doi.org/10.1177/002248710005100205

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In A. D. Grouws (Eds.), *Handbook of* research *on mathematics teaching and learning* (pp. 127-146). Macmillan.

Torner, G. (2002). Mathematical beliefs-a search for common ground: Some theoretical consideration on structuring beliefs, some research questions, and some phenomenological observations. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 73-94). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47958-3_5

USDOE. (2004). IDEA, individuals with disabilities education improvement act of 2004. Public Law 108-446. *United States Department of Education*. https://www.govinfo.gov/app/details/PLAW-108publ446

Vacc, N., & Bright, G. W. (1999). Elementary preservice teachers' belief and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30*(1), 89-110. https://doi.org/10.2307/749631

Vygotsky, L. S. (1993). Introduction: Fundamental problems of defectology. In R. W. Rieber, & A. S. Carton (Eds.), *The collected works of L. S. Vygotsky: Volume 2. The fundamentals of defectology* (pp. 29-51). Plenum Press. (Original work published 1929). https://doi.org/10.1007/978-1-4615-2806-7_2

White, A. L., Way, J., Perry, B., & Southwell, B. (2005). Mathematical attitudes, beliefs and achievement in primary pre-service mathematics teacher education. *Mathematics Teacher Education and Development, 7*, 33-52.

Wilkins, J. L., & Brand, B. (2004). Change in preservice teachers' beliefs: An evaluation of mathematics methods course. *School Science and Mathematics, 104*(5), 226-232. https://doi.org/10.1111/j.1949-8594.2004.tb18245.x

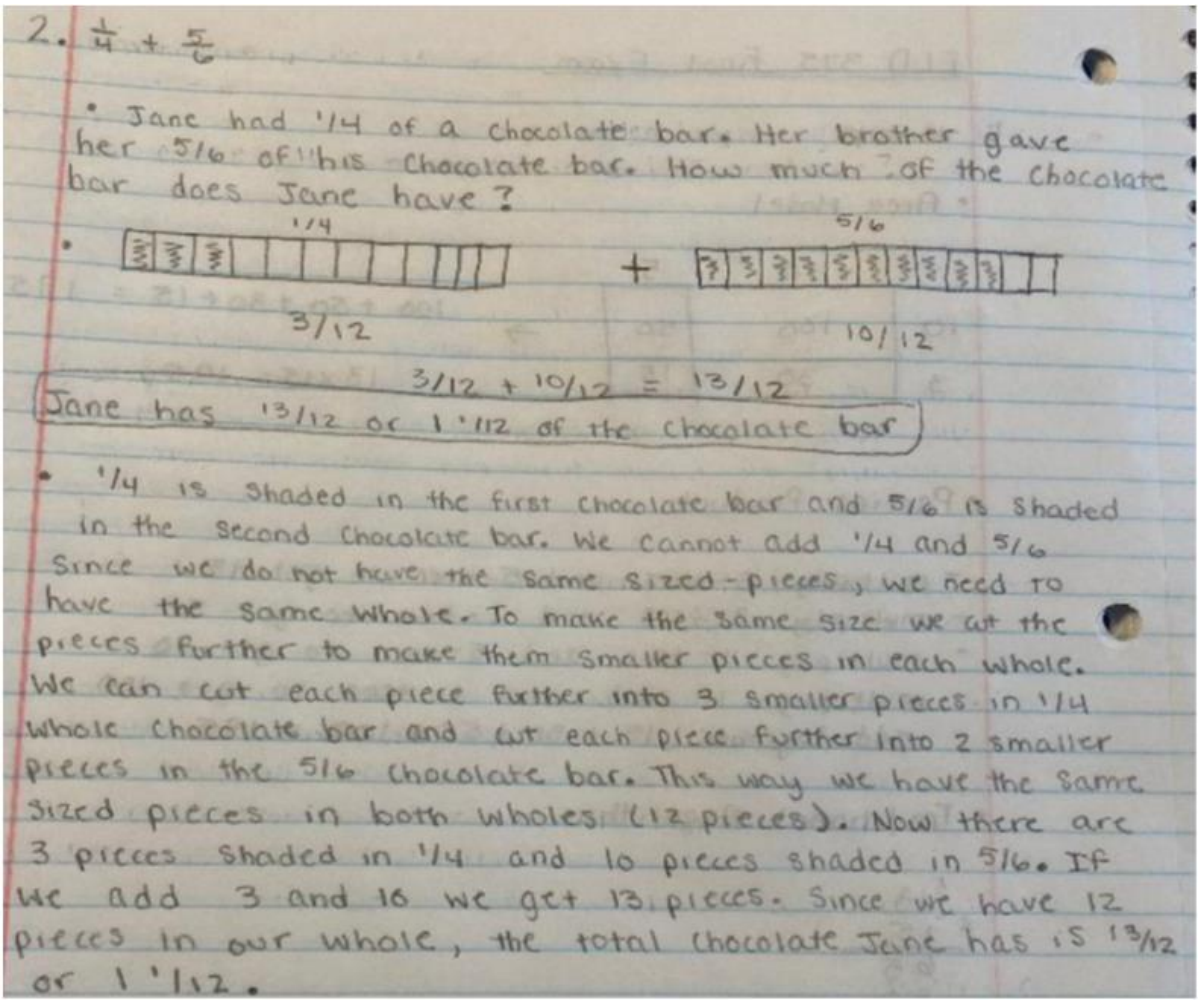
Wilson, M., & Cooney, T. (2002). Mathematics teacher change and development. Framing students' mathematics-related beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 127-147). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47958-3_8

Xie, S., & Cai, J. (2021). Teachers' beliefs about mathematics, learning, teaching, students, and teachers: Perspectives from Chinese high school in-service mathematics teachers. *International Journal of Science and Math Education, 19*(2), 747-769. https://doi.org/10.1007/s10763-020-10074-w

Yang, X., Kaiser, G., König, J., & Blomeke, S. (2020). Relationship between pre-service mathematics teachers' knowledge, beliefs and instructional practices in China. *ZDM-Mathematics Education, 52*, 281-294. https://doi.org/10.1007/s11858-020-01145-x

APPENDIX A

Sample Work



1. **Why U:** It contains animated video series designed for mathematics for k-12 and college level course. One of the videos presented in the class was "The dawn of numbers" (Link).
2. **Problem solving:** Math class needs a makeover (Link).