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How well do students in secondary school understand temporal development of dynamical systems?

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Abstract:
Despite difficulties understanding the dynamics of complex systems only simple dynamical systems without feedback connections have been taught in secondary school physics. Consequently, students do not have opportunities to develop intuition of temporal development of systems, whose dynamics are conditioned by the influence of feedback processes. We present a research study on students' understanding of temporal development of simple dynamical systems. Students participating in the study were attending the final two years of a technical secondary school (*gimnazija*) program. Based on written equations for the rate of change of some quantity students had to qualitatively determine dynamical development. The study confirmed the initial hypotheses with regard to poor knowledge in the area of dynamical systems, irrelevant of year in secondary school or final grade in physics or mathematics. The results showed that most students understood the development of linear systems without feedback and based on equations, students were able to forecast the dynamical development of changing quantities. Issues arose in understanding systems with feedback connections that influence the nonlinear dynamical development of changing quantity. Especially with negative feedback connections that provide stabilization of changing quantities. The reason could be that in most of the cases they turned towards linear dynamical development of changing quantities. Frequently, they incorrectly concluded that the temporal development of changing quantities is the same to temporal development of current, which determines the state of quantity. As a response in overcoming such issues, we recommend a geometrical consideration of one-dimensional dynamical systems.

Keywords: dynamical systems, secondary school physics education, empirical investigation

Introduction

In a modern globalized world there are different natural and social systems, which describe the interrelated and time-varying quantities. These systems are called dynamical systems, which are becoming increasingly important (Strogatz, 1994; Williamson, 1997). A large number of quantities and complex links among them lead to dynamical behaviour. This can be quite difficult for individuals to understand (Dörner, 1989; Sterman, 1989; Green, 1997; Jensen and Brehmer, 2003). For the purpose of teaching dynamical systems in the past decades, different approaches have been developed (Wiener, 1948; Bertalanffy, 1969; Vester, 1986; Rapoport, 1986; Odum, 1984; Ossimitz, 1998). Out of these approaches, system dynamics has been preferred namely because of the simple language system (Forrester, 1958; 1961, 1969). System dynamics analyses complex dynamical systems using computer simulations. The basic idea of system dynamics is to investigate in what way the behaviour of the system depends on its structure as well as the consequences of a system's response to changes in individual parts of the system or the connections between them. The first basic elements for analysis and description of the system in the field of system dynamics are the quantities which determine the state of the system and change through temporal development of the system, which we call state variables. The second basic elements are flows that change the quantities (Hannon and Ruth, 2001). To understand temporal development of the system, it is therefore crucial to understand the connection between the quantities which determine the state of the system and their flows. If original research studies suggest that the main reason for poor understanding of dynamical systems is in its complexity, the ground-breaking research from Booth-Sweeney and Sterman (2000) has shown that most people have difficulties with understanding and forecasting the development of the system even in the simplest dynamical systems, which are described only by one state variable. The authors of the study undertook an investigation with graduate students at the Massachusetts Institute of Technology (MIT) to determine to what extent they are able to understand basic systemic concepts, such as state variables, their flows and time delays and feedbacks. Despite the fact that the individuals participating in the study had above-average math skills, they did poorly with simple tasks that require only an understanding of the links between state variables and their flows, such as the link between the water level and the inflow and outflow in a tub. Subsequent studies (Sterman and Booth-Sweeney, 2002; Fisher, 2003; Zaraza, 2003; Kapmeier, 2004) have confirmed these results. To better visualize the structure of complex systems and for simulating their dynamics, graphically-based computer programs have been developed, such as Stella (Richmond, 2003), Dynasys (Hupfeld, 1997) and Berkeley Madonna (Macey et al., 2000). Such programs are based on the language of system dynamics. These programs allow for the integration of graphic elements on quantities, which determine the state of the system, the flows of these quantities and parameters in a simple and transparent manner, of which a flow diagram is created that represents the mathematical model, which is simulated by the computer (Fošnarič et al., 2003). These programs also allow us to graphically display time varying quantities, thereby making it easier to visualize the influence of connections among quantities on the dynamics of the modelled system. In the field of education we can find quite a few examples of the use of graphically oriented programs for describing real physical systems or to improve conceptual understanding of physics (Schecker, 1994, 1996, 1998, 1999; Lorenz, 1999; Leisen and Neffgen, 1999; Goldkuhle, 1999; Wilhelm, 2000; Heck and Ellermeijer, 2009; Trout and Sinex, 2011; Widmark, 2012). In some studies, graphically oriented programs were used to facilitate understanding of the temporal development of dynamical systems. Zuman and Weaver (1988) presented the results of their research study where they showed how eighth-grade students using the Stella program were able to understand in a relatively short amount of time how feedback leads to exponential growth and were also able to learn to predict the time behaviour of the system and consequently transfer this knowledge to other areas. Similarly, Stella was used by Diane Fisher (2009), who investigated whether students ranging from 14 to 17 years of age were able to determine from the flow diagram if the described system exhibits growth or decline, how the growth or decline on the time graph would look like, as well as whether a feedback loop was present in the system. The results showed that while students were largely able to identify whether there would be growth or decline, they did much poorer on the latter two tasks where they had to select the appropriate graph and recognize the type of feedback in the system.

The aforementioned studies were mainly confined to the study of linear feedback that lead to exponential growth or decline; however there is a gap in the research in examining the extent to which students are able to see how the dynamics of the system depends on various feedbacks. The study of equations from the viewpoint of feedbacks in the field of physics is important for students because in this way they learn to recognize the same concepts in a completely different physical systems and understand that the solution of the equation in one case is also useful in other situations with the same structure of feedbacks (Ruby, 1991; Bitensky, 1997). To check whether the current teaching method in schools encourages such an understanding, we conducted a study in which we assessed the extent to which third and fourth year general secondary school (in Slovenian: *gimnazija*) students (17 and 18 year old) were able, on the basis of given equation for the rate of change of the state variable, to predict the temporal development of dynamical systems with different feedbacks. We were primarily interested in whether students would be able to recognize if the quantity of the state grows or declines, and whether the growth or decline would be increasingly faster or slower. We also examined whether students understand that in some systems changing of the quantity will stabilize after a certain time. To confirm the initial hypothesis that an understanding of dynamical systems in secondary school students is not encouraged, resulting in what is weak understanding, we analysed how the results were dependent on the year in which the student is enrolled in and their final grades in physics and mathematics.

The structure of the article is as follows: first we present the general theory of feedbacks and describe to which point of temporal development feedback loops are developed, which are most commonly encountered in dynamical systems in secondary school physics. Then we show the basic features of secondary school physics curriculum in Slovenia and analyse the extent to which the physics curriculum and mathematics curriculum are making possible for students to develop the ability to understand dynamical development of dynamical systems as a result of their structure. Afterwards we describe the study, present the main results and in the discussion propose ideas for further research.

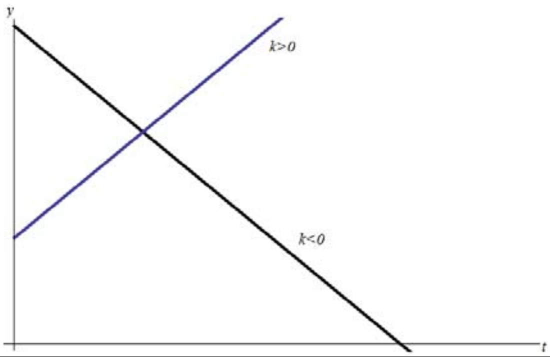
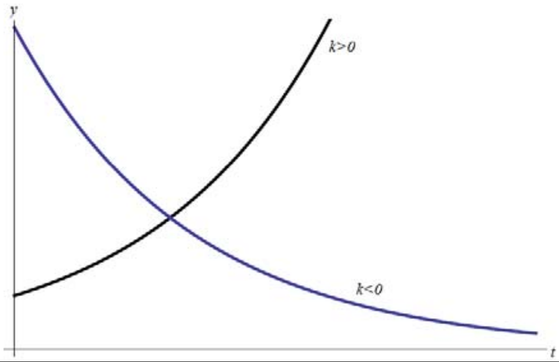
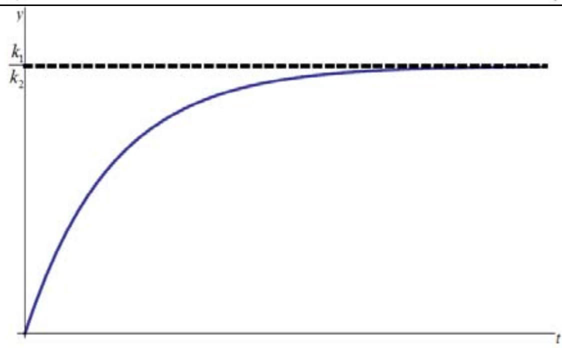
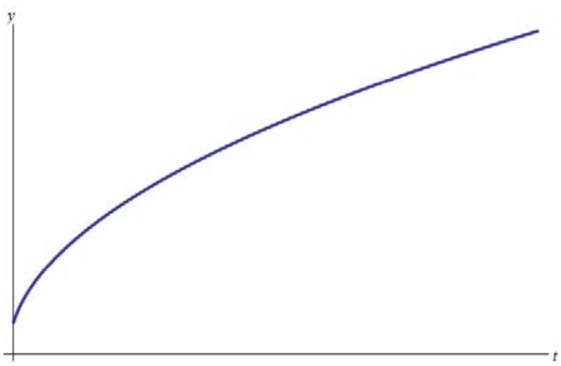
Feedbacks

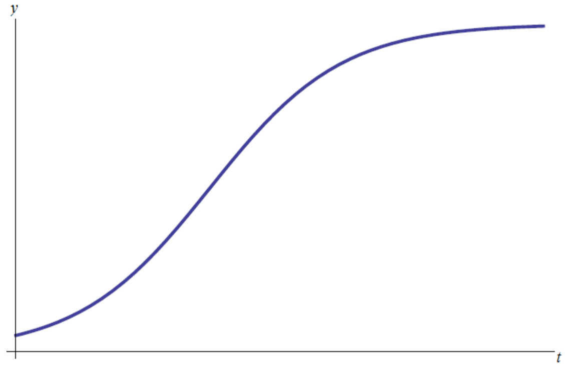
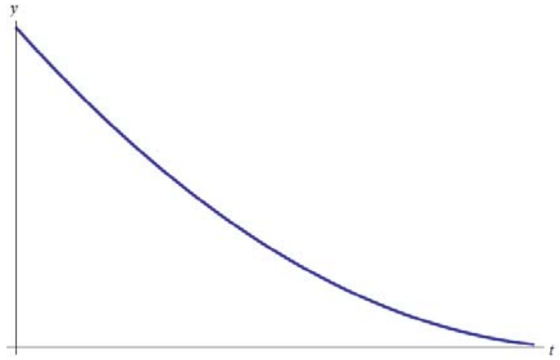
In addition to state variables and their flows, the basic elements of system dynamics are feedbacks, with which systems respond to changes and that represents processes in which the value of some quantity at some point, through cause and effect relationships, affects the value of this quantity in the next moment. Mathematically speaking, the feedbacks represent differential equations, where the rate of change of some quantity is dependent on the value of this quantity. Because it is quite difficult for secondary school students to solve differential equations analytically, it is reasonable to use numerical methods with the use of tabular computer tools, such as Microsoft Excel (Walter, 1989; Frank and Kluk, 1990; Severn, 1999, Lingard, 2003; Quale, 2012) or the aforementioned graphically oriented computer modelling programs. In this way, students can build different models of dynamical systems by themselves and explore which dynamical patterns can frequently be encountered and how they depend on the structure of the feedbacks. System sciences for such simple models of dynamical systems with feedbacks which causes frequently observed types of behaviour, uses the term “generic structure” (Senge, 1990). These are used to understand the links between the structure and behaviour of the system, namely because findings on the behaviour of these systems can be transferred to other systems with the same structure. Often the behaviour of the system is more evident than its structure and as such we can combine the systems according to their behaviour. Consequently, we can explore the whole spectrum of behaviour that is possible at the specified structure. Figure 1 presents the patterns of behaviour of such generic structures in the field of secondary school physics. Here we show only the simplest dynamical systems whose development is described only by one state variable and where the dynamics is sufficiently simple that the students with secondary school math skills are able to understand and solve what kind of feedback cause such dynamics. The presented examples can be found in the systems, which can be covered also with secondary school knowledge of physics.

Method

In Slovenia, general and technical secondary schools’ (in Slovenian: *gimnazija*) primary purpose is to prepare students for continuing education in higher education and in promoting as well as developing skills and competencies that every individual needs in her/his personal and professional life (Gimnazije Act, 2001). For the purposes of this study, we focused on a technical secondary school (in Slovenian: *tehniška gimnazija*), namely because the students not only receive general knowledge, but also obtain fundamental knowledge from the field of engineering. Typically, general and technical secondary schools are four years in length.

Table 1. Basic patterns of behaviour in one dimensional dynamical systems in secondary school physics.

Type of feedback	The equation for the flow of the state variable	The equation for time changing of the state variable	Time graph of state variable
No feedback	$\frac{dy}{dt} = k$	$y = y_0 + kt$ Linear growth or decline	
Flow of the state variable is proportion al to the state variable	$\frac{dy}{dt} = ky$	$y = y_0 e^{\pm kt}$ Exponential growth or decline	
Decrease of the flow of the state variable is proportion al to the state variable	$\frac{dy}{dt} = k_1 - k_2 y$	$y = \frac{k_1}{k_2}(1 - e^{-k_2 t})$ $+ y_0 e^{-k_2 t}$ Convergent growth	
Flow of the state variable is inversely proportion al to the state variable	$\frac{dy}{dt} = \frac{k}{y}$	$y = \sqrt{y_0^2 + 2kt}$ Root growth	

Flow of the state variable is proportion al to the square of the state variable	$\frac{dy}{dt} = yK(1-\frac{y}{k})$	$y = \frac{k}{1+(\frac{k-y_0}{y_0})e^{-K}}$ Logistic growth	
Flow of the state variable is proportion al to the root of the state variable	$\frac{dy}{dt} = -k\sqrt{y}$	$y = (\sqrt{y_0} - \frac{kt}{2})^2$ Declining by the square function	

In technical secondary schools, physics is one of the compulsory subjects, which is taught at the basic, elective, or matriculation exam level (called: *matura*), the latter representing the highest level of secondary school physics that provides a suitable basis for further study at universities in scientific and technical fields (Planinšič et al. 2008). At this level, the physics curriculum includes a qualitative understanding of basic physical laws, but also places an emphasis on quantitative treatment of simple physical systems. For this reason, we are primarily interested to see whether the curriculum includes dynamical systems of any kind. Review of the curriculum showed that in the *matura* level physics program, major emphasis is on stationary phenomena as well as simple dynamical systems in which there are no feedbacks. Throughout the *matura* level program, the students deal with only one dynamical system with feedback, i.e. time changing the number of radioactive nuclei in the sample. As such, students through their physics course do not receive experience with feedbacks in physical systems.

The focus of this research study is to examine whether students in secondary school can qualitatively predict the development of dynamical systems in physics and whether they can logically justify their prediction. Our study was conducted with 82 students from year three (17 year old students) and from year 4 (18 year old students) attending the technical secondary school in Slovenia. All the students participating in the study chose the *matura* level physics programs. Out of the 82 students, 39 students were in year three and 43 students in year four.

By the time this research study was conducted, all students already had topics in mechanics, heat, electricity and magnetism, to which the questions in the research study were related. According to the math curriculum all participating students have already had: linear, root, square, power, exponential and logarithmic functions, exponential functions, polynomials and rational functions and were able to use these functions to model simple examples from everyday life. In addition to the year the students were enrolled in, we also were interested in the final grade students received in physics and mathematics, namely to verify whether there were any statistically significant differences in solving tasks with the year students were enrolled in and final grade they received in physics or mathematics.

Prior to the study, we determined the following hypotheses:

I. Students recognize that the absence of feedback in the system means that the state variable will change with time according to a linear function. A review of the physics curriculum has shown that students in secondary school, except in one case, learn only about dynamical systems in which the rate of change of a quantity is constant; so we expected that students would be able recognize what form of equations leads to linear behaviour.

II. On the basis of written equations for the rate of change of the state variable, students are able to identify if the value of this quantity will increase or decline. Despite the fact that the concept of derivatives students learn formally in their fourth year, they already encounter in year one of secondary school through kinematics and learn about the importance of the tangent to the curve and know that if it is positive, the quantity increases, if it is negative, the quantity declines. Therefore, we expected that most of the students would be able to determine from the given equations, if the quantity would increase or decline.

III. Recognizing temporal development of dynamical systems does not depend on the year in which the student is enrolled in. Students attending physics in their fourth year discuss radioactive decay as a single dynamical system with feedback, but the nature of the feedback is not emphasized. Despite the additional hours of physics, students do not receive additional experience in the development of dynamical systems.

IV. Recognizing temporal development of dynamical systems does not depend on the final grade in physics. This hypothesis relates to the explanation in the previous paragraph.

V. Recognizing temporal development of dynamical systems does not depend on the final grade in mathematics. The curriculum for secondary school mathematics does not assume dynamical systems, and therefore we expected that the final grade in mathematics would not affect the understanding of temporal development of dynamical systems.

Results

Below we present the results of a test that consisted of six cases of physical dynamical systems (Appendix A), in which students from prepared graphs (Appendix B) were asked to choose the one graph that most accurately describes the temporal development of a certain quantity. In addition, they had to use the concepts which they had learned in physics and mathematics to briefly explain the reasoning of why a specific graph was selected. In doing so, we avoided some of the classic secondary school examples such as charging or discharging the capacitor, namely because some students covered such examples in other subjects and we wanted to avoid answers chosen based on memory, but on the basis of physical or mathematical reasoning.

On the first task, students had to choose a time graph of speed for a ball which moves under the influence of gravity and linear resistance. According to the physics curriculum students become familiar only with the movement under the influence of constant forces while movements with changing acceleration (except harmonic oscillations) are not covered. Figure 1 presents students' responses to the first task. Distribution of the responses shows that only 10% of students responded correctly (response *e*). Almost 40% of the students believed that the speed increases linearly (response *a*), which is attributed to the fact that in secondary school, students learn only about a freefall in which the velocity increases linearly with time (response *b*). More than 60% of the students thought the graph of velocity vs. time is either linear growth and decline (responses *a* and *b*), which testifies to the poor knowledge of negative feedback, which in this case causes smaller changes in velocity, leading to a stabilization of the falling bodies velocity. This is due to the fact that the equation for the time variation of the acceleration is linear and students just easily applied it to velocity, without doing any deeper thought in physics, whether such a motion even makes sense. With regards to the identification of the growth or decline of the quantity of the state on the basis of the written equations, in this case we see that most of the students (over 60%) correctly deduced the growing velocity (responses *a*, *d*, *e*, *f*, *g*). Only response *b* stands out as a result of the above-mentioned conclusions regarding linear course of time-varying quantities.

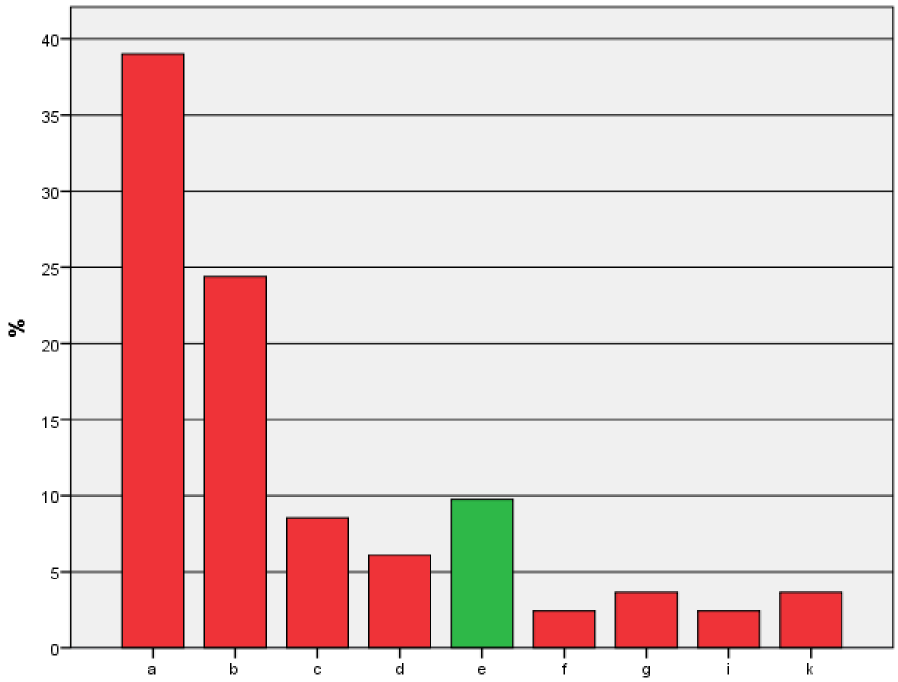


Figure 1. Distribution of student responses to the first task. Response *e* is the correct answer.

The second task was the only one in which there was no feedback in the described system, but the difference was between the outflow and inflow that was constant, which resulted in a linear increase of the height of the water level in the tank. Figure 2 presents student responses to the second task.

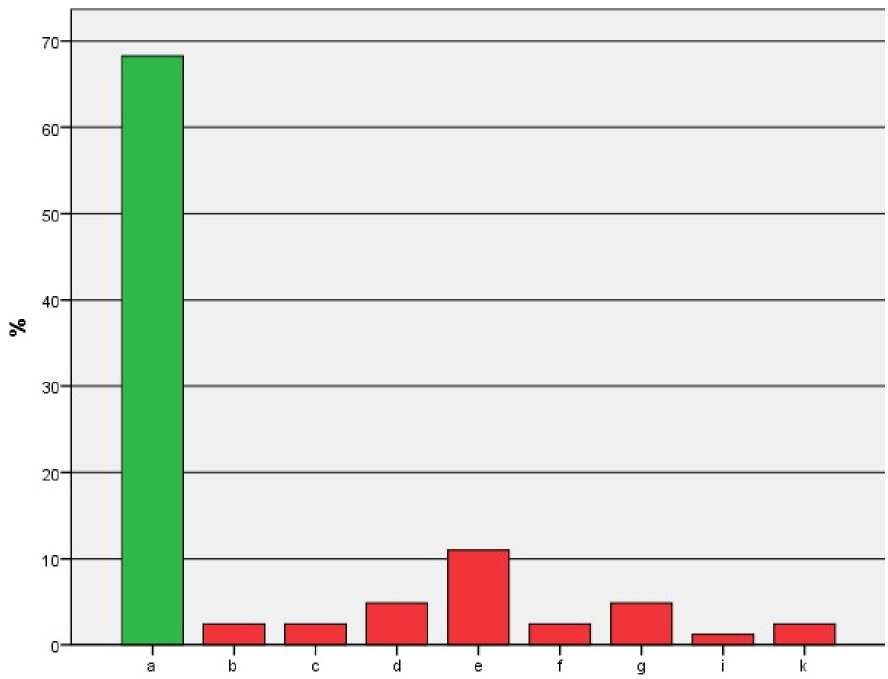


Figure 2. Distribution of student responses to the second task. Response *a* is the correct answer.

In this task almost 70% of students answered correctly (response *a*) and most of them also correctly reasoned their answers. A large percentage of correct answers was no surprise, because students in physics become acquainted with linear examples and know how to recognize and understand them. Among the wrong responses, response *e* stands out, which judging by the comments students made was mainly due to the fact that students misunderstood the task and assumed that once the tank is full the height cannot be changed any more.

The third task describes the cooling of water in a container, wherein the speed reduction of the temperature difference is proportional to the temperature difference itself, which means that the water temperature exponentially approaches an ambient temperature. Figure 3 presents student responses to the third task.

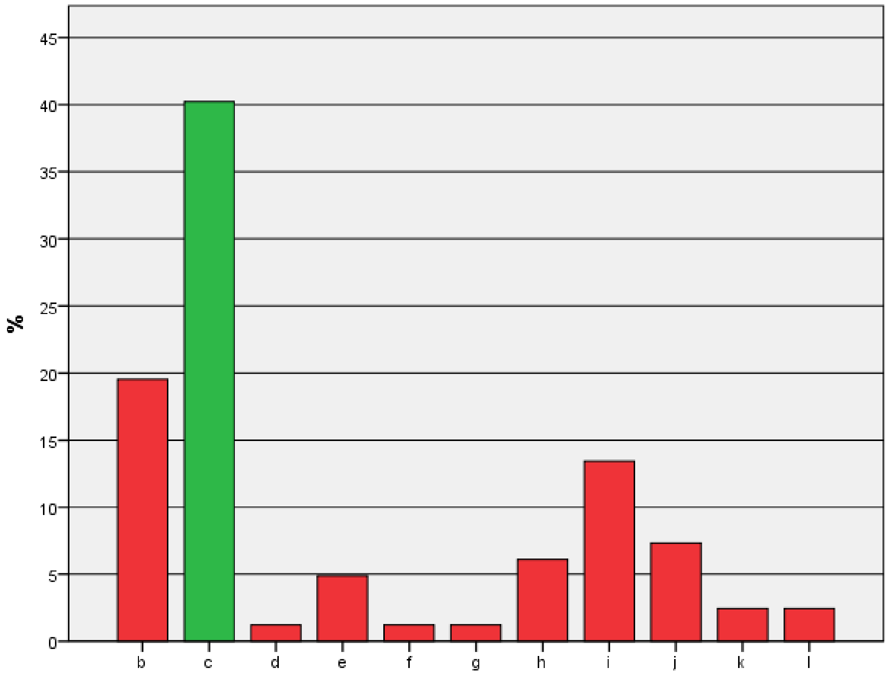


Figure 3. Distribution of student responses to the third task. Response *c* is the correct answer.

More than 40% of the students correctly identified the exponential time declining of temperature (response *c*). In their justification, they mostly reasoned with the phrase "temperature decreases more slowly, because the difference between the temperatures is decreasing" and "at a higher temperature water quickly dissipates heat to the surrounding area." The fact that when the rate of change of some quantity is proportional to the current value of this quantity, it changes with time according to an exponential function, judging by their responses, none of the students recognized this. A large percentage of students (nearly 20%) responded that the temperature decreases linearly (response *b*), which was attributed to the fact that the ambient temperature is constant but they overlooked that the conduction rate must be reduced because the water temperature decreases. More than 14% of the students correctly deduced that the conduction rate is declining but instead of exponential dependence they selected an inverse relationship (response *i*). They did not recognize that for this example, the initial result is not the initial value of the temperature difference, but an infinitely large value of this difference. It is interesting to note that in this case most of the students correctly identified the exponential decrease while in the first task the exponential convergent growth was not recognized. This only reinforces the already mentioned fact that students solve tasks trying to remember if they have already solved similar tasks while they do not pay attention to the nature of the feedbacks in the equation.

The fourth task described happenings on the surface of the lake which due to long-term low temperature freezes, with temperature difference between ambient temperature and water temperature being constant. Students had to respond to the question of how ice thickness changes over time with the rate of change of the ice thickness being inversely proportional to the current value of the latter. Figure 4 presents student responses to the fourth task.

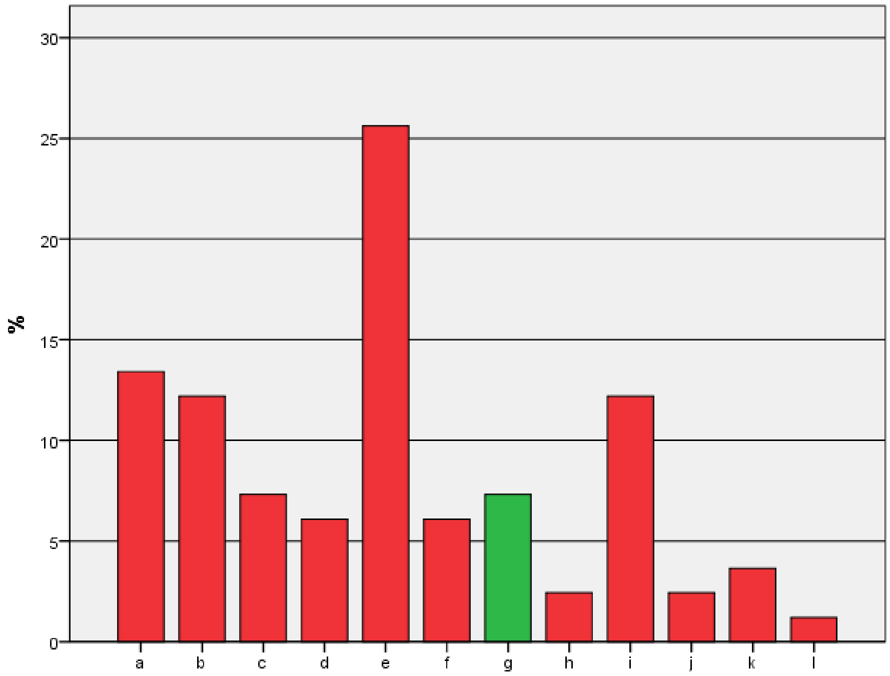


Figure 4. Distribution of student responses to the fourth task. Response *g* is the correct answer.

Only 6 students (7.3%, response *g*) answered correctly but none of the 6 students were able to justify why the chosen graph was correct. We could assume that the correct response was more the result of guessing than understanding. Among the incorrect responses, response *e* with 26% of responses stands out. The response deals with the graph that illustrates the exponential convergence to the final value. According to students arguments, we assume that most of them recognized that the growth rate of ice thickness must decrease but they argued that sooner or later the thickness would reach such a value that conduction rate drops to zero and the thickness would not change anymore. Among the other responses *a* and *b* stand out, which illustrate linear growth and decline. We also can see from the responses that quite a large number of students (nearly 40%) did not understand the physics of events, as they responded with decline (or reduction) in the thickness of ice (responses *b, c, h, i, j)*. More than 10% incorrectly responded with *i* which illustrates the inverse relationship is the result of written equation for the rate of change of the ice thickness, which is proportional to the inverse of it and the students only transferred it to the time changing of the thickness of ice.

The fifth task similarly, as the first, describes the movement with the presence of the resistance force but in this case this force is proportional to the square of the velocity. Figure 5 shows the distribution of student responses to this question and comparing this distribution to the distribution in Figure 1 we can see that the students interpreted two very similar physical situations completely differently.

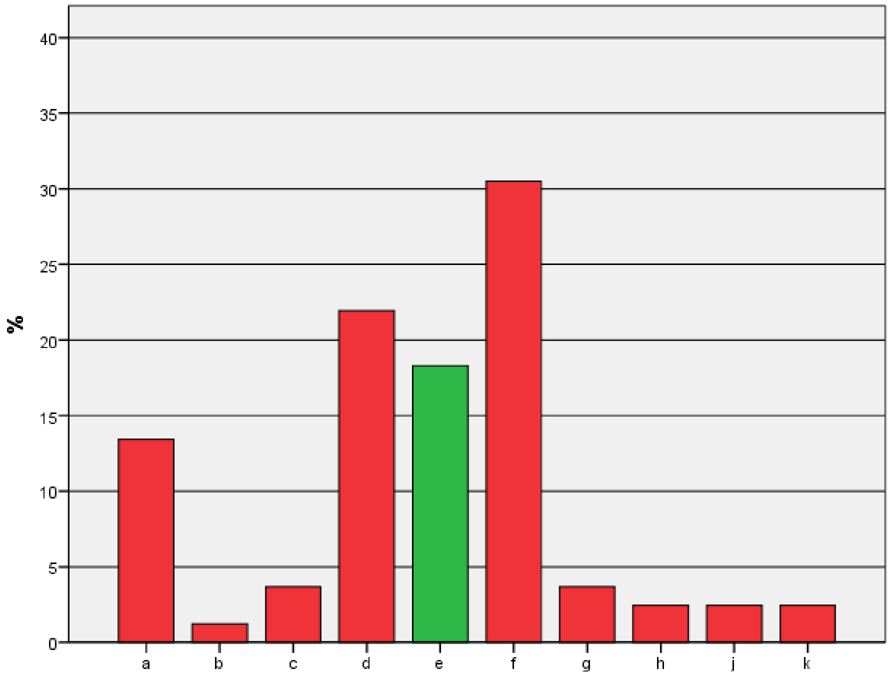


Figure 5. Distribution of student responses to the fifth task. Response *e* is the correct answer.

The correct response was given by 18% of the students who similarly, as with the first task, were able to correctly justify that the rate of change of the velocity is greatest at the beginning and decreases to zero when the weight becomes equal to the air resistance. Of the incorrect responses, response *f* with more than 30% and response *d* with more than 20% of the student responses stands out. Such a large number of *f* responses show that many students do not understand the physics behind it in that the slope of the tangent to the curve on the graph speed vs. time. In this instance, students made the wrong assumption that the acceleration is zero at the beginning and then it increases and then decreases and finally turns back to zero. From a physics point of view, when there is zero initial velocity and zero initial acceleration than no speed is changed to any bodies. With this reasoning students easily could have eliminated this response.

The sixth tasks describes pouring water from one container to another and then to a third one. The rate of water running out of the first two containers is proportional to the current height of the water level in the container. In the first container we are dealing with the exponential decreasing of the water level which was recognized by almost 32% of the students. More than half of them properly substantiated that the rate of decline in the height must also decrease. Despite this almost 48% of students circled response *b* and this is similar with the responses to the previous tasks where we determined that the linear view of the world is strongly rooted in students. Figure 6 presents student responses to the sixth task (item A).

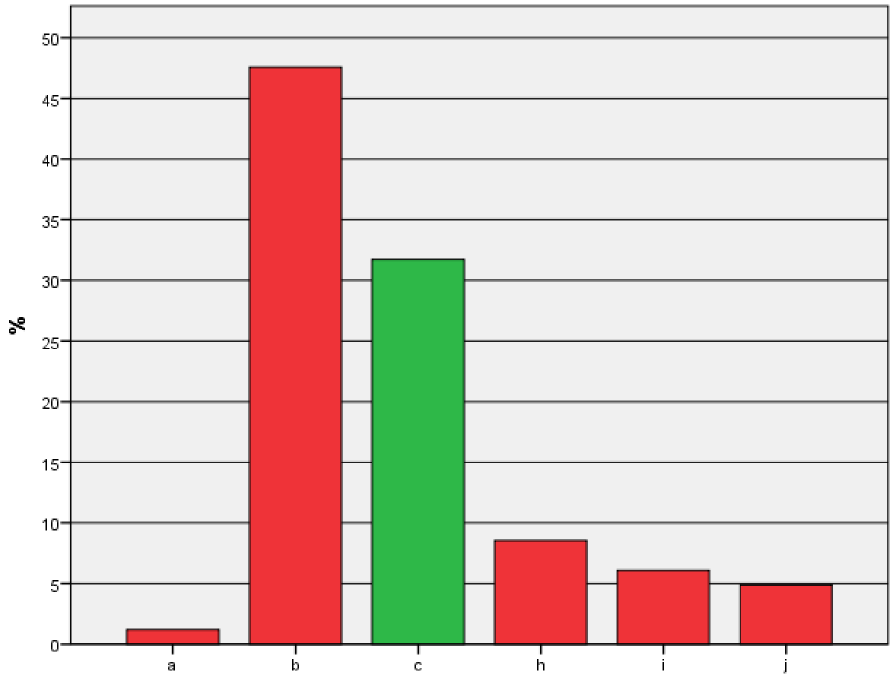


Figure 6. Distribution of student responses to the sixth task (item A). Response *c* is the correct answer.

The second container is initially empty and then begins to fill up, but at the same time the water is running out. Therefore, the second container is filling up slower and slower and after a certain time level reaches the maximum height. From that moment the outflow is greater than the inflow, but outflow also decreases with time. As such, the correct response is *l* and was recognized only by a little more than 10% of the students. Most of the students (44%) correctly determined that the height of the water level in the second tank should rise and then fall, but once again they choose a response that showed linear behaviour. Figure 7 presents student response to the sixth task (item B).

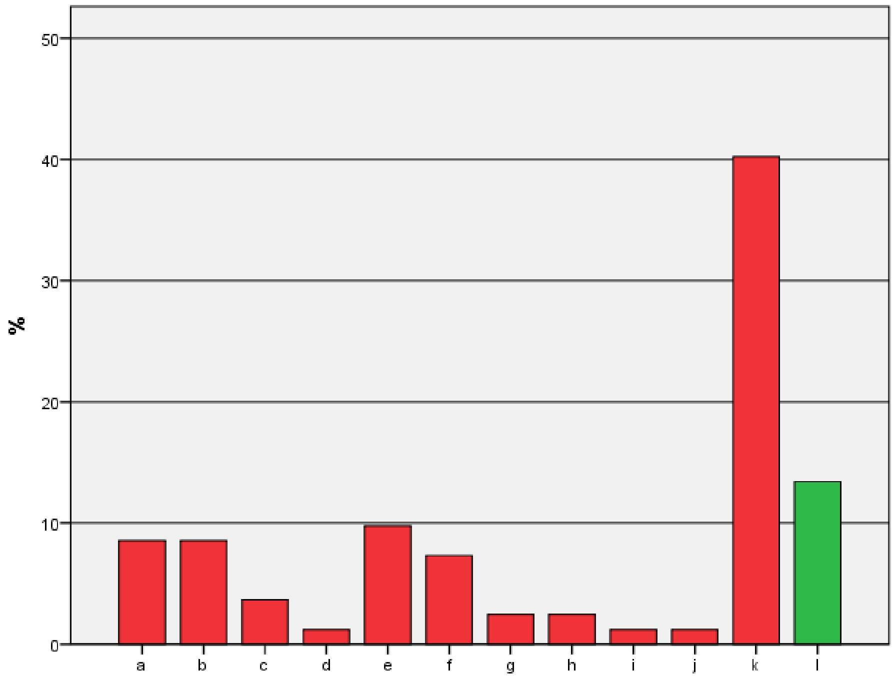


Figure 7. Distribution of student responses to the sixth task (item B). Response *l* is the correct answer.

At the start, the third tank is filling faster and faster. The filling is the fastest when the outflow from the second container is the greatest. From that point on the height is growing more slowly which means that the correct response is *f* representing an S-shaped graph. 18.3% of the students correctly answered this question but were unable to substantiate why this graph was correct. Among the incorrect responses dominates the linear increase of the height with 34% of responses, followed by the exponential growth with 25.6% of all responses. A relatively small number of correct responses was not a surprise, but it was expected that the majority of students would recognize that the third container must at some moment fill more and more slowly and would select the graph of convergent growth. However, there was slightly more than 10 % of students who choose response *e*. Figure 8 presents student responses to the sixth task (item C).

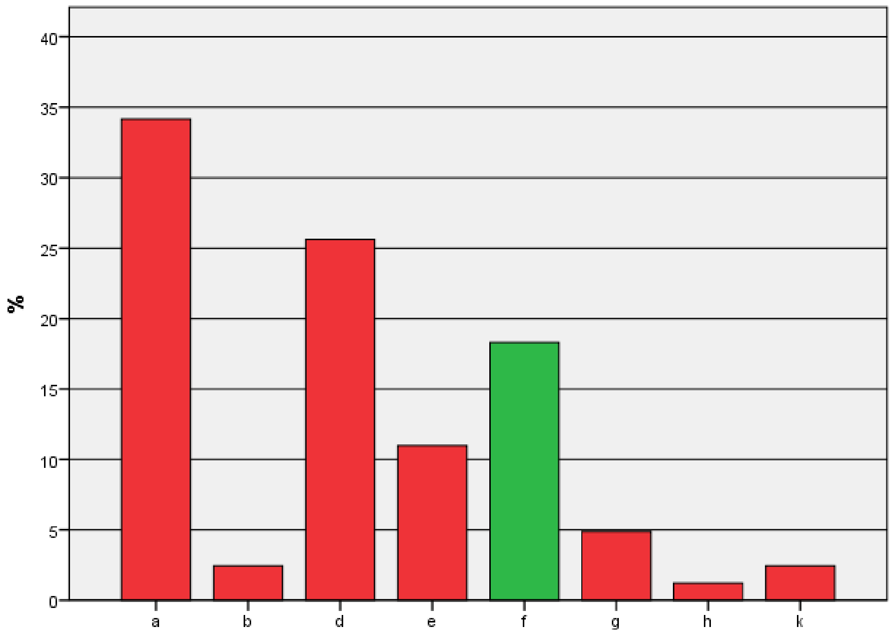


Figure 8. Distribution of student responses to the sixth task (item C). Response *f* is the correct answer.

To confirm or reject the third, fourth and fifth hypothesis, we used a χ^2 test to determine whether there were any statistical significant differences between student responses in relation to the year of secondary school enrolment and final grade in physics and mathematics. The results of the analysis are shown in Table 2.

Table 2. The value of chi-squared coefficient (χ^2) and its significance level (p)

	Year of secondary school the students attend		Final physics grade		Final mathematics grade	
	χ^2	p	χ^2	p	χ^2	p
Task 1	11.308	0.185	24.366	0.441	24.960	0.408
Task 2	8.468	0.389	26.138	0.346	22.467	0.551
Task 3	13.681	0.188	30.378	0.446	24.582	0.745
Task 4	17.901	0.084	38.024	0.251	38.920	0.221
Task 5	9.201	0.419	38.834	0.066	23.659	0.649

Task 6A	20.052	**0.001**	11.304	0.731	16.369	0.358
Task 6B	12.107	0.356	36.503	0.309	28.363	0.697
Task 6C	12.855	0.076	22.867	0.351	35.445	**0.025**

* *Note*: If p <0.05 then the variables are related to each other and we reject the null hypothesis.

Table 2 shows that in task 6 (item A) there were statistically significant differences in solving tasks according to the year of secondary school enrolment and with task 6 (item C) according to students final grade in mathematics. In all other cases there were no differences, which actually confirms the previously mentioned fact that students in secondary school do not have opportunities to develop intuition regarding temporal development of dynamical systems. Despite the fact that students in the fourth year had approximately 130 hours of physics and 170 hours of mathematics more than students in the third year and in addition, had had calculus in mathematics and radioactive decay in physics as the only secondary school example of a dynamical system with feedback; in seven of the eight tasks there were no differences between them. Moreover in task 6 (item A) most of the third year students responded correctly while the majority of the fourth year students circled a graph that was showing a linear decline. According to these results the topics covered in the fourth year does not help to improve understanding of time behaviour of dynamical systems.

Analysis of answers on task 6 (item C) showed that students who had a final grade in mathematics of 2 (ECTS=D) or 3 (ECTC=C) in most cases circled responses *a* and *d*, while students with a grade of 4 (ECTS=B) or 5 (ECTS=A) to a greater extent circled the correct response *f*. This indicates that students with a higher grade in mathematics should have been able to justify the correct answer, but nevertheless none of them were able to properly justify or at least indicate why response *f* is the correct response.

Discussion

The purpose of this research study was to investigate the extent to which students of third and fourth year technical secondary school in Slovenia are able to think about temporal development of dynamical systems. For this purpose, students received six tasks with different dynamical systems, where the equation for the rate of change of the state variable was presented and students were required to select the graph that best shows temporal development of this variable. Similar tasks are represented as a standard repertoire in textbooks of linear motion, where students learn the importance of slope of the tangent on the curve in graphs of time changes of speed and position, the students were equipped with basic knowledge needed to analyse the tasks for the research study. Results showed that students understood the examples with no feedbacks in the system, which was no surprise, as we have already discovered that the *matura* level physics curriculum program largely promotes such cases. This confirms the first hypothesis. We can accept the second hypothesis as well, because in all cases the majority of students correctly concluded, whether there would be growth or decline of the state variable. The results also pointed out a simplification of real situations and excessive treatment of linear cases as a significant problem in schools. Because of this, the students thought the nature is linear, because in five cases a linear decline or increase was the most common answer and in further two, the second most common. This research study also shows an interesting fact that was described in previous studies (e.g. Booth-Sweeney, 2000; Atkins et al., 2002; Ossimitz, 2002; Pala and Vennix, 2005; Cronin, 2009) where it was found that many people compare different cases by the conclusion that the time change of some quantity must be similar to the time change of rate of change. This fact was most obvious in all the questions except the second and fourth.

The results also confirm hypothesis III – V, which suggests that an understanding of feedbacks and their impact on temporal development of physical systems is very weak by all students regardless of year of schooling or grade obtained in physics and mathematics. This requires a longitudinal and more systematic approach to teaching feedbacks in physics. There are many dynamical systems whose physical background is not too difficult, which could be addressed in secondary school physics. Such examples include for instance falling with air resistance, charging and discharging the capacitor, nonstationary heat transfers, water rockets, leakage of water from a container, simple models of global warming, bungee jumping and the rise and fall of a current in a coil. Because analytical treatment of these cases in secondary school is not an option, a reasonable solution seems the numerical and geometrical treatments. While the numerical approach with tabular and graphically oriented computer tools teaching physics has been discussed already, a geometrical approach offers itself as a good introduction for understanding dynamical systems (Strogatz, 1994). With a geometrical approach we are looking for qualitative solutions of dynamical systems and we are not focused so much on quantitative results. Geometrical treatment of one dimensional dynamical systems from a mathematical point of view is sufficiently simple for students in secondary school to understand. As such, it represents a good way to help students to develop an intuition about relationships between feedbacks and dynamics of systems. Because studies on the use of geometrical methods in secondary schools have not been conducted, our initial results provide a good starting point for further research in this area.

References

Atkins , P. W. B., Wood, R. E. , and Rutgers, P.J. , (2002). The effects of feedback format on dynamic decision making. *Organizational Behavior and Human Decision Processes,* **88,** 587-604.

Bertalanffy, L. v., (1969). *General system theory*. New York: Braziller.

Bitensky, I. S., (1997). Look at equations heedfully and you'll win. *Physics Teacher,* **35,** 540-541.

Booth-Sweeney, L., and Sterman, J. D., (2000). Bathtub Dynamics: Initial Results of a Systems Thinking Inventory. *System Dynamics Review,* **16,** 249-286.

Cronin, M. A., (2009). Why don't well-educated adults understand accumulation? A challenge to researchers, educators, and citizens. *Organizational Behavior and Human Decision Processes,* **108,** 116-130.

Dörner, D., (1989). *The Logic of Failure: Recognizing and Avoiding Error in Complex Situations.* New York: Basic Books.

Fisher, D. M., (2003). Student Performance on The Bathtub and Cash Flow Problems. Proceedings of the 2003 International System Dynamics Conference. Albany, New York.

Fisher, D. M., (2009) "How well can students determine simple growth and decay patterns from a diagram?" http://www.systemdynamics.org/conferences/2009/proceed/papers/P1159.pdf, (accessed July 2014)

Forrester, J. W., (1958). Industrial dynamics: a major breakthrough for decision makers. *Harvard Business Review,* **36,** 37-66.

Forrester, J. W., (1961). *Industrial Dynamics.* Cambridge, Massachusetts: MIT Press.

Forrester, J. W., (1969). *Urban Dynamics.* Cambridge, Massachusetts: MIT Press.

Fošnarič, S., Marhl, M., and Grubelnik, V., (2003). Modeliranje kot usmerjena didaktična dejavnost pri pouku. *Pedagoška obzorja,* **18,** 35-45.

Frank, M. T., and Kluk E., (1990). Equations of Motion on a Computer Spreadsheet: The Damped Oscillator and More. *Physics Teacher,* **28,** 308-311.

Gary, M. S., and Wood, R. E., (2005). Mental models, decision making and performance in complex tasks. 23rd International Conference of the Systems Dynamics Society.

Gimnazija Act, O.G. RS, no. 12/96 and 59/01.

Goldkuhle, P., (1999). Bewegung mit Reibung. *Praxis der Naturwissenschaften Physik,* **48,** 21-26.

Green, D. W., (1997). Explaining and envisaging an ecological phenomenon. *British Journal of Psychology,* **88,** 199-217.

Hannon, B., and Ruth, M., (2001). *Dynamic modeling*. New York: Springer-Verlag.

Heck, A., and Ellermeijer, T., (2009). Giving students the run of sprinting models. *American Journal of Physics,* **77,** 1028-1038.

Hupfeld, W., (1997) "Dynasys Modellbildung und Simulation dynamischer Systeme" http://www.hupfeld-software.de/pmwiki/pmwiki.php?n=Main.Download?action=downloadman&upname=Dynasys-Handbuch.pdf, (accessed July 2014)

Jensen, E., and Brehmer, B., (2003). Understanding and control of a simple dynamic system. *System Dynamics Review,* **19,** 119-137.

Kapmeier, F., (2004). Findings From Four Years of Bathtub Dynamics at Higher Education Institutions in Stuttgart. Proceedings of the 2004 International System Dynamics Conference. Albany, New York.

Leisen, J., and Neffgen, M., (1999). Modellbuildungspraktikum: Fall von Körpern in Luft. *Praxis der Naturwissenschaften Physik,* **48,** 7-14.

Lingard, M., (2003). Using spreadsheet modelling to teach about feedback in physics. *Physics Education,* **38,** 418-422.

Lorenz, G., (1999). Der Fallschirmsprung, Ein Unterrichtsansatz im Fächerdreieck M-Ph-Inf. *Praxis der Naturwissenschaften Physik,* **48,** 15-20.

Macey, R., Oster, G., and Zahnley, T., (2000) »Berkeley Madonna User's Guide«

http://www.berkeleymadonna.com/BM%20User%27s%20Guide%208.0.pdf, (accessed July 2014)

Odum, H. T., (1994). *Ecological and general systems: an introduction to systems ecology*. Niwot, Colorado: Univ. of Niwot.

Ossimitz, G., (1998) »*The development of Systems Thinking Skills Using System Dynamics Modelling Tools*« http://www.fmd.uni-osnabrueck.de/ebooks/gdm/PapersPdf1996/Ossimitz.pdf, (accessed July 2014)

Ossimitz, G., (2002) »Stock–flow-thinking and reading stock–flow-related graphs: An empirical investigation in dynamic thinking abilities« http://www.systemdynamics.org/conferences/2001/papers/Ossimitz_1.pdf, (accessed July 2014)

Pala, O., and Vennix, J. A. M., (2005). Effect of system dynamics education on systems thinking inventory task performance. *System Dynamics Review*, **21,** 147-172.

Planinšič, G., Belina, R., Kukman, I., and Cvahte, M., (2008). *Curriculum of physics for secondary school*. http://portal.mss.edus.si/msswww/programi2008/programi/media/pdf/ucni_nacrti/UN_FIZIKA_strok_gimn.pdf, (accessed May 2014)

Quale, A., (2012). On the use of a standard spreadsheet to model physical systems in school teaching. *Phys. Educ.*, **47,** 355-365.

Rapoport, A., (1986). *General system theory: essential concepts & applications*. Tunbridge Wells, Kent: Abacus Press.

Richmond, Isee systems. Stella. Available from www.iseesystems.com/

Ruby, L., (1991). If You Understand Leaky Buckets, You Understand a Lot of Physics *Physics Teacher*, **29,** 44-46.

Schecker, H.P., (1994). System Dynamics in High School Physics Education *Proceedings of the 1994 International System Dynamics Conference*, 74-84

Schecker, H.P., (1996). Modeling physics: System dynamics in physics education. *The Creative Learning Exchange*. **5**, 1-8.

Schecker, H.P., (1998). *Physik-Modellieren, Grafikorientierte Modellbildungssysteme im Physikunterricht.* Stuttgart: Ernst Klett Verlag.

Schecker, H.P., et. al., (1999). *Physiklernen mit Modellbildungssystemen - Förderung physikalischer Kompetenz und systemischen Denkens durch computergestützte Modellbildung (Abschlussbericht für die Deutsche Forschungsgemeinschaft DFG)*. Universität Bremen, Institut für Bildungsforschung. Bonn

Senge, P. M., et. al., (1990). *The Fifth Discipline: The Art and Practice of the Learning Organization*. New York: Doubleday.

Severn, J., (1999). Use of spreadsheets for demonstrating the solutions of simple differential equations. *Physics Education*, **34,** 360-366.

Sterman, J., (1989). Modeling Managerial Behavior: Misperceptions of Feedback in Dynamic Decision Making Experiment. *Management Science*, **35,** 321-339.

Sterman, J. D., and Booth-Sweeney, L., (2002). Cloudy Skies: Assessing Public Understanding of Global Warming. *System Dynamics Review.* **18,** 207-240.

Strogatz, S. H., (1994). *Nonlinear Dynamics and Chaos With Applications to Physics, Biology, Chemistry and Engineering*. Reading, MA: Addison-Wesley.

Trout, C., and Sinex, S.A., (2011). Building Dynamic Conceptual Physics Understanding. *Physics Teacher*, **49,** 377-379.

Vester, F., (1986). *Unsere Welt - ein vernetztes System*. München: DTV.

Walter, R.K., (1989). Simulating Physics Problems with Computer Spreadsheets. *Physics Teacher*, **27,** 173-175.

Widmark, S., (2012). Modeling Physical Systems Using Vensim PLE Systems Dynamics Software. *Physics Teacher*, **50,** 105-108.

Wiener, N., (1948). *Cybernetics: or control and communication in the animal and the machine*. New York: Technology Press.

Wilhelm, T., (2000). Der alte Fallkegel-modern behandelt, Quantitative Auswertung eines klassischen Freihandversuchs *Praxis der Naturwissenschaften Physik*, **49,** 28-31.

Williamson, R., (1997). *Introduction to dynamical systems and differential equations*. Columbus, OH: McGraw-Hill.

Zaraza, R., (2003). *Bathtub Dynamics in Portland at SyMFEST*. Proceedings of the 2003 International System Dynamics Conference. Albany, New York.

Zuman, J.P., and Weaver, S.L., (1988). Tools for teaching problem solving: An evaluation of a modeling and systems thinking approach. Paper presented at the 1988 Annual NARST Conference, Lake Ozark, MO.

Appendix A: Test

RESEARCH ON UNDERSTANDING TEMPORAL DEVELOPMENT OF PHYSICAL SYSTEMS

Name and surname: Class:

Average grade in physics in current year (circle):	1	2	3	4	5
Average grade in mathematics in current year (circle):	1	2	3	4	5

1. **In the container with the oil a tiny metal ball with mass *m* starts falling from rest. During the fall the weight $F_g = mg$ and the linear resistance $F_u = kv$, which is proportional to the velocity of the ball are acting on it. Constant *k* is the proportionality factor depending on the size of the ball and the viscosity of the oil. We described falling of the ball with Newton's law: $F_g - F_u = ma$ or $mg - kv = ma$. For short time intervals *Δt* we can apply $a \approx \frac{\Delta v}{\Delta t}$ and can rearrange the above equation: $\frac{\Delta v}{\Delta t} = g - \frac{k}{m}v$.**

   a. Write down which of the accompanying graphs most accurately describes how the velocity of the ball changes with time.

   b. Briefly explain the reasoning why you chose the graph that you did. Try to describe your thinking by using the concepts you have learned in physics and mathematics. (*An example:* in movement with constant acceleration we would choose graph A for time changing of velocity since every second the velocity increase by the same value, which means that the steep of the graph is constant.)

2. **In an upright cylindrical container in which there is some water a <u>constant</u> volume flow of water Φ_{V1} flows in and outflows the <u>constant</u> volume flow Φ_{V2}, which is smaller than the volume flow Φ_{V1}. If *S* is the cross-section of the container, the water level *h* in the container changes over time according to the equation: $S\frac{\Delta h}{\Delta t} = \Phi_{V1} - \Phi_{V2}$.**

   a. Write down which of the accompanying graphs most accurately describes how the height of the water changes with time.

   b. Briefly explain the reasoning why you chose the graph that you did. Try to describe your thinking by using the concepts you have learned in physics and mathematics.

3. **In a container with a wall surface *S*, a wall thickness *d* and the thermal conductivity of the walls λ is the mass *m* of water at a temperature T_1. Then we shut down the heater that maintains a constant water temperature in the tank. In the surroundings of the container there is constant temperature $T_2 < T_1$, and because of that the heat flows in the surroundings:** $P = -\frac{\lambda S(T - T_2)}{d}$ **, where *T* is the current temperature of water. Due to loss of heat the water cools for ΔT:** $-\frac{\lambda S(T - T_2)}{d}\Delta t = mc\Delta T$ **or** $\frac{\Delta T}{\Delta t} = -\frac{\lambda S(T - T_2)}{mcd}$**, where** *c* **is the specific heat of water.**

   a. Write down which of the accompanying graphs most accurately describes how the water temperature in the container changes with time.

   b. Briefly explain the reasoning why you chose the graph that you did. Try to describe your thinking by using the concepts you have learned in physics and mathematics.

4. **Above the surface of the water in the lake on which the layer of ice is formed, there is the constant temperature T_1, which is lower than the melting point of water. The thermal conductivity of ice is λ, q_t is the heat of melting the ice, the density of ice is ρ and the temperature of the water is all the time equal to the melting temperature *Tm*. In the case of a constant temperature difference through the ice flows the conduction rate** $P = \frac{\Delta Q}{\Delta t} = \frac{S\lambda(T_t - T_1)}{x}$, **where *x* is the current thickness of the ice. Because of the conduction rate flowing out, the water is turning into ice:** $\Delta m = \rho S \Delta x = \frac{\Delta Q}{q_t}$**. Combining the two equations, we get** $\frac{\Delta x}{\Delta t} = \frac{\lambda(T_t - T_1)}{\rho q_t x}$.

   a. Write down which of the accompanying graphs most accurately describes the ice thickness with time.

   b. Briefly explain the reasoning why you chose the graph that you did. Try to describe your thinking by using the concepts you have learned in physics and mathematics.

5. **A skydiver jumps from a hovering helicopter and free falls with a closed parachute. During the fall there are two forces acting on the skydiver: the constant force of gravity $F_g = mg$ and air resistance $F_u = \frac{1}{2} c\rho S v^2$, where *c* is the drag coefficient, *ρ* is the air density, *S* is the cross section of the skydiver in the direction of fall and *v* is the velocity of the skydiver. For the motion of the skydiver, Newton's Second Law of Motion is as follows: $F_g - F_u = ma$ and consequently $mg - \frac{1}{2} c\rho S v^2 = ma$. When this equation is rearranged, we get the relationship for the rate of change of velocity $\frac{\Delta v}{\Delta t} = g - \frac{c\rho S}{2m} v^2$.**

   a. Write down which of the accompanying graphs most accurately describes how the velocity of the skydiver changes with time.

   ☐

   b. Briefly explain the reasoning why you chose the graph that you did. Try to describe your thinking by using the concepts you have learned in physics and mathematics.

6. **A vertical cylindrical container is filled with water. At the bottom of the container wall we drill a hole and insert a tube in it. Through this tube the water flows out, whereby its velocity is proportional to the height *h* of water in the container: $\frac{\Delta h}{\Delta t} = -kh$. The proportionality constant *k* depends on the density and viscosity of the water, the length of the tube and the cross-sections of the tube and the container. Water flows from the first container to the second container which also has holes with the tube at the bottom, from which the water flows in a third container, which has no outlet. Rate of water flow from the second container is also proportional to the current height *h* of water in the second container. Second and third containers are initially empty.**

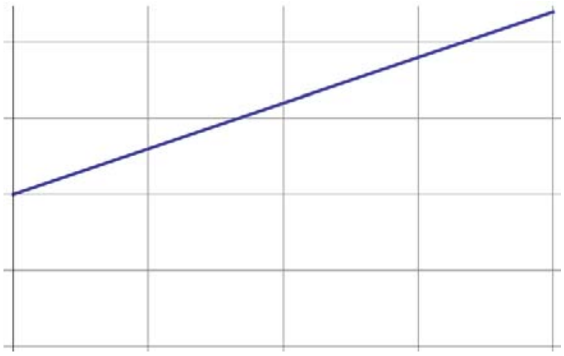
a. Write down which of the accompanying graphs most accurately describe how the height of water in all three containers changes with time.

First container	Second container	Third container
☐	☐	☐

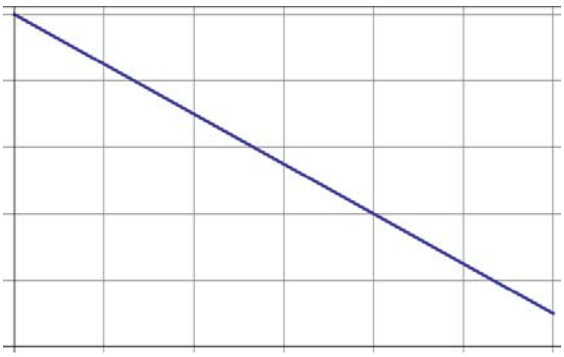
b. Briefly explain the reasoning why you chose the graph that you did. Try to describe your thinking by using the concepts you have learned in physics and mathematics.

Appendix B: Graphs attached to the test

a) Linear growth

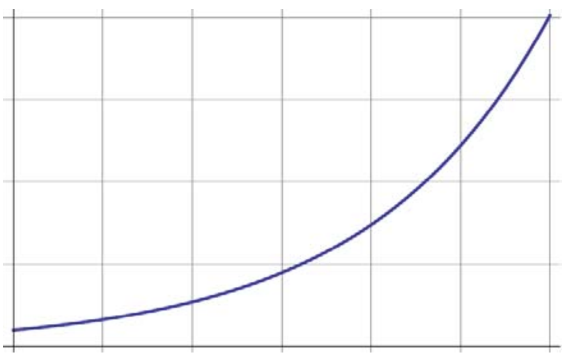


b) Linear decline

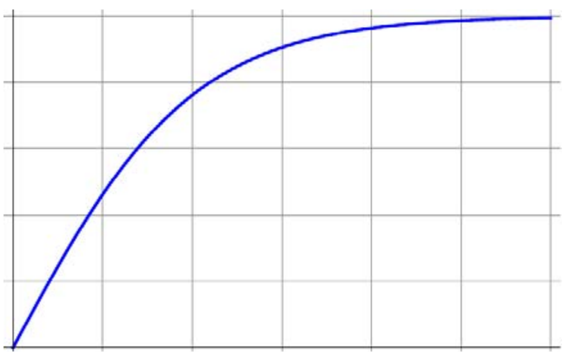


c) Exponential decline

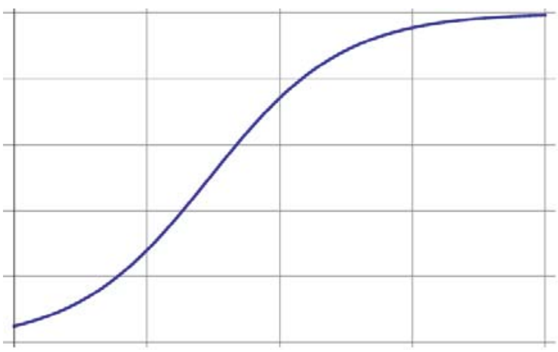
d) Exponential growth



e) Convergent growth



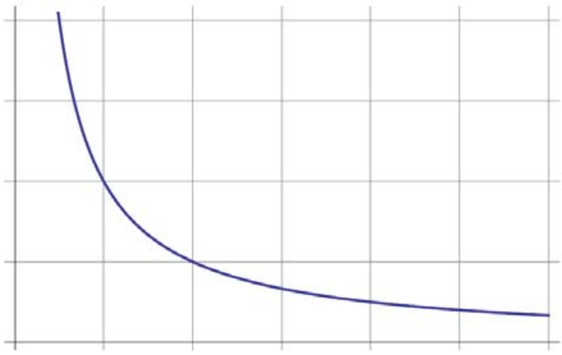
f) At first exponential growth and then exponential decline (S-shaped graph)



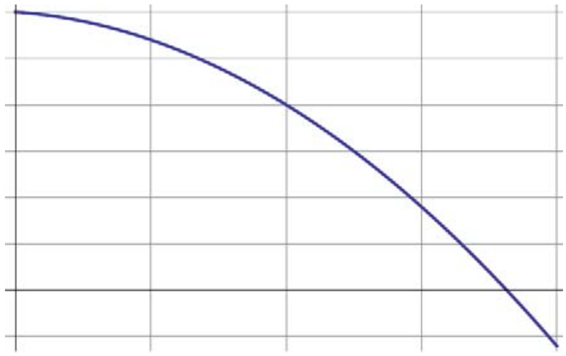
g) Root growth

h) Root decline

i) Function of reverse relationship



j) Decline by the square function



k) Linear growth and decline

l) Exponential growth and decline