**OPEN ACCESS** 

#### **Research Article**



# Helping puzzle-solvers find solutions missed by a famous puzzle author: Initial study on stimulated creativity

# Josip Slisko 1\*

© 0000-0002-5805-4808

- <sup>1</sup> Facultad de Ciencias Físico Matemáticas Benemérita Universidad Autónoma de Puebla, Puebla, MEXICO
- \* Corresponding author: jslisko@fcfm.buap.mx

**Citation:** Slisko, J. (2025). Helping puzzle-solvers find solutions missed by a famous puzzle author: Initial study on stimulated creativity. *European Journal of Science and Mathematics Education*, *13*(4), 385-394. https://doi.org/10.30935/scimath/17509

#### **ARTICLE INFO**

#### **ABSTRACT**

Received: 10 Mar 2025 Accepted: 16 Nov 2025 Many matchstick puzzles have multiple solutions and are ideal learning tasks for fostering and improving creative thinking. Nevertheless, many puzzle book authors exhibit a strange approach  $toward\ these\ multiple-solution\ puzzles.\ For\ some\ puzzles,\ the\ number\ of\ solutions\ is\ mentioned$ after puzzle formulation and these solutions are published in solution section. For other puzzles, information about the number of solutions is omitted, and only one solution is published. Such authors' behavior, certainly adverse to puzzle-solvers' creativity, is illustrated with puzzle examples taken from two most successful books on matchstick puzzles. The first was published by Tromholt (1889), and the second was authored by Obermair (1975). To demonstrate that information about the number of solutions stimulates puzzle-solvers' creative performance, in this initial small-scale study an arithmetic five-solution matchstick puzzle with Roman numerals was used. Its author Obermair (1975) published only one solution. This puzzle was given to three groups of students (N = 21), asking them to activate their creativity and to try find its five solutions. Students were allowed to manipulate the matchsticks. Thirteen students found Obermair's (1975) solution, and, additionally, between one and four solutions that Obermair (1975) missed. Most of these students were helped by a verbal hint "think about all arithmetic operations." Eight students did not find Obermair's (1975) solution. Nevertheless, they found between two and four solutions unknown to Obermair (1975). Students' reflections on this activity revealed that many gained a more positive view on their own creativity or puzzle-solving

**Keywords:** matchstick puzzles, puzzles solving, cognitive bias, spontaneous creativity, stimulated creativity

#### INTRODUCTION

The 21st century is characterized by many technological changes that are profound, rapid, and often unexpected. These changes generate challenges in all areas of life, from the personal and social to the professional and economic. Since the beginning of the century, it was suggested that facing these challenges successfully requires people possess what are known as "21st century skills." Among these skills, the most frequently mentioned are critical thinking, collaborative skills, communication skills, and creative thinking (Pellegrino & Hilton, 2012; Trilling & Fadel, 2009).

Keeling and Hersh (2011) were strong critics of failure of American higher education to provide opportunities for learning such important skills (p. 1):

"The truth is painful but must be heard: we're not developing the full human and intellectual capacity of today's college students because they're not learning enough and because the learning that does occur is haphazard and of poor quality. Too many of our college graduates are not prepared to think critically and *creatively*, speak and write cogently and clearly, solve problems,

comprehend complex issues, accept responsibility and accountability, take the perspective of others, or meet the expectations of employers. Metaphorically speaking, we are losing our minds."

With the passage of time, the list of skills was growing. In 2023, according to research surveys of business leaders on labor market demands, the top five most important skills were:

- (1) analytical thinking,
- (2) creative thinking,
- (3) resilience, flexibility, and agility,
- (4) motivation and self-awareness, and
- (5) curiosity and lifelong learning (World Economic Forum, 2023, p. 38).

However, for the period 2023-2027, based on their increasing economic importance, the predicted order is different:

- (1) creative thinking,
- (2) analytical thinking,
- (3) technological literacy,
- (4) curiosity and lifelong learning, and
- (5) resilience, flexibility, and agility (World Economic Forum, 2023, p. 39).

Despite this imperative need to foster creativity thinking of students, education systems still need adequate curricula and teaching changes at all education levels (Foster & Schleicher, 2022):

"In a world in which the kinds of things that are easy to teach and test have also become easy to digitize and automate, the capacity of individuals to imagine, to create and to build things of intrinsic positive worth is rising in importance. But this has not automatically led to corresponding changes in intended, implemented and achieved curricula."

The reasons for this situation are multifaceted and complex. A study, carried out with school principals, pointed out that the most frequent obstacle to creativity is teachers' lack of knowledge of pedagogical practices to favor the development of students' creativity (Alencar et al., 2015). Teachers themselves recognize as obstacles the lack of training and preparation for creative teaching practices and the pressure of standardized curricula (Gao & Hall, 2024) or standardized tests (Rubenstein et al., 2013). Students also describe various factors that are barriers to developing creativity: inhibition and shyness, lack of motivation, and lack of time and opportunities (Leire et al., 2024).

According to Nickerson (2010), the most important single reason for low presence or total absence of creativity learning in schools is a false belief that every problem has only one solution. In his description of design criteria for learning activities that foster creativity, Vincent-Lancrin (2021) also stress the importance of using problems with several possible solutions (p. 23). This idea is widely shared in mathematics education. Multi-solution mathematical tasks are used both for evaluating creative thinking (Leikin, 2013; Jukić Matić & Sliško, 2024) and for development of creativity (Levav-Waynberg & Leikin, 2012).

Creative solving puzzles and games help students learn better specific mathematics topics, like "parity principle" (Applebaum, 2025) or "periodicity" (Idika & Oluwaseyi, 2024).

To show convincingly the falsehood of belief "one problem-one solution," fostered by dominant use of algorithmic formula-based problems in mathematics education, solving matchstick puzzles with multiple solutions might be ideal learning tasks.

# MULTIPLE-SOLUTION MATCHSTICK PUZZLES: STRANGE APPROACH OF BOOK AUTHORS

Arithmetic and geometric matchstick puzzles consist in transforming an initial matchstick configuration into a sought new matchstick configuration by moving, removing or adding a specific number of matchsticks. Matchstick puzzles have two very important features:

- 1. Mathematical knowledge needed for solving them is formula-free (knowledge of basic arithmetic operations and of forms of simple geometric figures).
- 2. Big majority of these puzzles have multiple solutions.

Nevertheless, many puzzle book authors exhibit a strange approach toward multiple-solution matchstick puzzles. For some puzzles, the number of solutions is mentioned after puzzle formulation and these solutions are published in solution section. For other puzzles, information about the number of solutions is omitted, and only one solution is published. One possible reason might be that for some puzzles authors' spontaneous puzzle solution creativity is lower than spontaneous creativity for puzzle formulation.

Such enigmatic authors' behavior, certainly very adverse to puzzle-solvers' creativity, is illustrated with puzzle examples taken from two most successful books on matchstick puzzles.

#### Tromholt's (1889) Treatment of Multiple-Solution Geometric Puzzles

In 1889, Danish teacher and polar-light researcher Sophus Tromholt (1851-1896) wrote and published, in different languages (German, Danish, and Norwegian), the first edition of the book "Matchstick games. Mental exercise and entertainment" (Tromholt, 1889). The book had 123 pages and a modest dimension (11.5 cm x 15 cm). It contained 150 geometric matchstick puzzles, 103 games and amazing physics demonstrations with matchsticks. The number of 150 matchstick puzzles in Tromholt's (1889) book was in strong contrast with numbers of puzzles in previous books. Braun's (1876) book had 8 matchstick puzzles while in Mittenzwey's (1880) book their number was 19. It means that Tromholt (1889) made an enormous creative effort to expand the collection of possible matchstick puzzles to its greater extension.

Tromholt's (1889) book was sold very well and had second and third unchanged editions in the same year. In 1890 the fourth edition (Tromholt, 1890) came out with some changes (126 pages and 158 matchstick puzzles). The fifth edition, largely improved (Tromholt, 1892), was the last one Tromholt (1889) was involved with. It had 150 pages and 162 matchstick puzzles. All posterior editions were the same. Between 1892 and 1915, *twelve additional editions* were published. These 17 editions in the first 26 years, together with two posterior editions in 1986 and 2006, are an impressive success that no other book on matchstick puzzles has repeated until today!

Tromholt (1889) introduced another important feature to the world of matchstick puzzles: An announcement the number of possible different solutions readers are supposed to find which were published in solution section. In the first edition, this was done in three matchstick puzzles and in the fifth edition in twelve matchstick puzzles.

An example is a geometric puzzle with three announced and published solutions (Tromholt, 1892, p. 110, p. 147). Its formulations, initial matchstick configurations and three solutions are given in **Figure 1**.

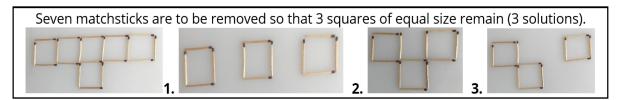
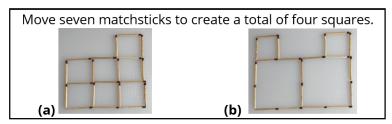


Figure 1. An example of puzzle with three announced and published solutions (the author's own creation)

An example of multiple-solution geometric puzzle with only one published solution is the first puzzle in Tromholt's (1889, p. 1, p. 101; 1892, p. 7, p. 127) books. Its formulation,

- (a) initial matchstick configuration and
- (b) single solution are given in Figure 2.

This puzzle with only one symmetrical Tromholt's (1889) solution was repeated in many posterior books (Bakst, 1954, p. 3, p. 183; Hansell, 1981, p. 49, p. 76; Troshin, 2022, p. 61, p. 323; Weaver, 1992, p. 41, p. 63; Williams, 2013, p. 11, p. 65) whose authors were unable to find any additional solution. Nevertheless, the first Tromholt's (1889) puzzle had four fundamentally different class of solutions with numerous variants.



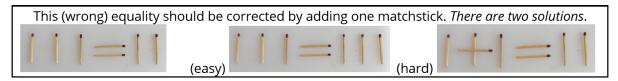
**Figure 2.** Formulation, initial matchstick configuration, and solution to the first Tromholt's (1889) puzzle (the author's own creation)

#### **Obermair's (1975) Treatment of Multiple-Solution Arithmetic Puzzles**

Gilbert Obermair (1934-2002) was an Austrian PhD holder, who designed and built various complex systems for teaching and gaming. His book on matchstick puzzles, tricks, and games achieved enormous success, no other puzzle book in 20<sup>th</sup> century and 21<sup>st</sup> century was able to reach even modestly. The book "Streichholz-spielereien [Matchstick games]" appeared in 1975 (Obermair, 1975) and had until 2000 thirteen editions in German with different publishers (Obermair, 1983, 2000). It was translated into several languages: French, Dutch, Czech, Polish, Hungarian, Portuguese, and English.

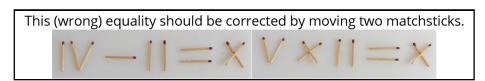
In his treatment of multiple-solution arithmetic puzzles Roman numerals, Obermair (1975), like Tromholt (1889), has shown two enigmatic approaches. In the first case, for some puzzles the number of solutions was announced after formulation, and those solutions were published in solution section. In the second case, for other multiple-solution puzzles only one solution was published.

The example of the first-case treatment is an arithmetic matchstick puzzle (Obermair, 1975, p. 87, p. 143; Obermair, 1983, p. 22, p. 42; Obermair, 1991, p. 107, p. 190; Obermair, 2000, p. 44, p. 57), whose formulation, initial matchstick configuration and two solutions (one easy and one hard) are given in **Figure 3**.



**Figure 3.** Obermair's (1975) multi-solution arithmetic puzzle with two announced solutions (the author's own creation)

The example of the second-case treatment is a multi-solution arithmetic matchstick puzzle (Obermair, 1975; Obermair, 1983, p. 23, p. 43; Obermair, 1991, p. 109, p. 207; Obermair, 2000, p. 45, p. 57), whose formulation, initial matchstick configuration and single solution are presented in **Figure 4**.



**Figure 4.** Obermair's (1975) multi-solution arithmetic puzzle with single published solution (the author's own creation)

Having for this multi-solution puzzle very low solving creativity, Obermair (1975) missed four additional solutions presented in **Figure 5**.



**Figure 5.** Four solutions missed by Obermair (1975) for his own multi-solution arithmetic puzzle with Roman numerals (the author's own creation)

#### AIMS AND METHODOLOGY OF THIS STUDY

Existence of four solutions missed by Obermair (1975) made this multi-solution arithmetic puzzle an excellent candidate for answering experimentally three research questions:

- 1. Does the information about the number of possible solutions stimulate puzzle-solvers' creative performance so that most would find more correct solutions than the original author?
- 2. Does the hint to think about all arithmetic operations help the participants find Obermair's (1975) solution?
- 3. What are students' reflections on the experience they had in solving Obermair's (1975) puzzle?

To achieve those ambitious aims, a worksheet was designed (**Figure 6**) and given to three different-size groups of students.

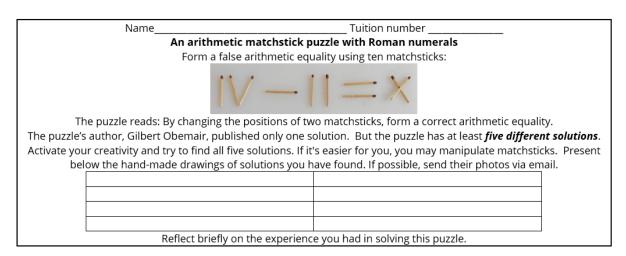


Figure 6. The worksheet used in the study (the author's own creation)

In the first group there were 14 first-semester physics students. With the researcher present in the classroom, they had 30 minutes to carry out the task. To eliminate a possible negative effect of cognitive bias "only arithmetic operation of adding or subtracting are allowed," after 20 minutes they received from the researcher a verbal hint: Think about all arithmetic operations.

In the second group there were 4 graduate students in mathematics education, one at doctoral level and three at Master level. With the researcher present in his office, they had 30 minutes to carry out the task. They didn't get the verbal hint.

The third group were 3 undergraduate students, one fifth-semester physics students and two mathematics students in their last semester. They received the worksheet via email and solved the puzzle as homework without time control and without the hint. They sent their solutions and reflections via email.

#### **RESULTS OF THIS STUDY**

The global results of this study are the following:

In all groups, the students presented 93 solutions. Among them, 71 (76 %) were correct and 22 (24 %) were incorrect.

Among 71 correct solutions, there were 13 Obermair's (1975) solutions (in what follows "O-solutions") and 58 correct solutions missed by Obermair (1975) (in what follows "M-solutions").

In the first group, one student found 5 correct solutions (1 O-solution and 4 M-solutions). Five students found 4 correct solutions (1 O-solution and 3 M-solutions). Four students found 3 correct solutions (1 O-solution and 2 M-solutions). Three students found 2 correct M-solutions. One student found 1 O-solution and 1 M-solution, and two only had 2 M-solutions.

In the second group, three students found 4 correct solutions. One student found 1 O-solution and 3 M-solution, and two only had 4 M-solutions. One student found 3 M-solutions.

In the third group, two students found 4 M-solutions and one student found 3 M-solutions.

All students' solutions are presented in the **Appendix A**.

According to these results, the answer to the first research question is:

All participants demonstrated better creative puzzle - solving performance than Obermair (1975), finding at least two correct solutions.

Analyzing the place of 12 O-solution in students' correct solutions in the first group which had the verbal hint (see the **Appendix A**), it is found that it appears two times as the second solution (likely without hint's influence), and ten times as the last solution (likely being induced by the hint). In the second and the third group who solved the puzzle without the hint, only one of seven students found the O-solution.

Being so, the answer to the second research question is:

The verbal hint "think about all arithmetic operation" helped ten students overcome cognitive bias "only arithmetic operation of addition or subtraction valid," and making them able to find Obermair's (1975) solution.

Students wrote a few revealing creativity-related or puzzle-solving reflections:

Guillermo (5 correct solutions): "I felt good about being able to find more solutions than the author of the puzzle, as that indicates that my creativity has increased."

Noé de Jesús (4 correct solutions): "It's good to know that I can reach that level of creativity, surpassing the creativity of certain scientists and mathematicians. Being able to achieve that is very satisfying."

Arturo (4 correct solutions): "I feel fulfilled knowing that my creativity has been improving, and that I can now solve puzzles more easily."

Oliver (4 correct solutions): "It feels satisfying to know that I found more solutions to this puzzle. One way to find more solutions was to use more types of arithmetic operations."

Héctor (3 correct solutions): I feel happy knowing that my creativity has improved during this course and I want to continue improving my creativity.

Alejandra (3 correct solutions): I felt incredibly satisfied to discover that I had surpassed the puzzle's author by finding more solutions than he published. It was a mixture of surprise, curiosity, and pride, because by moving just two matchsticks, I was able to see possibilities that initially seemed impossible. This challenge made me realize that creativity and perseverance can take us beyond the limit others have set.

Galilea (2 correct solutions): "It's satisfying to know that we too can be creative."

Claudia (4 correct solutions): "Perhaps if I hadn't been told there were up to five solutions, I wouldn't have kept trying. Being encouraged to come up with more solutions or being challenged to find them is a great incentive. It's possible that more solutions, different from mine, could be found, but I'm satisfied because initially I didn't think I could even come up with one."

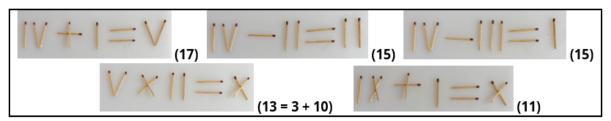
Sófia (4 correct solutions): "I have always found it comforting to discover more than one solution to matchstick puzzles, not only because it entertains me to think about these "mental exercises" during my free time, but also because it shows me that the same problem can be solved in different ways, something that I believe can be applied in the real world and daily life (I personally try to remember this every day)."

These reflections lead to the following answer to the third research:

Students' experiences in solving Obermair's (1975) puzzle were very positive, related either to their creativity or to their increased puzzle-solving skills.

#### **DISCUSSION**

It is interesting to see how 71 correct solutions found by the students are distributed over 5 correct solutions to Obermair's (1975) puzzle. The distribution is given in **Figure 7**.



**Figure 7.** The distribution of students' correct solution (the number after a particular solution indicates how many times it was found) (the author's own creation)

Among M-solutions, the first solution, transforming right-side "X" into "V", is obviously the easiest to find. It is in resonance with the fact that it was the first solution found by eight students. The last M-solution, transforming left-side "V" into "X" and left-side "- I" into "+," is the hardest to find. It was given as the first solution only by two students (see **Appendix A**).

Without an influence of verbal hint, the O-solution is globally the hardest to find. Nobody was able to find it as the first solution!

Only 9 of 21 students didn't present a single erroneous solution. The great majority of 22 erroneous solutions was presented by students with a smaller number of correct solutions. It shows that these students didn't carefully verify the correctness of their solutions. In this last step in puzzle-solving, the use of critical thinking is crucial to find out if more than two matchsticks were moved. The most frequent erroneous solutions are presented in **Figure 8**.



**Figure 8.** Three most popular erroneous solutions (the number after a particular solution indicates how many times it was found) (the author's own creation)

In all these incorrect solutions three matchsticks were moved. This result suggests a possible relationship between low creative and low critical thinking. It is similar to the findings in literature that support the connection between lower creative flexibility and a limitation in adaptive thinking (Leikin, 2013; Levav-Waynberg & Leikin, 2012).

# **CONCLUSIONS AND RECOMMENDATIONS**

The main result of this study is experimental evidence that students puzzle-solving creativity, clearly stimulated by the information about the number of possible solutions, exceeded that of the puzzle author. This result shows that multiple-solution arithmetic matchstick puzzles with announced number of solutions should be used more frequently in mathematics education to foster creative thinking and creativity-related self-confidence of students across education levels.

Mathematics textbook authors mainly use one-solution matchstick puzzles, or, following uncritical behavior of puzzle book authors, for multiple-solution puzzles publish only a single solution.

The limitations of this study are relatively small number of heterogenous participants and the use of only one multiple-solution arithmetic matchstick puzzles.

In future studies, large scale homogenous student groups should be involved with various multiple-solution arithmetic and geometric matchstick puzzles.

It would also be interesting to design different hints for helping students solve better multiple-solution arithmetic and geometric puzzles than puzzle authors.

Future research also could examine how announcing multiple solutions influences students' self-efficacy and persistence in matchstick puzzle solving.

Funding: The author received no financial support for the research and/or authorship of this article.

**Acknowledgments:** The author would like to thank all students who participated voluntarily in this study. The author would also like to thank Heinrich Hemme (University of Applied Sciences in Aachen, Germany) for a productive discussion in which four additional solutions to Obermair's (1975) arithmetic matchstick puzzle were found.

**Ethics declaration:** The study was conducted in accordance with the Guidelines of the Code of Ethics and Conduct of the Benemérita Universidad Autónoma de Puebla. The students voluntarily participated in the study and gave their consent for mentioning only their personal names in the results reporting.

**Declaration of interest:** The author declared no competing interest.

**Data availability:** Data generated or analyzed during this study are available from the author on request.

#### **REFERENCES**

- Alencar, E. M., Fleith, D. D. S., Boruchovitch, E., & Borges, C. N. (2015). Creativity in elementary school: Inhibiting and facilitating factors according to school principals. *Psicologia: Teoria e Pesquisa, 31*(1), 105-114. https://doi.org/10.1590/0102-37722015011849105114
- Applebaum, M. (2025). Proof by play: Teaching the parity principle with math games and puzzles. *Contemporary Mathematics and Science Education, 6*(2), Article ep25017. https://doi.org/10.30935/conmaths/17404
- Bakst, A. (1954). *Mathematical puzzles and pastimes*. D. Van Nostrand Company.
- Braun, F. (1876). *Der junge Mathematiker und Naturforscher* [The young mathematician and natural scientist]. Verlag von Otto Spamer.
- Foster, N., & Schleicher, A. (2022). Assessing creative skills. *Creative Education, 13*(1), 1-29. https://doi.org/10.4236/ce.2022.131001
- Gao, Q., & Hall, A. (2024). Early childhood teachers' beliefs about creativity and practices for fostering creativity: A review of recent literature. *Early Childhood Education Journal*. https://doi.org/10.1007/s10643-024-01816-4
- Hansell, S. (1981). 109 knifflige Streichholztricks [109 tricky matchstick tricks]. Otto Maier Verlag.
- Idika, M. I., & Oluwaseyi, M. P. (2024). The impact of puzzle game and video-based puzzle strategies on students' achievement and retention in periodicity. *Journal of Mathematics and Science Teacher*, *4*(2), Article em061. https://doi.org/10.29333/mathsciteacher/14366
- Jukić Matić, L., & Sliško, J. (2024). An empirical study of mathematical creativity and students' opinions on multiple solution tasks. *International Journal of Mathematical Education in Science and Technology, 55*(9), 2170-2190. https://doi.org/10.1080/0020739X.2022.2129496
- Keeling, R. P., & Hersh, R. H. (2011). *We're losing our minds. Rethinking American higher education*. Palgrave Macmillan. https://doi.org/10.1057/9781137001764

Leikin, R. (2013). Evaluating mathematical creativity: The interplay between multiplicity and insight. *Psychological Test and Assessment Modeling*, *55*(4), 385-400.

Leire, A., Morais, M. F., Cortabarria, L., & Lorea, M. (2024). Barriers to personal creativity in Spanish and Portuguese university students. *Educación XX1*, 27(1), 81-104. https://doi.org/10.5944/educxx1.35761

Levav-Waynberg, A., & Leikin, R. (2012). The role of multiple solution tasks in developing knowledge and creativity in geometry. *The Journal of Mathematical Behavior, 31*(1), 73-90. https://doi.org/10.1016/j.jmathb.2011.11.001

Mittenzwey, L. (1880). Mathematische Kurzweil [Mathematical amusement]. Julius Klinkhardt.

Nickerson, R. (2010). How to discourage creative thinking in the classroom. In R. Beghetto, & J. Kaufman (Eds.), Nurturing creativity in the classroom (pp. 1-5). Cambridge University Press. https://doi.org/10.1017/ CBO9780511781629.002

Obermair, G. (1975). Streichholz-Spielereien [Matchstick games]. Wilhelm Heyne Verlag.

Obermair, G. (1983). Streichholz-Spielereien [Matchstick games]. Honeywell Bull Computer.

Obermair, G. (1991). Streichholz-Spielereien [Matchstick games]. Wilhelm Heyne Verlag.

Obermair, G. (2000). Streichholz-Spielereien [Matchstick games]. Falken Verlag.

Pellegrino, J. W., & Hilton, M. L. (2012). *Educating for life and work: Developing transferable knowledge and skills in the 21st century*. National Academies Press.

Rubenstein, L. D., McCoach, D. B., & Siegle, D. (2013). Teaching for creativity scales: An instrument to examine teachers' perceptions of factors that allow for the teaching of creativity. *Creativity Research Journal*, *25*(3), 324-334. https://doi.org/10.1080/10400419.2013.813807

Trilling, B., & Fadel, C. (2009). 21st century skills: Learning for life in our times. Jossey-Bass.

Tromholt, s. (1889). *Streichholzspiele. Denksport und Kurzweil* [Matchstick games. Brain teasers and entertainment] (1st ed.). Otto Spamer Verlag.

Tromholt, S. (1892). *Streichholzspiele. Denksport und Kurzweil* [Matchstick games. Brain teasers and entertainment] (5<sup>th</sup> ed.). Otto Spamer Verlag.

Troshin, V. (2022). Don't mess around with matches. Litres (Published in Russian).

Vincent-Lancrin, S. (2021). *Skills for life. Fostering creativity*. Inter-American Development Bank. https://doi.org/ 10.18235/0003742

Weaver, J. (1992). Brain teasers. Watermill Press.

Williams, B. (2013). *Gymnastics for the brain. Matchstick puzzles*. Salim Bouzekouk.

World Economic Forum. (2023). Future of jobs report. *World Economic Forum*. https://www.weforum.org/publications/the-future-of-jobs-report-2023/

# **APPENDIX A**

In **Appendix A**, all students' solutions are presented in the order in which they were reported. Erroneous e-solutions are with red letters, and Obermair's (1975) solution is with green bold letters.

The solutions of the first group ( $N_1 = 14$ ) are given in **Figure A1**.

Guillermo (5 C-solutions)	Brisa Elizabeth (4 C-solutions)
	· · · · · · · · · · · · · · · · · · ·
(IV + I = V) (IV - II = II) (IX + I = X) (IV - III = I)	$(IV - III = I) (IV - II = II) (IV + I = V) (V \times II = X)$
(V x II = X)	
Noe de Jesús (4 C-solutions, 3 E-solutions)	Jesús (4 C-solutions)
( V - I  =  I ) ( V - V = -I ) ( V + I  = V)	$(IV + I = V) (IV - III = I) (IX + I = X) (V \times II = X)$
$( V - I  =  I ) ( V - I I = I) (V + V = X) (V \times II = X)$	
Juan Manuel (4 C-solutions)	Arturo (4 C-solutions)
$( X + I = X) ( V + I = V) ( V - I I = I) (V \times II = X)$	$( V + I = V) ( V - II = II) ( V - III = I) ( V \times II = X)$
Hector Ulises (3 C-solutions, 2 E-solutions)	Israel (3 C-solutions, 1 E-solution)
$( V - I   =  ) (V + V = X) ( V + I  = V) (V \times II = X)$	$(IV - III = I)$ $(V \times II = X)$ $(IV + I = V)$ $(V + V = X)$
(V – III = II)	
Oliver (3 C-solutions, 2 E-solutions)	Gloria (3 C-solutions, 3 E-solutions)
$( V -    =   ) (V \times    = X) (V + V = X) ( X +    = X)$	(IV + I = V) (V - II = III) (IV - II = II) (V - III = II)
(V – III = II)	$( V - I =  I ) (V \times II = X)$
Iván (3 C-solutions, 3 E-solutions)	Galilea (2 C-solutions, 2 E-solutions)
( V - I  =  I ) ( V + I  = V) ( IV - I  =  I ) ( V - I  =  I )	(IV - III = I) (IV - V = -I) (IV - II = II)
$(V + V = X) (V \times II = X)$	$(V - II \neq X)$
Diego (2 C-solutions, 2 E-solutions)	Luis (2 C-solutions, 3 E-solutions)
( V + I = V) ( V - I =  I ) ( V - I  =  I ) ( V - I I =  I )	(V - III = II) (V - II = III) (IV + I = V) (V + V = X)
	$(V \times II = X)$

Figure A1. All solutions presented by students in the first group (the author's own creation)

The solutions of the second group ( $N_2 = 4$ ) are given in **Figure A2**.

Alinne (4 C-solutions)	Claudia (4 C-solutions)
$( V - I   =  ) ( V - I  =  I ) ( X + I  = X) (V \times II = X)$	( V -   I =  ) ( V -    =   ) ( X +   = X) ( V +   = V)
María Fernanda (4 C-solutions, 1 E-solution)	Alejandra (3 C-solutions)
(IV + I = V) (IX + I = X) (IIV - II = I) (IV - II = II)	(IX + I = X) (IV + I = V) (IV - II = II)
(IV - III = I)	

Figure A2. All solutions presented by students in the second group (the author's own creation)

The solutions of the third group ( $N_3 = 3$ ) are given in **Figure A3**.

Sofía (4 C-solutions)	Vannessa (4 C-solutions, 1 E-solution)
( V - I  =  I ) ( V + I  = V) ( V - I I  = I) ( X + I  = X)	(IV - III = I) (IV - I = III) (IV + I = V) (IV - II = II)
	( X +   = X)
Yuritzi (3 C-solutions, 1 E-solution)	
(IV + I = V) (IV - II = II) (IX + I = X) (V - II = III)	

Figure A3. All solutions presented by students in the third group (the author's own creation)

