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Research Article

Fostering students' definitions and images in parallelism and perpendicularity: A paper folding activity

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ABSTRACT

This study investigated the effect of a paper folding activity prepared to develop the sixth-grade students' concept definitions and images of parallelism and perpendicularity concepts. The study also examined how the concept definition and images changed after the paper folding activity. A combination of quantitative and qualitative methods was used. A one-group pre-/post-test design revealed that the paper folding activity had a significant positive effect on students' concept definitions and images. In addition, the interviews after pre- and post-tests indicated that the students' personal concept definitions of parallelism and perpendicularity of two lines/line segments began to match the formal concept definitions of these concepts after the paper folding activity. Lastly, missing and mis-in concept image situations, encountered generally in the pre-test, were observed less after the paper folding activity.

Keywords: concept definition, concept image, paper folding, parallelism, perpendicularity

INTRODUCTION

The main focus of geometry, as one of the significant learning areas in mathematics (Clements & Battista, 1992), in primary and secondary schools is to teach basic geometric concepts such as lines, rays, and angles. Common Core State Standards for Mathematics (CCSSM, 2010) emphasizes reasoning with shapes and their attributes from the beginning of kindergarten. Similarly, two- and three-dimensional shapes and their classification by line and angle properties are given significant weight in Turkey's mathematics curriculum as early as grade 1 (Ministry of National Education [MoNE], 2018). Students are expected to learn basic geometric concepts such as angles and parallel and perpendicular lines from the beginning of grade 3. These basic concepts have a crucial role in understanding the geometrical concepts in the following years since many mathematical concepts depend on lower-order concepts (CCSSM, 2010).

Mansfield and Happs (1992) expressed that the topic of parallel lines has such a significant role in geometry since it forms a basis for the classification of polygons, an understanding of angle relationships, and geometric proofs. The findings of the latest study by Tirosh and Tsamir (2022) exemplify the role of parallel lines. Researchers observed that insufficient interpretation of parallel lines affected a student's decisions regarding parallelograms; she erroneously identified a non-example of parallelograms in the set of parallelograms. The topic of perpendicular lines has the same importance due to similar reasons. To illustrate, if a student cannot construct a perpendicular line segment to a given line segment, s/he cannot construct the altitudes of a triangle and compute the area of that triangle. Despite their significance, the studies regarding

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parallelism and perpendicularity concepts indicated that students have some misconceptions about them and have difficulty in the construction of parallel and perpendicular lines (Duatepe-Paksu & Bayram, 2019; Gal & Vinner, 1997; Mansfield & Happs, 1992; Ulusoy, 2016). Hence, in this study, we aimed to develop students' conceptions by offering alternative instruction on the concepts of parallelism and perpendicularity.

Studies About Parallelism & Perpendicularity

The available literature about parallelism and perpendicularity concepts dates back to the 1990s. For example, in an earlier study with eighth-grade students, Mansfield and Happs (1992) observed that even though some students could accurately describe parallelism, they depended on the appearance of the given line segments in identifying the parallel line segments. Similarly, Gal and Linchevski (2002) stated that ninth graders had difficulty explaining why the sides of a rectangle with one diagonal given were parallel. Besides, some studies about the right-angle concept indicated that students' difficulties with the perpendicularity concept might be because of their understanding of the right-angle. To illustrate, for students, a right angle may mean an angle that points to the right (Clements & Battista, 1992), or they cannot link the right-angle concept with a 90° angle (Gal & Vinner, 1997). In the same way, Hershkowitz (1989) stated that students had difficulty identifying right-angle triangles if the perpendicular sides were not vertical and horizontal. As such is the case, when the students were asked to check the perpendicularity of the diagonals of a square, they could not point out the right angle between the diagonals but focused on the angles of the square (Gal & Linchevski, 2010).

The latest studies also confirmed that students' images of parallelism and perpendicularity are limited to horizontal and vertical lines/line segments, and they do not depend on the definitions of the concepts in the identification or construction tasks (Duatepe-Paksu & Bayram, 2019; Ulusoy, 2016). Moreover, traditional instruction may not effectively eliminate students' misconceptions and difficulties regarding parallelism and perpendicularity (Mansfield & Happs, 1992) and enrich their images. Therefore, this study aims to develop students' definitions and images of parallelism and perpendicularity concepts.

Students may deal actively with geometric ideas and learn to reason about them through well-designed activities and concrete materials (National Council of Teachers of Mathematics [NCTM], 2000). Many research studies indicated that concrete materials or hands-on activities are helpful for the development of geometric concepts. The Turkish mathematics curriculum also recommends integrating concrete materials, more notably paper folding activities, into the instruction of many geometric concepts, such as the altitudes of triangles or congruent and similar polygons (MoNE, 2018). Haga et al. (2008) stated that paper folding provides students with practical manipulatives through which abstract mathematical concepts turn into concrete. In fact, paper folding is mathematics in action (Wares & Elstak, 2017), which is very important for understanding geometrical concepts. In the folding process, students can grasp many mathematical concepts, such as perpendicular bisector or properties of squares (Wares & Elstak, 2017), by discovering the relationship between folded lines and angles formed by those lines (Johnson, 1999). Various research studies indicated that paper folding activities could be a valuable instructional tool for geometry and different learning areas in mathematics, such as algebra (e.g., Boakes, 2009; Georgeson, 2011; Kandil & Isiksal-Bostan, 2019). However, in the available literature, we could not find a study mainly focusing on teaching parallelism and perpendicularity concepts through paper folding. In this respect, it was considered a worthwhile contribution to the literature to focus on developing students' definitions and images in parallelism and perpendicularity concepts through their engagement in a paper folding activity.

Theoretical Background

Among the various theories regarding the understanding of geometrical concepts, in this study, we focused on the model proposed by Vinner and his colleagues (Tall & Vinner, 1981; Vinner, 1983; Vinner & Hershkowitz, 1980). For these researchers, two constructs are employed in concept formation: concept image and concept definition. The first component, concept image, refers to "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (Tall & Vinner, 1981, p. 152). The concept image of individuals may differ from each other, and it may even be stated that the concept image depends on the cultures of the individuals (Vinner, 1983, 2011). Moreover, sometimes, only a part of the concept image may be stimulated. This portion is called the evoked concept image by Tall and Vinner (1981). Besides, concept definition is "a form of words used to specify that concept" (Tall & Vinner, 1981, p. 152). While a definition accepted by the mathematical community is regarded as a formal concept definition, an individual may form his own definition. In this case, it is called the personal concept definition, which reflects the individual's concept image (Vinner, 1983).

In a recent study, Tirosh and Tsamir (2022) extended this model by adding two more constructs, missing and mis-in concept images, to describe the students' concept images. The first construct describes a situation, where an example of a concept is regarded as a non-example. The second one, mis-in concept image, refers to "a non-example of the concept is *mis(takenly) in(cluded)* in the set of examples of the concept, and consequently, this non-example is erroneously identified as an example of the concept" (Tirosh & Tsamir, 2022, p. 3). For instance, if a student identifies a trapezoid as a parallelogram, a non-example is *mis(takenly) in(cluded)* in the set of parallelograms. Besides, if this student states that a rhombus is not a parallelogram, then s/he has a missing concept image (Tirosh & Tsamir, 2022). The researchers suggested analysing missing and mis-in concept image situations to place and study issues about them.

Vinner's model asserts that an amalgam of concept definitions and images is essential for a good learning process (Vinner, 1983; Vinner & Hershkowitz, 1980). However, various studies indicated that students generally depend only on their concept images (Clements, 2003; Gutiérrez & Jaime, 1999; Vinner, 1983, 2011). For instance, Mansfield and Happs (1992) observed that students relied on their concept images to identify the parallelism of the given line segments instead of their accurate definitions. Furthermore, students' concept images may be limited only to the prototype examples (Duatepe-Paksu & Bayram, 2019; Gutiérrez & Jaime, 1999; Ulusoy, 2016; Vinner & Hershkowitz, 1980). Prototype examples have strong visual characteristics because of some non-critical attributes (Hershkowitz, 1989). For the parallelism and perpendicularity concepts, two horizontal or vertical parallel lines/line segments or + shapes may be given as the prototype examples of parallel/perpendicular lines/line segments (Tirosh & Tsamir, 2022; Ulusoy, 2016). Although they may help initial concept formation due to their easy recognition, they may restrict the students' concept images (Hershkowitz, 1989; Wilson, 1990). When such is the case, students may regard the non-examples of the concepts as examples or the examples as non-examples (Wilson, 1990), missing and mis-in concept image situations (Tirosh & Tsamir, 2022). Although students can use prototype examples, the limitations or errors that prototypes can lead to should be understood, and their concept images should be enriched to include both prototype and non-prototype examples (Tsamir et al., 2008; Vinner, 2011).

On the other hand, students' concept images should align with their concept definitions (Tsamir et al., 2015) because definitions are a medium for a strong understanding of a given concept (Edwards & Ward, 2008). Accurate definitions lay the basis for mathematical theories and provide mathematical coherence (Tsamir et al., 2015). Also, concept images may be formed and flourish with the aid of definitions (Vinner & Hershkowitz, 1980). For these reasons, students should understand the significance of having precise definitions and learn to consult them in case of a conflict (Clements, 2003; Tsamir et al., 2015). Nevertheless, some pedagogical approaches are essential to support the development of concept definitions (Edward & Ward, 2008). Moreover, it is important to identify students' missing and mis-in concept images for different concepts, the sources behind them, and provide valuable ways to deal with them (Tirosh & Tsamir, 2022). Keeping these and the positive effects of paper-folding activities in mind, in the current study, we tried to foster students' concept definitions and images regarding the concepts of parallelism and perpendicularity through a paper-folding activity by investigating the research questions below. The study also allowed for identifying students' missing and mis-in concept image situations for the parallelism and perpendicularity concepts and illuminated a way of dealing with them.

1. What is the effect of a paper folding activity on sixth-grade students' concept definitions and images regarding parallelism and perpendicularity of two lines/line segments?
2. How do sixth-grade students' concept definitions and images regarding parallelism and perpendicularity of two lines/line segments change after participating in a paper folding activity?

METHODS

Research Design

A combination of quantitative and qualitative methods was utilized to analyze the effect of the paper folding activity. For the quantitative part, we used a one-group pre-test post-test design, where the performance of a group was assessed both before and after the treatment (Fraenkel et al., 2012). In addition, interviews with nine students were conducted after pre-and post-tests to examine the changes in the students' concept definitions and images.

Participants & Context of the Study

Data were collected through the convenience sampling method from sixth-grade students enrolled in a public middle school in a small district of Ankara, Turkey. In the school, where the first author works as a mathematics teacher, there were two sixth-grade classrooms, which included 43 students (20 boys and 23 girls), and the students were from lower- and middle-class families. As a school requirement, all the boys and girls were in separate classrooms. To eliminate the effect of gender, we considered all students in the two classrooms as one group. Therefore, this study does not involve a control group, and a one-group pre-test post-test design was performed. Mathematics grades of the two classrooms were similar to each other on average. Furthermore, although these two classes had different mathematics teachers in the previous years, there was no significant difference between their pre-test scores ($t[41]=1.34$, $p=0.19$). In the post-test, the average scores of the girls' class were higher; however, the difference was insignificant, as shown in **Table 1** ($t[41]=1.61$, $p=0.11$).

Table 1. Results of descriptive & inferential statistics for two classrooms

Test	Girls		Boys		*t*	*p*
	Mean	SD	Mean	SD		
Pre-parallelism & perpendicularity test (pre-PPT)	14.13	8.58	18.2	11.33	1.34	0.19
Post-parallelism & perpendicularity test (post-PPT)	33.78	9.12	28.8	11.13	1.61	0.11

Participants of this study have engaged with the basic geometric concepts from the beginning of the third grade (MoNE, 2013). Until the fifth grade, they are responsible for attaining point, line, line segment, ray, angle, and plane concepts. In the fifth grade, they learn the relative positions of two lines in a plane. While how to construct a line segment parallel to a given line segment is discussed in the fifth grade, the curriculum requires students to learn the construction of perpendicular lines/line segments in the sixth grade (MoNE, 2013). Therefore, we selected sixth-grade students as the current study's participants before they learned the construction of the perpendicular line segments. Although they completed most of the goals of the paper folding activity except for the construction of perpendicular line segments, we believed that they would have difficulties based on the studies in the literature, and the pre-test results confirmed this. As a final note, the students had not received any instruction regarding paper folding prior to the study. However, a few students expressed their experiences with origami activities.

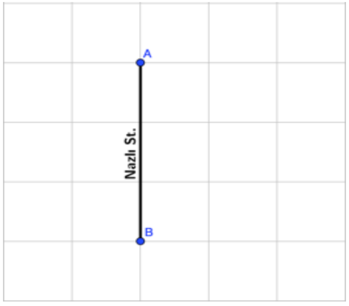
Data Collection Instruments

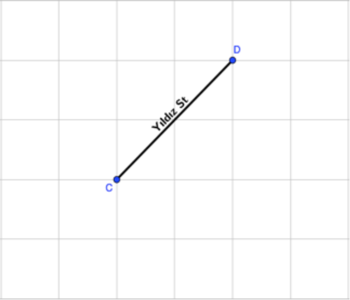
The present study used parallelism and perpendicularity tests (PPT) and interviews to collect data.

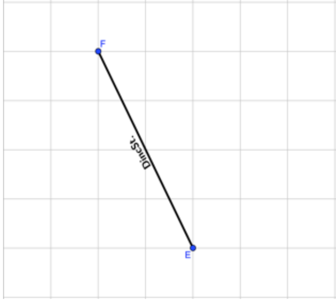
Parallelism & perpendicularity test

PPT measured the students' concept definitions and images regarding parallelism and perpendicularity. There were six questions with 12 sub-questions under three main categories (**Table 2**). The first category, definitions, was asked to examine students' personal concept definitions and prepared by the researchers. In these questions, students were expected to explain what parallelism or perpendicularity of two lines/line segments means to them. For the concept images, we used two cognitive tasks similar to the various studies in the literature (e.g., Tsamir et al., 2015; Vinner, 1983; Vinner & Hershkowitz, 1980). Namely, three identification and construction tasks for parallelism and perpendicularity were used. Moreover, two additional drawings in the definition tasks were requested to understand students' concept images better (Q1 and Q4 in **Table 2**).

Table 2. Parallelism & perpendicularity test categories & questions

<table>
<tr><th>Category</th><th colspan="3">Questions</th></tr>
<tr><td>Definitions</td><td colspan="3">Q1. Explain what parallelism of two lines/line segments means to you. Support your explanations with two drawings on squared paper below.
Q4. Explain what perpendicularity of two lines/line segments means to you. Support your explanations with two drawings on squared paper below.</td></tr>
<tr><td>Identifications</td><td colspan="3">Q2. & Q5. Based on given map, determine positions of given streets (parallel, perpendicular, & intersecting) according to each other. Explain your reasoning for each question.
For parallelism:
Q2a. Kartalcik St. & Demirhendek Ave. are _____________ each other.
Explanation:
Q2b. Tasdelen Ave. & Saricam St. are _____________ each other.
Explanation:
Q2c. Kocak St. & Dalboyu St. are _____________ each other
Explanation:
For perpendicularity:
Q5a. Fikir St. & Saricam St. are _____________ each other.
Explanation:
Q5b. Demirhendek Ave. & Altınay Ave. are _____________ each other.
Explanation:
Q5c. Kartalcik St. & Dalboyu St. are _____________ each other.
Explanation:</td></tr>
<tr><td>Constructions</td><td colspan="3">Q3. & Q6. Below, you will see three-line segments representing some streets in different neighborhoods in Ankara. Construct a parallel/perpendicular street to each street on squared paper provided. Explain how you ensured that your street is parallel to given one.</td></tr>
<tr><td></td><td>Q3a. & Q6a.


Explanation:</td><td>Q3b. & Q6b.


Explanation:</td><td>Q3c. & Q6c.


Explanation:</td></tr>
</table>

In the identification tasks, students were given a map of a specific district in Ankara, Turkey (**Appendix A**), which the researchers prepared based on the original district map. Since students do not know the relative position of lines in space but in a plane, we explained that this map is the two-dimensional representation of the district. It was also stated that while the line segments represented the streets and avenues they learned in previous years, dots represented the buildings. Then, students were requested to determine the positions of some streets according to each other. Both prototype (Q2c and Q5b) and non-prototype (Q2a and Q5c) examples were included in the identification tasks. Besides, in the construction tasks, students were provided three streets and asked to construct a parallel or perpendicular street to the given ones. Similar to the identification tasks, both prototype line segments (Q3a and Q6a) that involved the construction of a parallel/perpendicular line segment to a vertical one (Ulusoy, 2016) and the non-prototype line segments (Q3b, Q3c, Q6b, and Q6c) were placed in the construction tasks.

We also considered students' difficulties identified in the literature while selecting the streets. For example, since students may not think of extensions of the line segments in identifying parallelism or perpendicularity (Mansfield & Happs, 1992; Ulusoy, 2016), Q2b and Q5c were asked. The Turkish curriculum highly advises making identifications and constructions of geometric figures on squared or dot paper by emphasizing its crucial function in determining the attributes of the figures (MoNE, 2018). For example, a student can construct a perpendicular line segment using the angles formed by the squares' diagonals. Therefore, students were asked to answer the identification and construction tasks on the provided square paper. Moreover, they had some materials while answering the test, such as a ruler or protractor. It was stated that they could benefit from those materials if desired, but they should explain their reasoning.

Two mathematics educators were requested to examine and comment on the questions to ensure the content validity of PPT scores. They analyzed the questions in terms of their consistency with the research questions, relevant objectives in the Turkish curriculum, and clarity and comprehensibility. After making the essential revisions based on expert opinions, the instrument was piloted with ten seventh-grade students who met all the study requirements. We prepared PPT's final version based on the feedback during the implementation and analysis of students' answers and determined the test duration. For instance, an explanation part for each choice in the identification and construction tasks was added since most students had yet to explain their reasoning. Fraenkel et al. (2012) stated that the Cronbach's alpha coefficient indicates internal consistency if the test involves essay-type questions, which are not scored as right, one, versus wrong, zero, but allow for several alternative answers. In PPT, while the scores of the definition questions change between zero and three, the scores in the identification and construction questions range from zero to four. Cronbach's alpha coefficient was calculated using the scores of the students in the pilot test and was found to be 0.81. The interrater reliability coefficient was also calculated to indicate one more evidence of reliability. Another researcher scored students' answers in the pilot test according to the rubrics, and the level of agreement between the two researchers was found to be 0.83. Both coefficients indicated that PPT is reliable for the current study since it is above 0.70 (Fraenkel et al., 2012).

Interviews

Although all questions in PPT required an explanation, students generally wrote short statements. Therefore, interviews were used in addition to PPT to gain more detailed information about students' definitions, identifications, or constructions. The researchers developed interview questions based on the students' answers (**Appendix B**). The first author interviewed nine students selected based on their achievement levels in the pre-test following the weeks of pre-and post-tests. In particular, three students from each level (below, above, and on average) were selected to include all different responses. While the pre-test scores of students above the average were between 31 and 39 out of 54, the scores of the average students ranged between 13 and 17. On the other hand, the pre-test scores of students below the average were less than or equal to 10.

Paper Folding Activity

The researchers prepared the paper folding activity by considering the studies about the parallelism/perpendicularity concepts and paper folding. Although obtaining parallel and perpendicular line segments with paper is not a new idea, we transformed this idea into a complex activity, where students first explore the construction process by paper folding and then apply their observations to different line segments on the squared paper. The paper folding activity was implemented for eight lesson hours (four hours for perpendicularity and parallelism, respectively) and involved three sections.

The first section aimed to let students remember the basic geometric concepts of lines, line segments, rays, and relative positions of two lines/line segments through teachers' questions. Moreover, the previous objectives before the activity were about the angle definition and adjacent, complementary, and alternate angles. Thus, the concept of angle was reviewed with students before the paper folding activity. The first section was completed in one lesson hour.

In the second section, students tried to obtain parallel and perpendicular line segments individually in the given contexts with the waxed paper provided throughout the two lesson hours. We used waxed paper for its various benefits, such as transparency in observing the folding process (Olson, 1975). This component reflects the significant aspect of the activity in that instruction for obtaining perpendicular or parallel line segments was not given directly to the students by the teacher. Initially, students were given a line segment representing some objects and requested to construct a parallel/perpendicular line segment by just folding the paper. This part aimed to let students discover the process of parallel/perpendicular line segments themselves. When they needed it, the teacher asked some guiding questions. For example, when students had difficulty folding a parallel street to a given one, the teacher asked, "What if we first turn into a perpendicular street? What should we do next to turn into a street parallel to the one at the beginning?".

In the third section, which lasted one hour, students were expected to apply their observations in the second step to the various line segments on the squared paper. We also asked them whether they could construct parallel/perpendicular line segments through other strategies. Many line segments other than the prototype ones (Clements, 2003; Tsamir et al., 2008; Vinner, 1983) were presented to the students to enhance their concept images. The teacher also discussed wrong constructions since non-examples may decrease the effect of prototypes (Wilson, 1986). Lastly, students were required to make a justification after each construction. Tsamir et al. (2015) stated that when solving geometry tasks, students should be encouraged to base their decisions on the critical attributes of the figures, not only on their images. Therefore, the appearance of the line segments was not accepted as enough justification. We asked students questions such as "how can you be sure that these line segments are parallel/perpendicular to each other?" or "how can you be sure that the angle between those line segments is 90°?" or "how did you make the distance between two-line segments as equal?" In this way, we tried to guide students to make decisions based on their appearance and the definition of the concepts (Clements, 2003). Moreover, since students are expected to consult their definitions during their identifications and constructions, we believed the paper folding activity would enable development in students' definitions of parallelism and perpendicularity. Detailed lesson plans for the paper folding activity were provided as supplementary material.

Data Collection Procedure

After the preparation of the PPT, the pilot study and revisions were completed first. Then, it was administered as a pre-test in the first week of the implementation. The students were given 40 minutes to complete the test. The first researcher conducted pre-interviews in the second week with the selected students. Following the pre-interviews, the first researcher, the students' regular mathematics teacher, implemented the paper folding activity about perpendicularity and parallelism. Researchers administered PPT again as a post-test at the end of the activities. The duration between the pre-and post-tests was four weeks. Lastly, post-interviews were conducted with the same students in the pre-interviews to get more detailed information regarding the second research question.

Data Analysis

We used both quantitative and qualitative methods for the analysis. Firstly, descriptive and inferential statistics were used to quantitatively analyze the effect of the paper folding activity on students' concept definitions and images. For this aim, each student's answers were evaluated based on the rubrics prepared by the researchers. An example rubric for Q1 can be examined in **Appendix C**. Then, the mean, standard deviation, skewness, and kurtosis values of the pre-and post-test scores were calculated for the descriptive statistics. In terms of inferential statistics, a paired samples t-test was carried out. All these analyses were performed by using SPSS software. For the practical significance of results, Cohen's (1977) d was calculated.

For the qualitative part, content analysis was performed on pre- and post-tests and transcriptions of the interviews. To determine the change in students' concept definitions and images, all students' definitions and images in pre- and post-tests were compared. More specifically, firstly, each student's definitions and images of parallelism and perpendicularity and missing and mis-in-concept image situations were identified through open coding of the pre-test and pre-interviews. With the coding of the post-test and post-interviews and comparing these codes with the previous ones, the possible changes in students' concept definitions and images and the possible sources behind their missing and mis-in concept images were analyzed. For example, for the concept definitions, it was compared to how many students' personal concept definitions matched the formal concept definitions in pre-test and post-tests, which may be one of the reasons behind the difference in the quantitative results. Similarly, how many students had an inadequate understanding of the concepts related to parallelism and perpendicularity-extensions and relative positions of two lines/line segments in a plane, which might lead to missing and mis(*takenly*) in(*cluded*) concept image situations, were identified and compared in both pre- and post-tests.

FINDINGS

Descriptive and inferential statistics were conducted to answer the first research question. The obtained statistics are displayed in **Table 3**.

Table 3. Results of descriptive & inferential statistics for PPT

	Pre-PPT		Post-PPT		*t*	*p*	*d*
	Mean	SD	Mean	SD			
All students	16.02	10.04	31.47	10.29	9.46	0.000	1.44

Table 3 indicated that while students had a mean score of 16.02 (standard deviation [SD]=10.04) in the Pre_PPT, their mean score in the Post_PPT was 31.47 (SD=10.29) out of 54. In other words, descriptive statistics revealed an increase in PPT scores of the sixth-grade students after the paper folding activity. Inferential statistics were used to test whether this difference between pre and post-test scores is statistically significant. For the statistical analysis, a paired samples t-test was selected since there was only one group of participants in the current study. Before the analysis, assumptions of the paired samples t-test, independence of observations, and the normal distribution of difference scores (Gravetter & Wallnau, 2000) were checked. After assumptions were accepted to be satisfied, a paired samples t-test was conducted. The result of the analysis indicated that the effect of the paper folding activity on the sixth-grade students' concept definitions and images regarding parallelism and perpendicularity was statistically significant ($t[42]=9.47$, $p<0.001$). According to Cohen's (1977) guidelines for the paired-samples t-test, the d statistic was calculated as 1.44, corresponding to a large effect size. This indicated that the findings are practically significant in addition to the statistical significance.

After conducting quantitative analysis, students' answers in pre- and post-questionnaires, interviews with nine participants, and classroom discussions were analyzed to examine how the students' concept definitions and images changed after the paper folding activity. The qualitative analysis revealed that while the students' personal concept definitions of parallelism and perpendicularity of two lines/line segments began to match the formal concept definitions of these concepts, missing and mis-in concept image situations were observed less after the paper folding activity, which may explain the significant difference between the pre-and post-test scores. These changes and descriptives related to them in pre and post-tests will be explained in detail in the following sections.

Concept Definitions

Analysis of the students' answers showed that the number of students whose personal concept definitions of parallelism and perpendicularity of two lines/line segments correspond with the formal concept definitions of these concepts increased after the paper folding activity. **Table 4** displays the number of students who could present a formal concept definition for parallelism and perpendicularity in the pre and post-test. In the pre-test, only 13 (28.30%) students could present a personal concept definition that matches the formal concept definition of parallelism of two lines/line segments. In other words, these students defined the parallelism of two lines/line segments as either "lines/line segments that do not intersect" (20.93%) or "lines/line segments that have equal distance between them" (2.33%) or "lines/line segments, which are in the same direction" (4.65%). One of the students combined these three definitions.

Table 4. Distribution of students who correctly defined parallelism & perpendicularity of two lines/line segments in pre- & post-test

		Pre-test	Post-test
Definition of parallelism	Lines/line segments that do not intersect	9 (20.93%)	4 (9.30%)
	Lines/line segments that have equal distance between them	1 (2.33%)	5 (11.63%)
	Lines/line segments that are in same direction	2 (4.65%)	0 (0.00%)
	Combination with above three definitions	1 (2.33%)	26 (60.47%)
Total		13 (30.23%)	35 (81.39%)
Definition of perpendicularity	Lines/line segments that intersect at a right or 90° angle	16 (37.20%)	20 (46.51%)
	Lines/line segments that form a right or 90° angle	0 (0.00%)	2 (4.65%)
Total		16 (37.20%)	22 (51.16%)

The remaining students presented definitions such as "lines/line segments that are side by side" (9.30%) or "lines/line segments that are opposite to each other" (11.63%). These personal definitions resulted in mis*(takenly)* in*(cluded)* a non-example of parallel line segments. For example, in the pre-test, S6 defined the parallelism of two lines as "lines that are opposite to each other" and provided drawings in **Figure 1**. As a result of this definition, she identified in the pre-test that Tasdelen Ave. and Saricam St. in the given map (Q2b in PPT) are parallel. Moreover, she explained, "this is because these two lines are opposite." In other words,

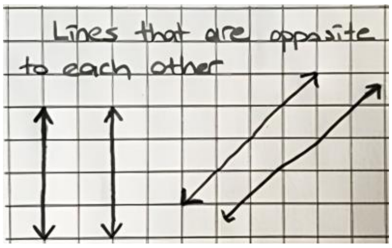


Figure 1. Parallelism definition of S6 in pre-test (Source: Authors)

she mis*(takenly)* in*(cluded)* non-parallel line segments in the set of parallel line segments because of her inaccurate personal definition.

Similarly, analysis of the pre-test results revealed that less than half of the students could present a definition of the perpendicularity of two lines/line segments that match the formal concept definition. Namely, these students defined the perpendicularity of two lines/line segments as "lines/line segments that intersect at a right or 90° angle" (37.20%). The other definitions included "one or two vertical line segments" (9.30%) and "lines/line segments that intersect at any point" (4.65%), or "to be 90°, right or 90° angle" (16.28%). Moreover, because of these definitions, students inaccurately categorized non-examples of perpendicular lines/line segments as examples of that concept, a mis*(takenly)* in*(cluded)* concept image situation.

For instance, when S43 was requested to construct a perpendicular line segment to Nazli St. in **Figure 2**, he constructed a parallel line segment instead of a perpendicular one and explained that both are perpendicular. Based on his drawing and explanation, this student might confuse perpendicularity with verticality. In other words, for S43, for a line segment to be perpendicular, it should be vertical to a horizontal base. Since he constructed his perpendicular line segment according to this definition, which is invalid, he incorrectly constructed parallel line segments as an example of a perpendicular line segment, a mis*(takenly)* in*(cluded)* concept image situation.

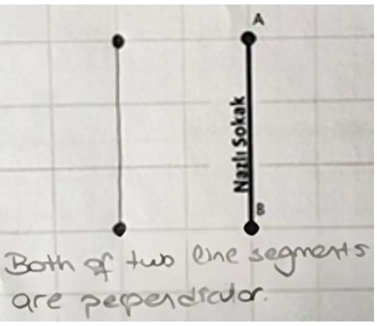
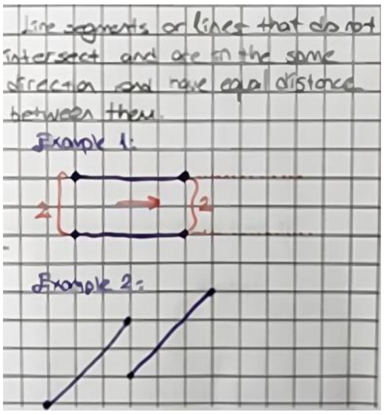


Figure 2. Perpendicular line segment construction of S43 in pre-test (Source: Authors)

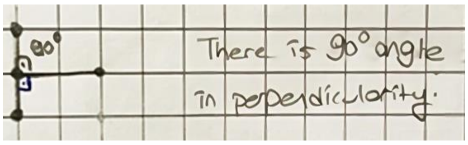
Besides, the study's findings indicated that the number of students whose personal concept definitions match the formal concept definitions of parallelism and perpendicularity of two lines/line segments increased in the post-test. The numbers were 35 (81.39%) and 22 (51.16%) for parallelism and perpendicularity, respectively. When the students' definitions in the post-test were examined separately for parallelism, it was observed that rather than stating one of the three definitions in **Table 4**, most students (60.47%) combined at least two of them as S3 in **Figure 3**. S3 stated that for two lines or line segments to be parallel, they should not intersect and should be in the same direction. She added that the distance between them should be equal and showed the distance between her parallel line segments as two units. Surprisingly, the number of students providing the correct definition of perpendicularity did not increase as much as the number for the parallelism definition. This situation was observed because some students wrote 90° or right angle or lines/line segments with a 90° or right angle (27.90%) as their answers. State differently, while they made their definitions or explained their identifications or constructions, they either did not use the verb intersect, or it was unclear which angle they referred to. Thus, their definitions were not evaluated as accurate. However, the ones who provided an accurate definition showed the right or 90° angle in their drawings and explained how they measured that angle, as S42 showed (**Figure 4**) and explained in his interview, as follows:



Line segments or lines that do not intersect and are in the same direction and have equal distance between them.

Figure 3. Parallelism definition of S3 in post-test (Source: Authors)

> Interviewer: You said there should be a 90° angle for the perpendicularity of two lines/line segments. What do you mean by this statement?
>
> S42: I drew this (he indicates the vertical line segment in his drawing) and a horizontal one. Then, I used the square.
>
> Interviewer: How? Can you explain?
>
> S42: Since the interior angle of a square is 90°, here (he draws the blue angle in **Figure 4**) is 90°.



There is 90° angle in perpendicularity.

Figure 4. Perpendicularity definition of S42 in post-test (Source: Authors)

Concept Images

Analysis of the students' answers indicated that missing and mis*(takenly)* in*(cluded)* concept image situations, frequently observed in the pre-test, decreased after the paper folding activity. As mentioned in the above section, one source behind students' missing and mis-in concept images was students' inaccurate personal definitions. The analysis of students' inaccurate identifications and constructions revealed two more possible sources behind their decisions: Prototype line segments and inadequate understanding of the concepts related to parallelism and perpendicularity.

(Non)Prototype line segments

As remembered, PPT involved prototype examples in identification and construction questions (Q2c, Q3a, Q5b, and Q6a in PPT). As expected, many students could answer these questions correctly in pre and post-tests. More particularly, almost all students could answer those questions correctly after the treatment. Although students could answer those questions correctly, students' answers in pre and post-tests revealed that these prototype examples in students' minds led to mis*(takenly)* in*(cluded)* concept image situations. Some students' answers to the questions that do not involve prototype examples implied that they made their identifications or constructions depending on the prototype examples they knew. For example, when S1 was requested to construct a parallel line segment to the Dinc St. in the pre-test, she constructed the line segment in **Figure 5** ([KL]), which seems parallel but indeed not because the distance between the two line segments was not equal. Also, in her explanation, she drew two vertical line segments whose lengths are different and explained, "I draw such a shape since parallel means this shape." In other words, because this student wanted to observe the prototype parallel line segments in her mind, she mis*(takenly)* in*(cluded)* a non-example of parallel line segments in the set of parallel line segments.

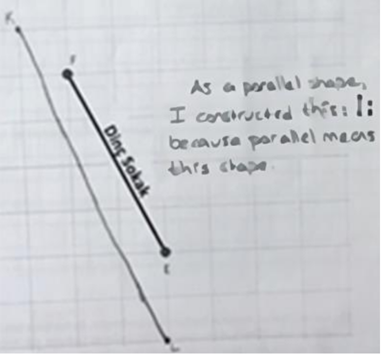


Figure 5. Parallel line segment construction of S1 in pre-test (Source: Authors)

Similarly, S39 identified in the pre-test that Fikir and Saricam streets in the given map (Q5a in PPT) are perpendicular. He stated his reasoning, as follows:

> Interviewer: What did you say regarding the positions of Fikir and Saricam streets?
>
> S39: Fikir and Saricam Streets. Saricam St. is here, and Fikir is here (he mentions the streets on the map). They intersect; that is, they intersect at a right angle.
>
> Interviewer: How did you understand that?
>
> S39: Here is horizontal, and here is vertical (he shows the streets on the map).
>
> Interviewer: Which one is horizontal? Can you show it again on the map?
>
> S39: Saricam St. is horizontal, and Fikir St. is vertical.

In the above excerpt, S39 verbally expressed the prototype example in his mind. For this student, two line segments are perpendicular to each other "if one is horizontal and the other is vertical." In other words, since he wanted to observe the + image for the perpendicularity of two-line segments, he inaccurately categorized a non-example of perpendicular line segments as an example, a mis*(takenly)*-in*(cluded)* concept image situation.

On the other hand, findings indicated that students depended less on their prototype examples in the post-test than they did in the pre-test. As such, mis*(takenly)* in*(cluded)* concept image situations were observed less in the post-test. For example, the construction of S19, which depended on the prototype example in her mind in the pre-test, to the Yildiz St. can be examined in **Figure 6**, and she explained that "to make the distance same, I used a perpendicular." During the paper folding activity, students discovered that to construct a parallel line segment to a given one by only paper folding, they can benefit from perpendicular line segments, which enable them to keep the distance between the line segments the same. In the post-test, S19 made her construction correctly using this strategy, not the prototype example in her mind.

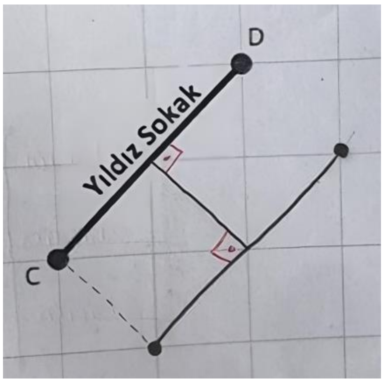


Figure 6. Parallel line segment construction of S19 in post-test (Source: Authors)

As explained in the methods section, the students tried other strategies to construct parallel/perpendicular line segments in the third section of the paper folding activity. For instance, to construct a perpendicular line segment, S10 suggested using an object with a 90° angle, like a protractor, to form the angle between two line segments as 90°. In the post-test, S3 constructed a perpendicular line segment to the Dinc St. with this strategy discussed in the class during the last section of the activity. In the interview, she explained her strategy, as follows:

> Interviewer: How did you construct a perpendicular street to the Dinc St. given?
>
> S3: I had a paper. I used it to construct a perpendicular street.
>
> Interviewer: How did you use it? Can you use it again?
>
> S3: I align the paper with the given street like that (she puts her paper so that one of the sides of the paper is on the street and the other is on the green line segment, which she constructed) (**Figure 7**).
>
> Interviewer: Well, what is the purpose of using a paper?
>
> S3: Imm. This side of the paper (she shows the angle on the paper, which is between the given and constructed line segments) is 90°. The paper is a rectangle.

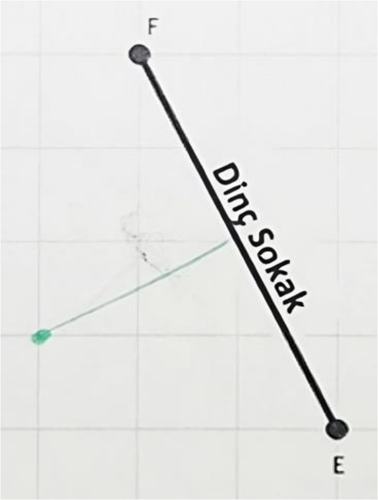


Figure 7. Perpendicular line segment construction of S3 in post-test (Source: Authors)

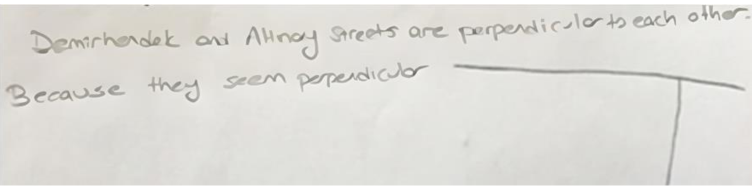
The explanation of S3 suggests that she did not depend on the prototype example in her mind while constructing a perpendicular line segment to a given line segment. Instead, she used an object with a 90° angle, which allowed her to end up with an accurate construction.

Concepts related to parallelism & perpendicularity

The second source behind students' decisions about identifying two-line segments or constructing a line segment as a non-example or example of parallel/perpendicular line segments was related to the students' inadequate understanding of the concepts related to parallelism and perpendicularity. Those concepts were extensions and relative positions of two lines/line segments in a plane. It was observed that their inadequate understanding led to both missing and mis*(takenly)* in*(cluded)* concept image situations. More particularly, the analysis in the pre-test indicated that 19 (41.30%) students could not think about the extensions of the line segments while identifying parallel line segments in Q2b. Similarly, 17 (37.00%) students showed they do not mind the extensions of the given line segments while identifying the perpendicularity of two-line segments. This resulted in missing an example of parallel/perpendicular line segments from the set of those concepts. For instance, in the pre-test, S3 identified that Kartalcik and Dalboyu streets in Q5c are not perpendicular. She explained her reason: "I think that two-line segments should touch each other to be perpendicular." In other words, since she could not think about the extensions of the line segments, she erroneously categorized an example of the perpendicular line segments as a non-example, a missing concept image situation. However, during the interview after the pre-test, she changed her answer and said that the two streets were perpendicular. The following exchange illustrates her reasoning:

> Interviewer: What did you say regarding the positions of Kartalcik and Dalboyu streets?
>
> S3: Kartalcik and Dalboyu streets? Dalboyu St. is here, and Kartalcik St. is here (she indicates two streets on the map). They can be perpendicular to each other.
>
> Interviewer: Why do you think so?
>
> S3: Because they may not need to touch each other.
>
> Interviewer: In your test, you said that for two-line segments to be perpendicular, they should touch each other. Now, you say, do they not have to touch each other?
>
> S3: Yes (it sounds like she is unsure about her answer).
>
> Interviewer: Well, why do you think that they are perpendicular?
>
> S3: Why did we say perpendicular to these streets (she shows the streets in Q5b)? Because they seem perpendicular. These streets (Kartalcik and Dalboyu streets) also seem perpendicular.
>
> Interviewer: Do you think Kartalcik and Dalboyu streets look like Demirhendek and Altinay avenues (streets in Q5b)?
>
> S3: If we combine Kartalcik and Dalboyu streets, they look like each other.
>
> Interviewer: Should we combine them?
>
> S3: I think not necessarily.

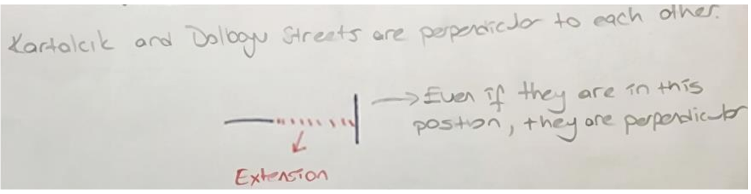
Although S3 accurately identified the positions of the given line segments during the interview after the pre-test, she still did not use the extensions of the line segments to make her identification. Instead, she depended on the prototype example in her mind, one horizontal and one vertical line segment, as indicated for Q5b in **Figure 8**. However, this belief resulted in mis *(takenly)* in*(cluded)* a non-example of perpendicular line segments as an example in Q5a.



Demirhendek and Altınay streets are perpendicular to each other.
Because they seem perpendicular

Figure 8. Perpendicular line segment identification of S3 in pre-test (Source: Authors)

On the other hand, the analysis in the post-test indicated that only three (7.00%) and six (14.00%) students ignored the extensions of the given line segments in Q2b and Q5c, respectively. This resulted in a decrease in missing concept image situations after the paper folding activity. To illustrate, S3, who depended on the prototype example in her mind in the pre-test and could not think about the extensions of the given line segments in Q5c, correctly identified the positions of the given line segments in the post-test (**Figure 9**). Her reasoning is presented, as follows:



Kartalcik and Dalboyu Streets are perpendicular to each other.

→ Even if they are in this position, they are perpendicular

Extension

Figure 9. Perpendicular line segment identification of S3 in post-test (Source: Authors)

> Interviewer: What did you obtain when you drew the extension? Can you draw it on the map?
>
> S3: If we extend Kartalcik St., then they (she refers to the Kartalcik and Dalboyu streets) are perpendicular.
>
> Interviewer: How do you know that they are perpendicular?
>
> S3: I know, thanks to the squares. Here is 90° (she indicates the angle of the square, which exists at the intersection of Dalboyu St. and the extension of Kartalcik St. on the map).

State differently, S3 explained that in the post-test, she drew the extension of the Kartalcik St. and made her identification based on her definition, not on the appearance of the given line segments as she did in the pre-test.

Another concept the students had insufficient understanding of was the relative positions of two lines/line segments in a plane. More specifically, in the pre-test, seven (16.28%) students did not know that two lines in a plane could be either parallel, intersect, or coincident. However, they had some naïve ideas regarding the relative positions of the two lines. This inadequacy resulted in mis*(takenly)* in*(cluded)* non-examples of parallel/perpendicular line segments in the set of those concepts. For instance, in the pre-test, S43 had confusion between verticality and perpendicularity (**Figure 1**). Hence, when he was requested to support his perpendicularity definition with a drawing in the pre-test, he provided the parallel line segments in **Figure 10**. Also, he explained, "these two line segments are both parallel and perpendicular to each other since they do not intersect." State differently, this student understood that two-line segments in a plane could be both perpendicular and parallel to each other, so he inaccurately categorized parallel line segments as perpendicular lines, a mis-in concept image situation.

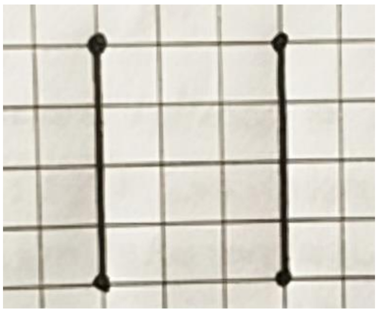
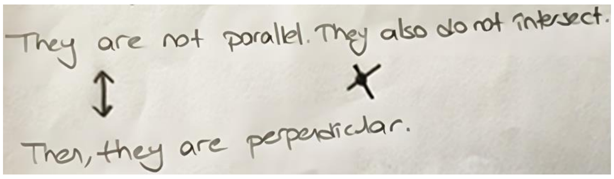


Figure 10. Perpendicular line segment drawings of S43 in pre-test (Source: Authors)

Similarly, S10 had an understanding in the pre-test that two lines/line segments are perpendicular to each other if they are neither parallel to nor intersect with each other. Her understanding resulted in mis*(takenly)* in*(cluded)* a non-example of the perpendicular line segment as an example in Q5a. As illustrated in **Figure 11**, by drawing the images of parallel and intersecting lines for her, she explained that "they (she refers to the Fikir and Saricam streets) are not parallel. They also do not intersect. Then, they are perpendicular." It can also be stated that S10 based her decision on the prototype examples of parallel and intersecting line segments. However, this belief led her to mis*(takenly)* in*(cluded)* a non-example of perpendicular line segments as an example.



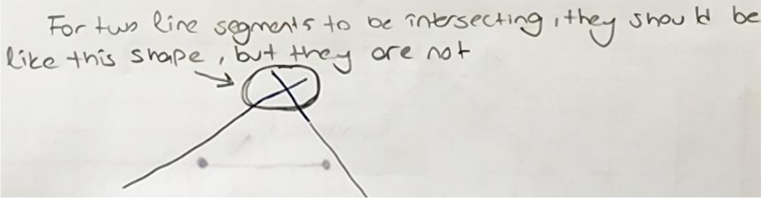
They are not parallel. They also do not intersect.

↕ ✗

Then, they are perpendicular.

Figure 11. Perpendicular line segment identification of S10 in pre-test (Source: Authors)

Besides, in the post-test, although some students (6.98%) continued depending on such naïve ideas, most (9.30%) gave up making their identifications and constructions with the naïve ideas exemplified above. Hence, mis*(takenly)* in*(cluded)* concept images were observed less after the paper folding activity. However, some students (6.98%) continued using such incorrect ideas. For instance, S18 identified that Tasdelen Ave. and Saricam St. in Q2b are neither parallel, perpendicular, nor intersecting. In her explanation in **Figure 12**, she indicated what intersecting lines mean to her by extending the given line segments and explained, as follows:



For two line segments to be intersecting, they should be like this shape, but they are not

Figure 12. Parallel line segment identification of S18 in post-test (Source: Authors)

> S18: For two-line segments to be parallel, they should be in the same direction, but they (she refers to Tasdelen Ave. and Saricam St.) are not. The angle between the two-line segments should be 90° for the two-line segments to be perpendicular. However, it is not. For two-line segments to intersect, they should be like this shape (she indicates her drawing), but they are not in this shape.

DISCUSSION & CONCLUSIONS

The main aim of the current study was to examine the effect of a paper folding activity prepared to develop sixth-grade students' concept definitions and images regarding the parallelism and perpendicularity of two lines/line segments. The findings and inferences revealed in previous studies regarding the effectiveness of paper folding on student achievement were confirmed once more by the present study's findings (e.g., Boakes, 2009; Georgeson, 2011; Kandil & Isiksal-Bostan, 2019).

More specifically, the findings obtained from the pre-test analysis were similar to those obtained previously about parallelism and perpendicularity concepts (e.g., Ulusoy, 2016; Duatepe-Paksu & Bayram, 2019). However, during the paper folding activity, students actively obtained parallel and perpendicular line segments and discussed their observations with their classmates. Moreover, in the last part of the activity, they worked individually to implement their observations in the paper folding to the line segments on the squared paper. They reached various strategies for constructing parallel and perpendicular line segments through class discussions and used them in their constructions in the post-test (see **Figure 7** for an example strategy). Thus, it can be inferred that students' concept definitions and images regarding parallelism and perpendicularity could be developed through active exploration in the paper folding and a whole class discussion of mathematical ideas observed.

The analysis regarding the second research question gave us information about how students' concept definitions and images changed after the paper folding activity, which may explain the significant difference observed in the quantitative results. The findings showed that the students had many missing and mis-in concept image situations at the beginning regarding the parallelism and perpendicularity concepts. We observed that one of the causes behind those situations could be the students' incorrect personal definitions. Before the activity, the definitions of many students did not match the formal definitions of the concepts. As consistent with the previous studies (Mansfield & Happs, 1992; Tirosh & Tsamir, 2022), even if they could present an appropriate definition, their definitions were on the surface, i.e., they did not use them as a basis for their identifications or constructions. Instead, they depended on their concept images, usually restricted to the prototype (horizontal and vertical) lines/line segments. This dependence and inaccurate definitions usually resulted in mis*(takenly)* in*(cluded)* a non-example of parallel/perpendicular line segments as an example. During the activity, students were requested to justify their identifications or constructions in the class discussions, whether they were either on the paper folded or on the squared paper. However, to lessen the dominance of prototype images and to guide them to the critical attributes (Tsamir et al., 2015), the appearance of the line segments was not accepted as enough justification. In this way, the students had a chance to repeat and apply their definitions after their identifications and constructions. This might have a role in developing students' concept definitions after the paper folding activity.

Constructions of parallel/perpendicular line segments to various oblique line segments in the last part of the activity could be one reason underlying the development of students' concept images (Clements, 2003; Tsamir et al., 2008). To state it differently, since students constructed parallel and perpendicular line segments to the several non-prototype line segments on the squared paper, their dependence on the prototypes might reduce after the activity. Furthermore, as stated previously, the appearance was not accepted as the sole justification following both true and false constructions during class discussions. For instance, if a student justified his/her construction by stating the angle is the right angle, it was asked which angle is right and how they can be sure it is right. Such discussions could have enabled students to enlarge their images to the non-prototype ones and consult their definitions to make their constructions.

Students' inadequate understanding of the concepts related to parallelism and perpendicularity was observed as another source behind students' missing and mis*(takenly)* in*(cluded)* concept image situations. It was revealed that the inability to think about the extensions of the line segments resulted in missing an example of parallel/perpendicular line segments. Besides, their insufficient understanding regarding the relative positions of two lines/line segments in a plane led to mis*(takenly)* in*(cluded)* non-examples of parallel/perpendicular line segments in the set of those concepts. Moreover, we believe that students' incorrect definitions and dependence on prototype images might not allow them to think about extensions of the line segments or could lead them to naïve ideas regarding the relative positions of two lines/line segments. Thus, the development in students' definitions and images regarding parallelism and perpendicularity after the paper folding activity might indirectly affect students' understandings of extensions and relative positions of two lines/line segments, which in turn could decrease the number of missing and mis-in concept image situations.

Although the paper folding activity developed students' definitions and images regarding parallelism and perpendicularity, we observed that their definitions and dependence on prototype images resist change. For instance, some students presented such a definition of parallelism: The distance between two line segments is equal, and they should be opposite. Although their definitions of distance were accurate, they could not give up the definition of being opposite to each other after the activity. They made their identifications and constructions based on this definition. If we look at it from the perspective of van Hiele's (1986) theory, it can be stated that these students could not reach the Descriptive/Analytic level; they still show the characteristics of the first level, Visual. This finding could stem from the fact that the duration of the intervention was relatively short. Thus, more time may be allocated to the proposed paper folding activity to include more line segments, which can also affect students' definitions. Furthermore, to form correct concept images and definitions, such activities should continue throughout the whole period of learning (Vinner, 1983).

Another issue arising from this study's findings regarding students' definitions was that students generally provided all three definitions of parallelism. Thus, it may be stated from the perspective of van Hiele's (1986) theory that since they did not provide only the necessary and sufficient conditions, they could not reach the Relational level. This could be because all the definitions of parallelism were emphasized in the class discussions after each identification or construction. However, definitions should contain only necessary and sufficient conditions (Tsamir et al., 2008; Winicki-Landman & Leikin, 2000). Besides, many students did not use the verb 'intersect' in defining the perpendicularity, or it was unclear which angle they referred to in their identifications or constructions of a perpendicular line segment. This could lead to a slighter increase in the number of students who provided the correct definition of perpendicularity compared to the number for the parallelism definition. Hence, we suggest revising the paper folding activity so that minimalism in the parallelism definition or the verb 'intersect' in the definition of perpendicularity is focused more.

In conclusion, the findings showed that the paper folding activity positively affected the sixth-grade students' concept definitions and images. While their personal concept definitions began to match the formal concept definitions of parallelism and perpendicularity of two lines/line segments, missing and mis-in concept image situations, encountered generally in the pre-test, were observed less after the paper folding activity. Therefore, we recommend teachers and teacher educators use this activity as an instructional tool by considering the abovementioned issues. Similarly, curriculum developers or textbook writers may give more space to paper-folding activities in mathematics textbooks. Moreover, this study presents an example of how to create an active learning environment, which provides observations by students with cheap material-paper- and enhances them through classroom discussions. Thus, similar activities to teach different concepts based on parallelism and perpendicularity, such as the altitudes of triangles or quadrilaterals, can be designed, and their effectiveness may be measured. In addition, similar studies can be performed at different grade levels or in international contexts to understand the effect of paper folding activity better and compare students' definitions and images. However, this research is limited to the questions in PPT. Therefore, much detailed information regarding students' concept images may be obtained through the revision of PPT, including more non-prototype examples and non-examples of parallelism and perpendicularity. Also, the present study suffers from the deficiency of a control group because of a school condition. Although this limitation was tried to be eliminated by adding qualitative data-interviews-, it is undeniable that some external factors might have an explanation for the outcomes because one group pre-test post-test design is one of the weak experimental designs (Fraenkel et al., 2012). Therefore, further research with a control group is suggested to observe how the scores of the experimental group change with respect to the control group. Lastly, more complex research designs, including random sampling methods and a delayed test, can allow further group comparisons and eliminate the limitations caused by a one-group pre-test post-test design, and convenience sampling method.

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Declaration of interest: The authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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APPENDIX A

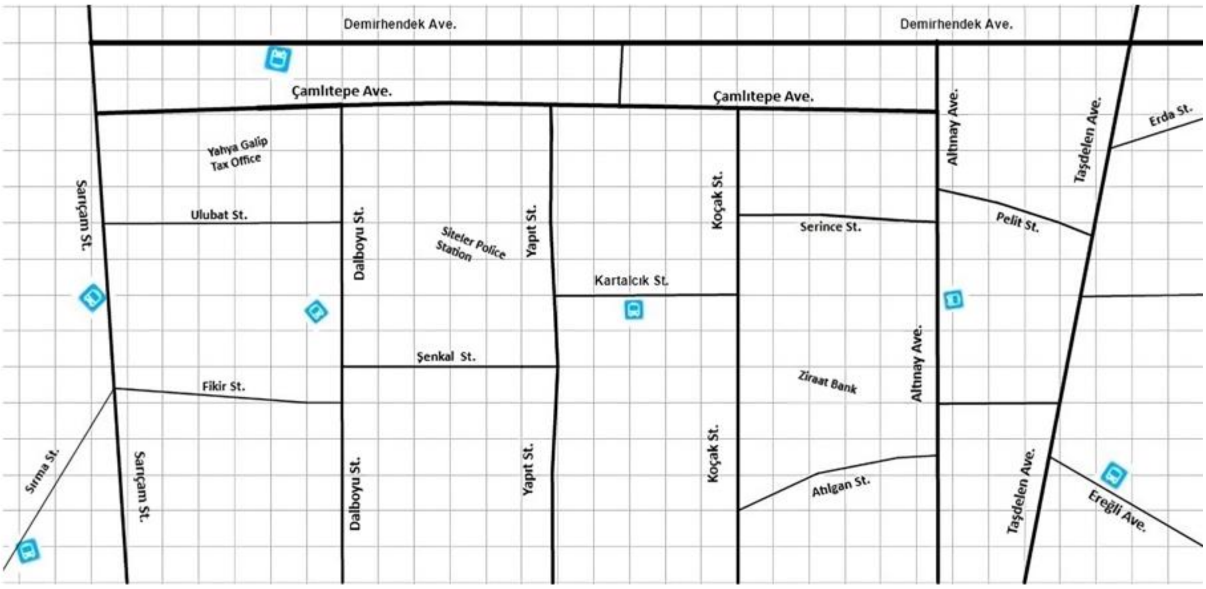

Figure A1. Map of Ankara Siteler District (Source: Authors)

APPENDIX B

Table B1. Interview questions

Questions	Interview questions
Definitions (Q1 & Q4)	If student explains parallelism of two lines/line segments with statements such as “lines/line segments that are opposite to each other,” but his/her statements are not understood from his/her drawings, ask: - What do you mean by this statement? - Can you explain what you mean by using your drawings? If student provides just some drawings, ask: - Why do you believe these lines/line segments are parallel/perpendicular? - Based on explanations, how can you describe parallelism/perpendicularity of two lines/line segments? If student provides just an explanation or an explanation with only one drawing, ask: - Can you show what you mean by a drawing? - Can you draw different parallel/perpendicular lines/line segments? If student provides just a line/line segment in his/her drawings, ask: - Why do you believe this line/line segment is parallel/perpendicular?
Identifications (Q2 & Q5)	If student does not provide an explanation, ask: - Why do you believe that … St./Ave & … St./Ave are parallel/perpendicular to each other? If student provides explanations such as “do not intersect,” or “Intersect at a right or 90° angle,” or “angle between lines/line segments is right or 90° angle”, ask: - How do you know these lines/line segments do not intersect or intersect at a right or 90° angle? If student just says right or 90° angle for perpendicularity of two lines/line segments, ask: - What do you mean by right or 90° angle? Where do you see/need to see this angle? If student provides explanations such as “I measured & found them as parallel/perpendicular,” ask: - How did you measure? Can you explain? If student did not identify parallelism/perpendicularity of lines/ line segments on explanations in Q1 & Q4, ask: - Based on answers to these questions, how can you describe parallelism/perpendicularity of two lines/line segments in Q1 & Q4? Do you change your previous explanations? Why?
Constructions (Q3 & Q6)	If student does not provide an explanation, ask: - Why do you believe the constructed St./Ave is parallel/perpendicular to given one? - How did you construct this line segment? If student provides explanations such as “do not intersect,” or “intersect at a right or 90° angle,” or “angle between lines/line segments is right or 90° angle”, ask: - How do you know these lines/line segments do not intersect or intersect at a right or 90° angle? - How did you construct this line segment? If student just says right or 90° angle for perpendicularity of two lines/line segments, ask: - What do you mean by right or 90° angle? - Where do you see/need to see this angle? - How did you construct this angle? If student did not identify parallelism/perpendicularity of lines/ line segments on explanations in Q1 & Q4, ask: - Based on answers to these questions, how can you describe parallelism/perpendicularity of two lines/line segments in Q1 & Q4? Do you change your previous explanations? Why?

APPENDIX C

Table C1. Rubric for Q1

Point	Criteria
3	- Student explains parallelism of two lines/line segments as lines/line segments that do not intersect,” or “lines/line segments that have equal distance between them,” or “lines/line segments which are in same direction” AND supports his/her explanation with two different drawings. Note. Students’ constructions in Q3b & Q3c should be examined for drawings.
2	- Student does not explain parallelism of two lines/line segments. But s/he provides at least two different drawings. - Student explains parallelism of two lines/line segments as “lines/line segments that do not intersect,” or “lines/line segments that have equal distance between them,” or “lines/line segments which are in same direction BUT does not support his/her explanation with two different drawings. S/he does not construct a parallel line/line segment to an inclined line/line segment.
1	- Student does not explain parallelism of two lines/line segments. But s/he draws either horizontal or vertical parallel lines/line segments. - Student explains parallelism of two lines/line segments as “lines/line segments that are side by side” or “lines/line segments that are opposite to each other” AND supports appropriate drawings with his/her explanation. - Student does not explain parallelism of two lines/line segments. But s/he draws two lines/line segments, which seem parallel, but indeed not.
0	- Student draws two intersecting lines/line segments. - Student draws just a vertical, horizontal, or inclined line/line segment.

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