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Research Article

Flipped learning and affect in mathematics: Results of an initial narrative analysis

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ABSTRACT

Flipped learning is a constructivist methodology that uses various active learning strategies, drawing on the potential of digital technologies whenever possible. Several research studies have shown the many benefits that flipped learning can bring to students' learning process and their affective sphere, but studies in the specific area of mathematics are still limited. The present article aims to examine the possible impact of this methodology on affective factors in mathematics. To achieve this goal, it was designed a 20-hour experiment involving about 300 secondary school students, during which conic sections were studied using a flipped approach. At the end of the experiment, textual material was collected on which a narrative analysis was conducted. The analysis took a content-categorical approach, and the reference theory for interpreting the data was the three-dimensional model for attitude toward mathematics by Di Martino and Zan (2010, 2011). The results show that a positive, rather than a negative, emotional disposition prevails, as does a view of mathematics as useful, rather than useless. Students' perceptions of their own competence are equally distributed between low and high. Initial findings suggest that flipped learning is a methodology that can create a learning environment that has a positive influence on affective factors.

Keywords: flipped learning, affect, three-dimensional model for attitude towards mathematics, narrative analysis, mathematics

INTRODUCTION

In mathematics education, the limitations of a traditional approach to learning difficulties have been known for several decades now. Nevertheless, this approach has been predominant in the teaching practice for a long time (Zan, 1996) and it is still used today in a variety of schools. This awareness has led to the proliferation of multiple initiatives, both institutional and spontaneous, inspired by the need to innovate this model. Among them, over the past decade, one in particular has led to the development of flipped learning. This is a constructivist teaching method that exploits various active learning strategies, using whenever possible the potential of digital technologies (Bergman & Sams, 2012). Despite its very recent origins, its spread around the world has been rapid thanks to its success among teachers and students (Bergman & Sams, 2012). The methodology immediately attracted our interest because of its characteristics that seemed to adapt well to the education of mathematics.

The analysis of the art state reveals the existence of a large number of experimental research, but they often use different types of flipped approaches, making it difficult to compare them. The number of research in the specific field of mathematics education, however, is significantly more limited (Jdaitawi, 2020), the difficulty of comparing them also remains because of the different flipped approaches used (Guler et al., 2022). Nevertheless, some findings of some relevance have emerged, prompting further study of this methodology. There are several benefits that the flipped approach seems to have on mathematics learning, for example, improved performance (Guler et al., 2022; Wei et al., 2020), increase level of engagement

(Cevikbas & Kaiser, 2021) and acting on students' motivation, commitment, and self-efficacy (Carotenuto & Sbaragli, 2018).

A significant exploration, useful for better defining the direction of the research proposed here, was implemented in 2016/17 school year. This consisted of the proposal of a flipped learning unit on the topic of number systems and a final self-report questionnaire to investigate students' perceptions of the experience. The goal was to research whether there were any benefits that can be added to the results already known to researchers of flipped learning. The aspect that emerged most prominently was related to affective factors (Lazzari, 2017), which thus became one of the focuses of this research.

Affect is a relatively young field of research in mathematics education. Only in recent decades it has been accepted that affective factors are relevant in mathematics education (Di Martino & Zan, 2011). The need to explain the failures of students with sufficient cognitive resources in problem solving tasks has led researchers to recognize the influence on such processes of metacognition, belief systems and emotions (McLeod & Adams, 1989; Schoenfeld, 1983; Silver, 1982). Some more recent studies (Hannula, 2012) have also shown that each affective aspect possesses an independent stability dimension, meaning that each factor can either vary rapidly (called *state*) or be stable (called *trait*)[1]. The aim of this research is to study the possible influence that the use of flipped learning in mathematics might have on students' affective state factors. To do this, a two-month experiment was organized in fifteen classes of 11 and 12 grade, during which self-report narrative material was collected to obtain data on which interpretive analysis could be conducted.

THEORETICAL FRAMEWORK

Flipped Learning

Flipped learning is an innovative constructivist teaching methods that involves the organized use of various existing educational strategies that have been studied for decades, with the aim of promoting significant and lifelong learning. It is inductive teaching, which traces the scientific process from observation or case study, through the formulation of hypotheses and concluding with the deduction of formulas and principles (Sams & Bergmann, 2013). The distinctive element of flipped learning is the *flipping* of the teaching process, which does not begin with the teacher's exposition of content but with the active participation of students who act to construct their own knowledge. It then ends with the teacher's restructuring of knowledge and not the students' passive application of information. It is thus evident how we move away from the transmissive teaching approach and adopt one inspired by the principles of constructivism (Cecchinato & Papa, 2016).

The approach used in the present work is the one proposed by Cecchinato and Papa (2016), which revolves entirely around the concept of a "challenge", with the goal of getting students interested and involved in their own learning process. The idea is to appeal to the emotional aspects produced by engaging students in challenges that excite them, test them, and enable them to prove their worth. And so, it is fundamental to select an activity level that is appropriately challenging in relation to the skills possessed by the students. Connection can be seen with Brousseau's (1986) thought, which states that it is not possible for a student to construct significant knowledge without real interest and involvement in it. Therefore, the construction of concepts must be motivated and stimulated by placing it in contexts related to the everyday life of young people (D'Amore & Sbaragli, 2011).

Cecchinato and Papa's (2016) proposal consists of a new learning-teaching cycle divided into three phases:

1. *launching of challenge*, which is intended to attract students' attention, activating their desire to learn;
2. *conducting of challenge*, during which learners should be prompted to go through the processes that leads them to obtain the knowledge they want to acquire; and
3. *closing of challenge*, which consists of a collective process of reflection and comparison on what they have learned, guided by the teacher, involving the whole class.

[1] For example, emotions are defined as *states* when they appear in a certain situation, thus contextualized and with the potential to change rapidly. On the other hand, they are called *traits* when talking about a more stable individual characteristic that refers to someone's own experience and to the repetition of the same state emotion under similar conditions for some characteristic (Hannula, 2014).

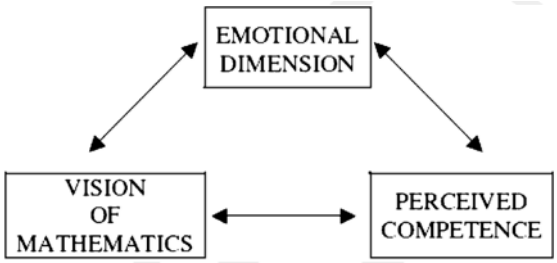

Figure 1. Three-dimensional model of attitude toward mathematics (Di Martino & Zan, 2010, p. 43)

The use of active learning strategies, such as cooperative learning[2] and problem-based learning[3], is planned during the conducting of the challenge.

In such a context as the one just described, it is no longer necessary for the teacher to play the role of a knowledge disseminator, but to be a facilitator of learning processes and a guide to rely on (King, 1993). The teacher thus has a way of assisting students in the knowledge-building phase, giving special attention to struggling learners, and providing greater stimulation to brighter learners, making a personalization of teaching feasible and realizing a deeper and more rewarding educational relationship (Bergmann & Sams, 2012). Just as the role of the teacher is being changed, the role of assessment is also being changed. Assessment shifts from being summative, final, and one-sided, to being formative, integrated into the instructional process, and participatory. The goal is providing timely information that is understood and shared by the student and thus can be used to improve learning.

Making use of technological tools in flipped learning is important, although not necessary. In fact, these can be exploited at home, in the phase preceding the lesson, but also in the classroom, simplifying organizational aspects, enhancing the teaching processes, and promoting student involvement.

Three-Dimensional Model for Attitude Towards Mathematics

In the field of mathematics education there is a general agreement in seeing the affective domain as divided into beliefs, attitudes, and emotions (McLeod, 1992), but agreement is not as unanimous when it comes to the definition of these constructs. Among the many theories that have developed in this context, the one of reference for the present research is the Three-dimensional model for attitude toward mathematics (TMA) by Di Martino and Zan (2010). The two researchers conducted a study with the aim to construct a characterization of attitude toward mathematics based on students' narratives about their own experiences. In other words, a study aimed to construct a grounded theory (Glaser & Strauss, 1967) with respect to attitude toward mathematics, namely a theory that emerges from the collected data through a cyclical analytical process.

This model is characterized by three strictly interconnected, but at the same time independent dimensions: emotional disposition toward mathematics, vision of mathematics, perceived competence in mathematics (**Figure 1**). In this way, we can outline profiles of attitude depending on the dimension. This is possible if each dimension is reduced to a dichotomy:

1. positive or negative emotional disposition;
2. relational or instrumental vision of mathematics; and
3. high or low perceived competence.

Emotional disposition coincides with the degree of affectivity expressed toward the discipline. This is considered positive or negative, respectively, in concurrence with expressions such as "I like/dislike mathematics". The perceived competence corresponds to the perception of being or not being able to

[2] Cooperative learning is an active learning approach in which students work together in small groups to achieve common goals, in terms of disciplinary, interdisciplinary, and social skills, generating positive interdependence and respecting specific roles (Johnson et al., 1994).

[3] Problem-based learning is an active learning approach in which students, guided by a facilitator, face problems, often set in real-world contexts, with the goal of acquiring or consolidating new knowledge, as well as developing disciplinary and transversal skills (Barrows & Tamblyn, 1980).

succeed in mathematics, concisely expressed with "I can/cannot do it". Such statements are respectively regarded as high or low perceived competence. The vision of mathematics is concisely expressed with "Mathematics is…". It is considered instrumental when the focus is on the products to be remembered (rules without reasons), juxtaposed to a relational vision, when focus is on the processes and their relations rather than on products.

An emotional disposition is closely related to emotional reaction (Di Martino & Zan, 2010), for which reference is made to the emotion theory of Ortony et al. (1988), that moves away from basic emotion theories and makes the origin of emotions clear. Researchers classify emotions according to the direction in which the subject's reaction is directed: objects, events, and agents. Emotions in response to objects can be considered variations of the affective like/dislike reaction. Therefore, they depend on the individual's subjective preferences, a classic example being love and hate. Emotions that emerge as an effect of events correspond to those classes of pleasure/displeasure reactions. These are influenced by the subjects' goals, and examples are joy, hope and fear. Emotions in response to agents are equivalent to affective reactions of approval/disapproval. They depend on individuals' beliefs and values, an example being pride, shame, and admiration.

EXPERIMENT DESCRIPTION

Hypothesis

Generally, in narrative studies there are no a priori hypotheses, the specific direction of the study emerges from reading the material. Narrative research, being interpretive, is always personal, partial, and dynamic. Hypotheses are generated during the reading and analysis of the text and can be further enriched by rereading and redefining theoretical statements in an ever-increasing circle of understanding (Lieblich et al., 1998).

This is what also happened during the research presented here. However, as anticipated in the introduction, we chose to focus the research to the field of affect as a consequence of the first exploration results acquired in the 2016/17 school year (Lazzari, 2017). For this reason, the hypotheses will not be listed in detail in the following but will be referred directly to the analysis of the results obtained.

Experiment and Participants

The experiment, entitled "Flipped math! Discovering conic sections", was implemented during the 2017/18 school year in fifteen classes of 11 and 12 grade, involving a total of 356 students[4]. The total duration of the project was about 20 hours and the methodology chosen was flipped learning, using the new learning cycle proposed by Cecchinato and Papa (2016). Among the different teaching strategies that flipped learning involves, cooperative learning and problem-based learning were chosen to be combined, considering the positive feedback from some research (Fredriksen & Hadjerrouit, 2020; Heo & Chun, 2016). The groups created contained a maximum of four members, heterogeneous with respect to mathematical skills[5], nationality, and gender, and remained unchanged throughout the experiment.

The introduction of the new topics took place through stimuli that showed the applicability of the conics' properties in real-life contexts, and then proceeded to the autonomous discovery of the analytical and geometrical properties of each curve through the support of videos, GeoGebra animations and guideline sheet. At the end of each lesson, the processes implemented, and the results obtained by each group were shared in a collective discussion, thus arriving at the definition of the knowledge covered by the learning unit. The lessons were conducted by the researcher, always supported by the class teacher. The evaluation was summative at the end of the course (to ensure achievement of the set learning objectives) and, above all, formative.

[4] The sample consisted of students who attended a scientific lyceum (22%), a classical lyceum (35%), an art lyceum (14%), a technical institute (24%), and a vocational institute (5%).

[5] Math skills were graded through the administration of a prerequisite test, as well as from assessments given to students during the school year.

Methods and Tools

In the present work, the self-report research tool chosen is the structured essay. This is usually called narrative material, which is becoming more common in interpretive research, particularly in the field of affect. Indeed, in this field it is important to give as much freedom of expression as possible, allowing the writer to make his or her own decisions about what is relevant to say and what is not. Analysis of these decisions can yield interesting information, such as psychological centrality, non-logical connections, and a temporal dimension (Di Martino & Zan, 2011). The goal is to understand how students interpreted the mathematics learning experience using flipped learning. The proposed essay submission was:

> "How have you experienced the Flipped math! project (What feelings and emotions did you experience? How was your relationship with math? Did you feel motivated? How much, how, and why?). Also specify what you think are the most positive and negative aspects of this path."

Being aware of the risk of influencing the students' response by providing pointed indications of the aspects to be covered in the essay, we also know how in this kind of analysis the parts of narrative material related to the research context do not always emerge spontaneously; sometimes a more detailed question from the researcher is needed (Lieblich et al., 1998)[6].

Paper writing was assigned to all classes at the end of the entire project. They were asked to do the task individually outside the classroom, in an attempt that everyone had a quiet environment and appropriate time frames for reflection and self-analysis. This choice, however, led to some students not turning in the final essay. Out of a total of 341 pupils who were assigned the final essay, only 64% (i.e., 217 students[7]) returned the paper. Although sometimes in this kind of research the number of participants is not especially high, the number of sentences extracted from the texts can become large.

An analysis of the narrative texts collected at the end of the experiment described above was carried out. The aim is not to identify significant differences, but rather to try to understand how flipped learning has conditioned affective factors, remaining open to the possibility of aspects influenced by the methodology that have not yet been considered so far. In fact, there are often no a priori hypotheses in narrative studies; the specific direction of the analysis emerges from reading the material. Hypotheses can also be generated during the reading and analysis of the text and can be further enriched by rereading and redefining theoretical statements in an ever-increasing circle of understanding (Lieblich et al., 1998).

For the analysis proposed here, we have chosen to use a categorical-content approach, which is implemented by dissecting the original texts into parts, which are then placed into categories according to the explicit content present in the individual "sections". This choice is particularly suitable when the phenomenon studied is shared within a group of people, as in our case. In fact, the phenomenon studied is the influence of flipped learning on affective factors during mathematics learning processes, and the group of people corresponds to the students joining the experiment. In the following it goes into more detail by describing the stages of the analysis.

The first step was to select the relevant parts of the topics, called *units* (words, phrases, or groups of phrases), and then assemble them together to create a new text containing all the statements concerning the area studied. The criterion for selecting the relevant parts was based on the identified focus, affective factors, and the broader goal of understanding what features of flipped learning influence these. Therefore, we focused on those parts of the text that expressed positive and negative opinions about the methodology, to identify "which features", and on the perceptions expressed by students regarding their own learning and affective sphere, to determine "which influences".

[6] The essay proposed to the first class that joined the experiment was vaguer, omitting the questions included in parentheses. The papers produced were found to be very impersonal, comparable to a report rather than a narration. For this reason, it was decided to modify the essay, specifying which aspects to deepen in the narration, to bring out considerations related to the personal and affective sphere of the students.

[7] The sample consisted of students who attended a scientific lyceum (24%), a classical lyceum (34%), an art lyceum (9%), a technical institute (28%), and a vocational institute (5%).

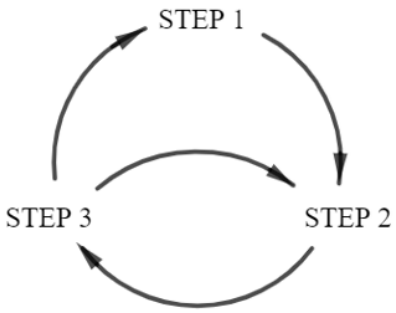

Figure 2. Narrative analysis stages

The second step consisted of the creation of categories, defined according to the content of the units they would later go on to include. In general, the label assigned to individual categories corresponds to a topic that cuts longitudinally through the previously selected portions of text. In addition, categories can be defined a priori, based on reference theory, or empirically from reading the text (Lieblich et al., 1998). The third step was to reorder the selected units within the categories. In the present research, the last two steps are closely interconnected through a circular procedure (**Figure 2**). In fact, after the categories were defined a priori, they were revised and adapted as a result of subsequent readings and classifications.

The categories thus identified were then organized hierarchically into categories and subcategories, which were subsequently divided into groups, here called *domains*. In addition, some categories were assigned a positive or negative valence, depending on the content. Such domains are, as follows:

1. opinions (positive/negative), which pertain to the opinions expressed by students about the method and its features;
2. cognitive aspects (positive/negative), which pertain to the perception of having achieved or not achieved the cognitive goals set; and
3. affective aspects (positive/negative), which refer to having experienced certain feelings attributable to particular affective factors.

Units in each category were counted, tabulated, and sorted according to frequency. In addition, the categories were used descriptively to formulate a representation of a factor in a certain group of people.

From reading the essays, recurring relationships also emerged between some of the categories and subcategories that could provide interesting information about flipped learning. In some cases, these connections emerged explicitly through various types of conjunctions, such as “because”, “therefore”, “but”, etc., while in other cases they emerged implicitly but could be inferred from the context. It was also observed that some sentences linked to the same categories contained explicit signs of occurrence of change, using phrases such as “more than before”, “better than usual”, etc., which might suggest that some categories were more subject to transformation than others.

NARRATIVE TEXT ANALYSIS

The goal of the analysis is to understand *which features* of flipped learning may *influence* students’ affective factors. The focus was on those parts of the text that expressed positive and negative opinions about the methodology, to identify *which features*, and on considerations regarding affective aspects to determine *which influences*.

In order not to deviate from the focus already stated above, we circumscribe the analysis of text portions referring to *which influences* to the categories related to affective factors, and those related to *which features* to the categories that were found to be more related to such factors.

Attitude Towards Mathematics

According to the TMA of Di Martino and Zan (2010), no single category was identified into which to place the text items referable to students’ attitudes toward mathematics during the experiment. Instead, three

independent categories related to emotional disposition, view of mathematics, and perceived competence were created.

Positive emotional dispositions emerged in 63% of the subjects and negative in 12%. Sentences considered positive emotional dispositions were those in which one expressed an appreciation for mathematics, a good relationship with it (example 1) or positive emotions according to the distinction of Ortony et al.'s (1988) theory. Three subcategories were created to enclose the most frequently stated emotions, in order: interest (example 2), enjoyment (example 3) and other emotions such as tranquility (example 4), satisfaction and joy. Compared to the total number of students who expressed a positive emotional disposition, 60% expressed interest, 45% amusement, and 23% one of the other positive emotions.

1. "I hope that my relationship with Math will remain as enjoyable as in these lessons and not return to being a *burden*."

2. "The Flipped Math project was very interesting and engaging. I felt very engaged compared to the usual lessons we do in class in which it is easy to get distracted."

3. "[…] I enjoyed doing the exercises and didn't skip a single assignment because it was fun and challenging."

4. "During the activity I would arrive at class with a calm heart, without the worry about the upcoming test."

Conversely, sentences considered to be negative emotional dispositions are those in which one expresses a lack of appreciation for mathematics, a bad relationship with it, or the presence of negative emotions, such as anxiety, frustration (example 5), discomfort, despondency, and anger.

5. "[…] I felt frustration and anxiety when I did the exercises that I sometimes failed to do."

With less frequency the other two dimensions of attitude appear. High perceived competence (example 6) or low perceived competence (example 7) occurs in 7% and 8% of the material, respectively.

6. "During group work, I often felt myself up to the exercises I had to do."

7. "Maybe, being not mathematically inclined compared to someone else, the explanations were not immediately clear to me."

No views of mathematics emerged from the essays that could refer to the rational/instrumental classification proposed by Di Martino and Zan (2010); rather, there seems to be a tendency to judge it useful, in 6% of the essays, or not useful, in 1% of the essays. The phrases expressing the usefulness of mathematics refer mainly to the possibility of identifying connections between the discipline and real life (example 8). Conversely, those claiming not usefulness assert that it has no repercussions in everyday life (example 9).

8. "This project allowed me to approach the whole world of mathematics in a different way. With the proposed activities I realized that mathematics surrounds us every day in every situation, not only in the classroom."

9. "I think math is useful until a certain point, the topics we covered in class will never be useful in real life, but this is not a criticism of the project in of itself, but of math in schools."

In some parts of the text concerning cognitive and affective aspects, it was possible to reveal some indices of an occurrence of positive change during experimentation.

Among the categories with a higher incidence of references to positive change were precisely those related to the three dimensions of attitude in mathematics: emotional disposition (examples 10[8] and 11), perceived level of competence (example 12), and view of mathematics (example 13).

[8] To graphically emphasize explicit indices of a change, we will highlight certain key words with the "underlined" style.

10. "My relationship with mathematics was less difficult compared with other instances."

11. "[...] we learned mathematical things not with the usual explanation but in a more fun way."

12. "Now I feel more [...] confident in learning the subject."

13. "This project has made me even more aware of the importance of mathematics connected to many objects present in everyday life."

Opinions on Flipped Learning Features

At the beginning of this overview, it was anticipated the presence of relationships between the different categories, identified through causal conjunctions ("because", "since", etc.), some of which recurring within the sample. Thanks to these relationships, it was possible to identify a link between some flipped learning features and the categories related to affective factors outlined above. To understand *which features* of the method influenced these factors, some categories referring to opinions expressed by students regarding the use of flipped learning in mathematics were also considered.

One of the categories that appears with the highest frequency, at 56%, is the one that contains positive judgments about the use of cooperative learning, identified by students as "group work". Being aware of the difference between the two terminologies, what is expressed by learners in the selected items can be traced back to this teaching strategy. Indeed, several key features of cooperative learning emerge, for example, positive interdependence (example 14) and the development of social skills (example 15).

14. "During the Flipped math! project, but especially during group work, we were able to cooperate to achieve various goals, and some people's knowledge gaps could be filled with the help of others."

15. "Having to relate to my classmates made me realize the importance of being able to listen, to accept and question my point of view, in order to achieve and obtain the best result at the end."

It also emerges how using this approach made students experience positive emotions while studying mathematics (example 16), especially that of fun (example 17).

16. "Being able to compare and work with my groupmates [...] made carrying out this experience in a truly exceptional way, easy and exciting."

17. "This method helped me a lot, it was easier for me to learn math by getting help from other group members and I enjoyed helping others."

Negative opinions about cooperative learning are among the highest compared to the other negative-value categories as well, appearing in 9% of the papers. These are mostly due to a personal predisposition to individual work (example 18), perceived uselessness for learning, or lack of collaboration within the group.

18. "[...] I personally do not like to work in groups, because I prefer to think on my own and take my own time (fast or slow)."

Less impactful was the second learning technique used, problem-based learning, which is referred to, with positive valence, in 16% of the essays. Of course, references to it are not direct, but rather occur through its description, mentioning reasoning and deduction processes that characterize it (example 19).

19. "In the end, we can say that we were a bit like our own teachers: by reasoning calmly we always arrived at the solution by ourselves."

Although the incidence of problem-based learning is not comparable to that of cooperative learning, the reasons underlying such appreciation are relevant. These include the positive emotion of interest (example 20) and the high perceived competence (example 21).

20. "The activities carried out and the topics covered were interesting also because of the way in which they were examined. In fact, I believe that starting from the example and then getting to the theory, with the active participation of the student, is an effective method of teaching and learning."

21. "It was also satisfying to understand that I was able to come to the definition of concepts independently, in this case about conic sections."

In particular, the proposal of contextualized problems within the real world, especially at the challenge stage, led to an appreciation in 24% of the essays. This was found to be related mostly to the positive emotion of interest (example 22).

22. "It was also very interesting to see how conic sections are a constant in our daily lives, and how mathematics materializes in reality and how useful this is to us in close connection with art."

Some negative opinions were also noted with respect to this teaching strategy, about 4% of the samples did not like it. A difficulty emerged for some students to adapt to the new method, considered "more difficult", because they were used to a passive position in which it was sufficient to memorize notions. Flipped learning, being an active approach, requires more cognitive engagement and effort.

One feature that distinguishes flipped learning from other methodologies is that it also proposes creative activities. Only one was contained in our course, and it consisted of creating a video lesson for a hypothetical new classmate to explain to her the last covered topic. Although the opportunity to approach this type of activity was unique, it was positively referred to by 19% of the students, while negative considerations did not even reach 1% of the topics. This positive valence in the category was found to be related to the positive emotion of fun. This depended on the ability to use imagination and to work with peers on an authentic project relevant to their interests, that is the world of videos and the web (example 23).

23. "What I absolutely enjoyed the most was making the video. I had fun with my group and from a simple idea we, even as a joke, created a real short film that always made me come home with a smile."

This very last fact introduces us to the next category: the use of technology. The use of digital tools in our educational approach was not limited to the use of computers, interactive whiteboards, or smartphones in the classroom, but consisted mainly of extracurricular communicative choice. This was through the web, in the proposal of videos as stimuli or reminders of prerequisites, and in the use of educational software.

Positive opinions with respect to the use of technology are present in 20% of the essays, referring especially to the use of educational software (example 24) and online videos (example 25), mainly because of the triggered interest.

24. "We approached these topics with the help of GeoGebra, and I think this attracted our attention, especially seeing how the variation of certain parameters affects the graphical representation of the figure."

25. "We were allowed to gradually approach the concepts through a digital path and not just paper. Watching videos is a stimulating and functional way because it captures attention really well and leaves what I learned during the hour firmly in my mind."

Negative opinions about the use of technology in education are also present, but the frequency does not exceed 2%.

A last characteristic of flipped learning that will be examined here is the pursuit of the optimal difficulty level of the challenges proposed. In 11% of the essays, an appreciation for such pursuit emerges (example 26), while in 8% it was not considered effective in some cases, because the proposed activities were perceived to be too difficult (example 27), others because they were considered too easy (example 28).

26. "Finally, the division with which homework assignments were given was very helpful. Since they were always in the right amount and of the correct difficulty, they played a positive role."

27. "[...] the downside was that [the conics] were a lot and it was hard to study everything for the test."

28. "One area for improvement could be the excessive simplicity of some exercises that, made more difficult, would have been more enticing and challenging for students."

Finally, this category, with positive valence, was often found to be related to high perceived competence.

DISCUSSION OF THE RESULTS AND CONCLUSIONS

Some conclusions can be drawn from the results presented above. Students' attitudes in mathematics were influenced by the use of flipped learning. In particular, the component of emotional disposition, which includes emotions, is the most frequently presented category within the essays. It is found with positive valence about 5 times more often than it is in the negative. In addition, in some cases, there are signs in papers of change having occurred during experimentation. The fact that such methodology can promote positive emotions during learning in mathematics, rather than negative ones (which usually prevail) is in line with the results obtained from other research, such as that of Jdaitawi (2020). The cited study also highlights the need to determine how the flipped learning environment can better support students' emotions, and this was one of the goals of the present study. From the results obtained, almost all the features of flipped learning listed above contributed to the prevalence of positive valence in this category: the use of cooperative learning, the use of technology, the proposal of creative activities and real problems.

In terms of frequency of occurrence in the essays, the component of the perceived competence, which are considered high or low, follows. The positive and negative valence in the presented categories is seen with an almost comparable percentage, but again for the first ones there are references to a change having taken place. The perception of one's abilities as adequate was influenced mainly by the use of problem-based learning and the proposal of optimal level activities. The relationship between the methodology investigated and perceived competence in mathematics is a specific area that has not yet been investigated in detail, so it is not possible to have feedback outside the present research.

We conclude the list with the component of the view of mathematics, which is divided into useful and useless. Although the percentages for the category are quite low, we can assert that the one with positive valence is 6 times more frequent than the one with negative valence. The contribution to a view of mathematics as a useful discipline comes mainly from the proposal of real and contextualized problems in everyday life. A study on the influence of flipped learning on the vision of mathematics was conducted by Lee et al. (2017), from this it appears that students developed mathematical views and epistemological beliefs closer to those of experts and less superficial.

There was no absence of negative opinions, as shown, although in much lower percentages than positive ones. Note, however, that while links were found between flipped learning features and positively valence attitude components, the same was not found for negatively valence attitude components. Thus, there are criticisms about flipped learning in the essays, as well as phrases expressing components of the negative valence attitude, but there is no recurring link between these two types of categories. This suggests that flipped learning did not negatively affect students' attitudes toward mathematics.

In conclusion, we hypothesize that a use of flipped learning, which takes advantage of cooperative learning and problem-based learning strategies, could continue to foster in students a positive valence of the three components of attitude toward mathematics, which, if repeated over long periods of time, could turn into a stable trait of the subject.

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