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Extreme Apprenticeship – Engaging undergraduate students on a mathematics course

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Abstract : This paper describes how an educational method called Extreme Apprenticeship has been used in teaching mathematics to undergraduates. The aim has been to facilitate the transition from secondary to tertiary education and to teach students the kind of skills they need in further studies and professional life. We report how the Extreme Apprenticeship method has been implemented in the course Linear algebra and matrices I at the University of Helsinki. This first year mathematics course has approximately 400 students each year. We compare the method with the traditional lecture-based approach that was in use before introducing the Extreme Apprenticeship method. We focus on how these two methods engage students. The results show that the Extreme Apprenticeship method managed to engage the students better than traditional teaching, as more students completed their coursework than before. Even though the new teaching method demanded a lot of personal effort from the students, they did not think that the workload was too big, and were pleased with the course.

Keywords: mathematics education, Extreme Apprenticeship, tertiary education, linear algebra

Introduction

Transition from secondary education to tertiary education is challenging for many students. The way teaching is organised and the amount of responsibility expected can come as a surprise for the first year undergraduates. This could be addressed by making the first year more like school, but this would only push the difficult transition one year further. Clark and Lovric (2008) see the transition as an initiation rite, where the students become members of a new community. It is not to be easy, but the community acknowledges the difficulty and supports those willing to participate in it.

If transition is viewed as an initiation rite, what kind of a community is it initiating to and why is the rite important for the community? In the university setting, a grim and simplified answer is that the rite is about selecting those students who succeed in memorising necessary bits of information by sitting in crowded lecture halls and are able to write that information down in exams. Being good at these kind of skills might correlate with succeeding in later studies and professional life, but at least it is a very different set of skills than the one used by professionals. For example, mathematicians do not spend most of their time listening to mathematical presentations (even if they do that from time to time), but work hard thinking about mathematical theories. They play with ideas and discuss them with others.

In order for the initiation rite of transition to tertiary education to support the students in becoming mathematicians, the Department of Mathematics and Statistics at the University of Helsinki has employed a new teaching model called Extreme Apprenticeship method. The central idea of the method is to develop experts in the field of study, and create teaching arrangements that foster this development. In addition to theoretical knowledge, also the practices of an expert are important. The aim "is not only learning about expertise, but becoming an expert in the practised skill" (Keijonen et al., 2013). This is done by making the students participate in meaningful activities which resemble those carried out by professionals. These activities require a lot of personal effort from the students, but at the same time the teaching staff offers scaffolding for the students' progression in learning mathematics. Passive activities, such as sitting in lectures, are reduced to the minimum.

Emphasis on the learning situation

For students, the purpose of the first year courses is to learn knowledge and skills necessary for their future studies. Therefore, successful participation in these courses should lead to successful participation in following courses, and eventually professional careers.

We could study the learning of mathematics as a process of acquiring mathematical knowledge, but to be a mathematician is more than to know mathematics. The situative view on learning incorporates the social, cultural and physical environment as inseparable parts of the learning process (Brown et al., 1989; Lave and Wenger, 1991; Collins and Greeno, 2010). Therefore, the theory to be taught and actions used in teaching it are inseparable. According to the view, an effective learning situation leads the learner to successfully participate in a socially esteemed practice of a community (Greeno, 1997).

In this light, the objectives of a mathematics course become more complex than just to deliver the mathematical theory. In mathematics, students should learn to successfully participate in practices of professional mathematicians. This participation involves not only gathering and managing mathematical knowledge and strategies, and using these skills in e.g. understanding mathematical proofs, but also persistence in problem-solving, expressing oneself by mathematical writing, and helping others in gathering mathematical knowledge (Wood et al., 2012; Selden and Selden, 2012).

Students in universities do not study their subjects in isolation, but form various communities and become members in the existing ones. The social norms and culture of these communities affect the learning of the students (Lave and Wenger, 1991; Lave, 1996; Collins and Greeno, 2010). The practices of the communities form *locally negotiated regimes of competence*, which define successful or competent participation of its members (Wenger, 1998). The teaching arrangements of university education should direct the regimes of competence in the communities of practice of its students towards appreciation of the development of expertise, not towards passing courses with minimum effort. External motivators such as summative assessment play a big role in this (Biggs and Tang, 2007), but even greater is the need for intrinsic motivation (Lindblom-Ylänne et al., 2009).

It is worth noting that in a large university course, students do not form a single community of practice, but a constellation of communities of practice. In the communities of this constellation, different regimes of competence are formed, and not all can be affected by the arrangements of educational practices. In some communities, the regime of competence might emphasise knowing mathematics, in some membership in open source projects and in some participation in student parties.

Extreme Apprenticeship

The Extreme Apprenticeship method is an educational model based on the concept of Cognitive Apprenticeship, where characteristics of the traditional apprenticeship-based learning are used in teaching and learning cognitive skills such as thinking and problem-solving (Collins et al., 1991). The model was designed for use in computer programming (Vihavainen et al., 2011a,b; Kurhila and Vihavainen, 2011) and subsequently in mathematics with the first large scale implementation in autumn 2011 (Hautala et al., 2012).

The label ”Extreme” comes from the software engineering industry, where Extreme Programming (Beck, 1999) is a method where good programming practices are taken to the extreme. In Extreme Apprenticeship, the same is done to the educational best practices. For example, as monitoring student progress is seen as a good teaching practice, the method enforces continuous monitoring of students' progress (Vihavainen et al., 2011a).

Recent trends in education, such as variants of the flipped classroom, have emphasis on the active involvement of students. Passive activities, such as listening to the teacher are done through videos, and teachers can use their time in class supporting the students' learning. Extreme Apprenticeship shares many of these ideas, but the focus in its design is not on the embedded technology used to grasp material. The focus is on how to arrange the one-on-one instruction, especially in large university courses with several hundred attendants.

Traditionally, mathematics has been taught at our university through lectures and take-home assignments that relate to the lectures. The lectures form the structure of the course by pacing the schedule and delivering the content. Solutions to the take-home assignments are demonstrated in demo classes in the guidance of teaching assistants, with some possibilities for discussion. At the end of the course, students are assessed in a final exam. In Extreme Apprenticeship, the role of the lectures and assignments is changed: the assignments are the primary structure of the course pacing progress and guiding towards necessary information. The amount of exercises is substantially larger and the number of lectures smaller than traditionally.

The students are not expected to solve all assignments on their own. The main method of teaching is one-on-one instruction provided by the teaching team of the course. The team consists of a responsible teacher (a member of staff) and teaching assistants (usually older students). The personal instruction is based on the concepts of coaching and instructional scaffolding in Cognitive Apprenticeship (Collins et al., 1991).

Instructional scaffolding refers to temporary support given to students (Wood et al., 1976). Scaffolding is interlinked with the concept of zone of proximal development introduced by Vygotsky (1978), where a person is able to work on tasks that are slightly more difficult than the ones that they could handle on her own. As the students become more skilled in the task, the scaffolding is faded away until the students are performing the tasks on their own. Coaching refers to the broader perspective of regulating pace and difficulty of assignments in the course. A necessity for coaching students' progress is a bi-directional feedback between the students and the teaching team (Kurhila and Vihavainen, 2011). This feedback is transmitted, for example when the teacher is scaffolding a student's learning, or when the teacher reads the students' answers to assignments and responds to them.

Contrary to the traditional model where student's progress was only assessed at the end of the course, in Extreme Apprenticeship teaching team receives information on the progress of the students during the whole course, and can react accordingly. More importantly, the students receive support and information on how they are performing throughout the course. Feedback is not only about the solutions being correct or incorrect, but also about the quality of the solution (e.g. readability, clarity of thought and language). As the Extreme Apprenticeship method is designed to be implemented in courses with several hundred students, the role of the responsible teacher as a coordinator of the teaching team is important. The teacher has to make sure that the assignments, scaffolding, course literature and lectures are all aligned (Vihavainen et al., 2011b). The teaching team has to share the same views on instructional practices, learning objectives of assignments and grading of the students' answers.

Linear algebra and matrices I in 2013

Educational setting. University studies in Finland resulting in a master's degree are intended to last five years with three years of bachelor's studies and two years of master's studies. There are no tuition fees. The University of Helsinki is the biggest university in Finland with over 35000 students and almost 4000 members of staff. The Department of Mathematics and Statistics at the University of Helsinki is the biggest department in its field in Finland with over 1300 students. The students of the department are selected by their performance in the upper secondary school matriculation examination, an entrance exam or both.

In 2011 a first year course "Linear algebra and matrices I" was transformed from a traditional lecture course to a course taught with the Extreme Apprenticeship method. In this paper we investigate how the transformation affected the activity of the students, and what the students thought about the new teaching method.

Learning objectives. Previously the learning objectives for the linear algebra course were a set of mathematical topics. As utilising the Extreme Apprenticeship method meant focusing more on the activities and situation of teaching, this needed to be communicated to all stake holders, both students and staff. The learning objectives were written down in a matrix form influenced by the Unit Intended Learning Objectives matrix by Biggs and Tang (2007). The objectives were split into two, one matrix consisting of objectives regarding mathematical topics such as "is able to determine if a matrix is invertible using the definition of invertibility", and another describing more general skills needed in mathematics such as reading and writing (see Table 1). Especially the second had been lacking in previous curriculum considerations.

Table 1. Learning objectives of the course Linear algebra and matrices I in 2013.

	Skills approaching the learning objectives	Skills reaching the learning objectives.
Reading mathematical text	• Reads the course literature • Compares own solutions with provided model solutions	• Familiarises oneself with the topic of the lecture beforehand
Oral communication	• Asks for assistance when facing mathematical difficulties	• Is able to form precise questions • Uses correct vocabulary for mathematical concepts • Takes part in conversations during instruction and lectures
Producing solutions	• Writes solutions that other people can read • Uses terminology defined on the course	• Defines all the symbols used, such as variables and constants. • Writes complete sentences and uses mathematical symbols only when necessary
Proving	• Knows the difference between example, definition and theorem	• Reads proofs and is capable of following their logical structure. • Solves simple tasks of the form "Show that..."

Delivering the course. Arranging teaching activities on a large scale with the Extreme Apprenticeship method in linear algebra was conducted for the first time in autumn 2011 (Hautala et al., 2012). Even though the results were promising, alignment of activities was not sufficient, which led to excessive working hours and hence cost. The management and structure of the course, as well as the course material were improved in autumn 2012. The iterative process of improving the course was continued in autumn 2013, when the course implementation was carried out with virtually the same monetary resources as a traditional lecture course. In 2013 there were 484 students enrolled for the course, 42 % of whom were major students in mathematics. Other common major subjects were computer science and physics.

As mentioned earlier, the role of the lectures and assignments was reversed. New mathematical concepts were presented first through assignments and then discussed in lectures. The phasing is described below:

1. A new topic is introduced through relatively easy course assignments.
2. In order to solve the tasks, students have to read the course material by themselves, but they receive guidance from the teaching team.
3. After the students have worked on the tasks, the topic is discussed in the lectures.
4. After this, the students are given more challenging tasks regarding the topic.

Each week 3 hours of lectures were taught by the teacher responsible of the course. Teaching assistants of the course, consisting of 2 senior teaching assistants and 6 junior teaching assistants, were recruited amongst the students of the department. The senior teaching assistants had been junior teaching assistants during previous course iterations.

The teacher and the senior teaching assistants prepared 10–20 take-home assignments per week. Students were offered around 20 hours of guidance by the teaching team to complete the assignments. This guidance formed the main part of teaching in the course, and was conducted in a classroom fitted for the purpose. The guidance was not compulsory to the students. Of the assignments, 1–2 were selected for inspection each week. The whole teaching team took part in evaluating the solutions. Students had two weeks to improve their solutions, if necessary.

In Extreme Apprenticeship, it is important that the teaching team shares practices regarding scaffolding and evaluating solutions, as well as all the information on the progress of the students. Therefore, the teaching team met every week for one hour. Half of the meeting was spent reflecting on general pedagogical topics, such as how to interact with the students. During the other half of the meeting, the responsible teacher discussed the course assignments of the week with the teaching assistants.

The teaching team agreed on the following guidelines for instruction and evaluating solutions were agreed upon by the teaching team:

1. Leave the joy of discovery to the student. Do not give answers, but lead subtly towards the right solution.
2. There is usually more than one right way of thinking. Do not obtrude your own solutions.
3. Be supportive, especially when the student has had difficulties.
4. (New) students are shy. Approach them, and do not necessarily wait for them to ask for advice.
5. Teaching assistants are not information depositories. The students must learn to read the course literature
6. Teaching assistants do not need to know everything, and they can let the students see this.
7. When evaluating answers, be concise. Underline the parts of the solution that are wrong and if necessary write a short comment. If there is much awry, do not spend time evaluating the assignment, but recommend asking an instructor for help.
8. Having the right idea is not enough. The answer has to be written correctly. The aim is to practise expressing oneself in a precise and readable way.
9. If in doubt about rejecting a solution, assess if the student will benefit from improving the answer.

Results

Engaging the students. We investigated the students' engagement on the course Linear algebra and matrices I during years 2008–2013 by monitoring how many students completed course assignments. The course was taught with a traditional lecture-based approach in 2008–2010 with varying teachers. In 2011–2013 the Extreme Apprenticeship method was used. During these years, the first author was the responsible teacher of the course.

Table 2 shows how many students enrolled for the course, and how many of them completed course assignments. In the lecture-based model completing a task means that the student has attended the demo class and indicated that he/she has completed an assignment. In the Extreme Apprenticeship method, completing an assignment means that a student has done the assignment and submitted it.

Table 2. Number of students completing course assignments each year. The Extreme Apprenticeship method was used in 2011 – 2013.

Year	Number of students enrolled for the course	Students who completed assignments during the course	Students who completed assignments on week 5	Passing rate
2008	379	71 %	48 %	68 %
2009	372	66 %	50 %	61 %
2010	348	69 %	47 %	66 %
2011	419	80 %	60 %	69 %
2012	434	82 %	60 %	73 %
2013	484	83 %	62 %	67 %

We considered two things: (1) How many of the students completed tasks at least once during the course? (2) How many of the students completed tasks during week 5? Week 5 was chosen for study, because it is the second to last week of the course. By doing this, we wanted to see how persistently the students worked during the course. The final week of the course did not come into question because in years 2011–2013, the last week was substantially different from the other weeks: the students did not receive any feedback from the coursework submitted in the last week, which affected the submission rates.

From Table 2 we see that when the course was taught with the Extreme Apprenticeship method, a greater percentage of the students completed assignments during the course. In traditional teaching, the percentage ranged from 66 % to 71%. When applying the Extreme Apprenticeship method, the percentage rose to 80 %–83 %. Table 2 also shows that with the Extreme Apprenticeship method, a greater percentage of students completed assignments on week 5. The change was from 47 %–50 % in traditional teaching to 60 %–62 % with the Extreme Apprenticeship method.

Table 2 shows also the passing rates of the course each year. It can be seen that they have not changed considerably during the years.

Feedback from the students. After the course in autumn 2013, students were asked to fill in an online questionnaire for course feedback. Answering the questionnaire was voluntary, and 97 students filled in the form. As the course had 484 students, this amounts to a 20 % response rate.

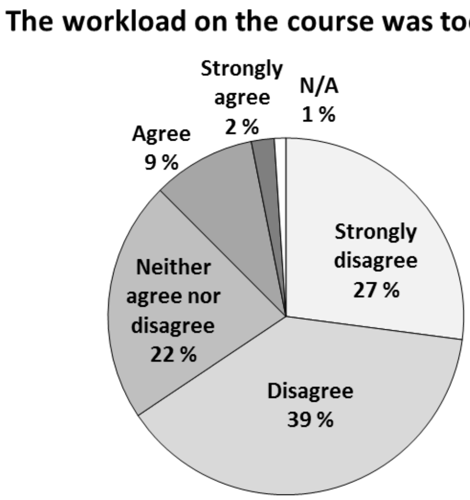

Figure 1. Students' answers regarding the workload of the course (n=96).

The questionnaire consisted of 51 Likert-type questions and 9 open questions. Here we present the students' answers to two of the questions. The Likert scale used ranged from (1) strongly disagree to

(5) strongly agree. The students were posed the statement "The workload on the course was too big." 66 % of the students disagreed with this statement. (See Figure 1.) Another statement posed to the students was "I feel that instruction has benefited me a lot in my studies." 60 % of the students agreed with this statement (See Figure 2.).

In the questionnaire, the students were also asked to give a grade for the course on a scale from 1 to 5, the best grade being 5. The average grade given by the students was 4.3, with standard deviation 0.59.

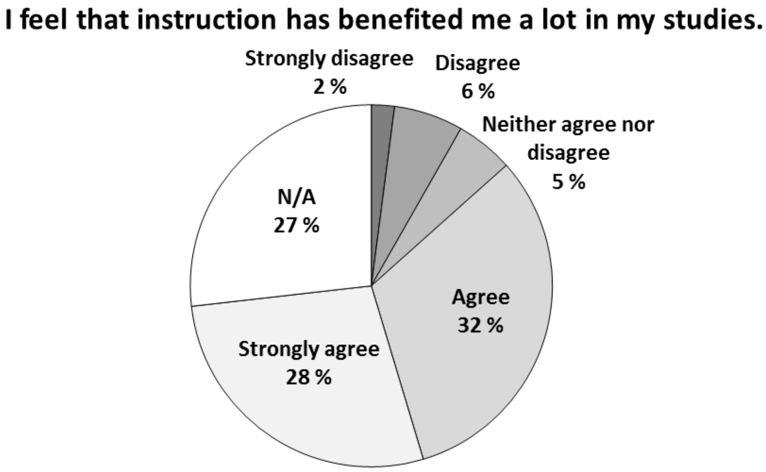

Figure 2. Students' answers regarding the usefulness of the instruction (n=97).

Discussion

We believe that as in other areas of expertise (Ericsson et al., 1993), mathematical expertise comes through deliberate practice. As mathematics is learned by doing, it is important that students work with course assignments throughout courses. It seems that Extreme Apprenticeship manages to encourage students to work more persistently than traditional teaching methods. One cause to this is probably the scaffolding offered to the students: if the students are not able to solve the course assignments by themselves, they do not need to give up. Another reason could be the design and placement of the tasks, which makes course assignments a meaningful activity for students. As there are not as many lectures as before, and the students have to study new topics by doing course assignments, they are more likely to realise that working hard is the only way to learn.

There has not been a significant change in the passing rates on the linear algebra course since introducing the Extreme Apprenticeship method. However, the passing rate was not a major problem on the course, as most of the students who did the course assignments and took part in the final exam passed the course. Also, there is no point in comparing the passing rate over the years, as the educational goals have changed. Our aim has been to move emphasis from memorising mathematical facts and algorithms towards doing and applying mathematics in more demanding ways.

The feedback from the students seems to show that even though studying with the Extreme Apprenticeship method proposed several challenges to the students, namely: (1) it was new to most of them (2) they had to do a lot of course assignments and (3) and they had to read the course material on their own, they did not seem to think that the workload was too big. Also, the students thought that the scaffolding provided by the teaching team was useful. However, when analysing students' feedback, it has to be taken into account that the number of students answering the questionnaire is relatively low compared to the number of students enrolled for the course.

Conclusion

We have managed to implement successfully the Extreme Apprenticeship method on a mathematics undergraduate course with several hundred students. Our results show that the students find the new teaching method pleasing. Also, the Extreme Apprenticeship method engages the students better than the traditional lecture-based method. However, we do not yet know much about how the Extreme Apprenticeship method affects the students' learning, and further investigation is required. Some results will hopefully be seen when the students who have been taught with the method take more advanced courses. Then it will possible to compare their performance with the students who have been taught with the traditional lecture-based approach.

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