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Examining teaching based on errors in mathematics amongst pupils with learning disabilities

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Abstract
Teaching mathematics while learning from students' mistakes, errors and misconceptions, is most important for meaningful learning. This study was based on intervention programs prepared by preservice teachers. It aimed to examine their knowledge of assessment of errors in mathematics amongst pupils with learning disabilities, and their use as a basis for didactically adapted teaching. The research methodology was qualitative. Three pairs of preservice teachers (a total of six students) studying special education participated. Content analysis was performed using the Individualized Education Program and according to Hiebert and Lefevre's cognitive learning model.

The research findings support and indicate the importance of coordinating an analysis of the pupil's errors and the choice of a didactically adapted teaching strategy, in order to achieve a more effective focus on the mathematical difficulties and the misconceptions, and thus to contribute to improving the therapeutic effectiveness and realize the student's abilities to the maximum. The current study is thus innovative in examining the assessment of the errors in mathematics made by students with learning disabilities, not only in regards to the disability in cognitive functioning and difficulties in mathematical functioning, but also in regards to the use of conceptual and procedural mathematical knowledge by the preservice teachers and the teacher.

Keywords: conceptual knowledge, procedural knowledge, learning from errors, didactically adapted teaching, pupils with learning disabilities, preservice teachers.

Introduction

The number of students with learning disabilities is significantly growing over the past two decades. Different studies show that students who have difficulties acquiring mathematical knowledge have a wide variety of learning disabilities (Geary, 1993, 2005; Strang & Rourke, 1985).

The definition of learning disability in mathematics is not agreed upon within professionals, and it creates an ongoing debate about how much difficulties in calculation and numeral understanding are specific difficulties in mathematical functioning (Augustyniak et al., 2006; Landerl et al., 2004; Mazzocco & Thompson, 2005), or do they reflect difficulties in general cognitive functioning, that are at the basis of mathematical functioning (Ardila & Rosselli, 2002; Geary, 1993; Shalev et al., 1997), such as language, memory, sequence perception and abstraction. The research about the causes of learning difficulties in mathematics is in progress, and mostly documents difficulties in specific subjects (National Mathematics Advisory Panel's task group, 2008). The current research may contribute and enrich the existing knowledge about teaching based on mathematical errors of students with learning disabilities.

According to the literature, the causes for learning disabilities can be divided into three paradigms: 1. The neuro-psychology paradigm (Orton, 1937); 2. The cognitive paradigm (Werner, 1937); 3. The behavioral paradigm (Koorland, 1986). The current research is based on the cognitive paradigm, which focuses on researching damaged cognitive processes in learning disabilities. The current treatment approach is that understanding the unique pattern through which the child processes information may assist correcting the problem.

According to the cognitive approach, which emphasizes the importance of mathematical understanding, difficulties in understanding number stem from unsuitable teaching and inefficient intervention programs (Baroody 2006; Gersten et al., 2009). One of the questions that researchers and educationalists deal with is: "What instructional strategies for teaching mathematics to students with learning disabilities and to low-achieving students show the most promise?" (National Mathematics Advisory Panel‘s task group, 2008). The current research is based on intervention programs planned by preservice teachers. The research aims to examine the knowledge of the preservice teachers about assessing mathematical errors of students with learning difficulties and using them as a basis for didactically adapted teaching.

Literature Review

Students with Learning Disabilities in Mathematics

A learning disability is a neurological syndrome, congenital or acquired through accident or disease, which interferes with acquiring basic learning skills such as: reading, writing, mathematics, time/space organization and attention focusing. As a result, a gap is created in all learning subjects, even though intelligence is normal or above normal (Reid & Kirk, 2000). A Learning disability in math is called "developmental dyscalculia". It is a disturbance in the numeral mathematical ability, among children of normal intelligence without other cognitive disabilities (Butterworth et al., 2011). This definition is common in the literature, however, it is still not agreed by many researchers, due to findings that show 17% of children with dyscalculia also suffer from dyslexia and 25% of them also suffer from attention difficulties and hyperactivity (Cawley & Parmer, 2003; Geary, 2007; Landerl et al., 2004).

However, it is estimated in the literature that 5–10% of school children have learning disabilities in different levels and characteristics, which affect the learning in math (Mazzocco and Thompson 2005). According to the cognitive approach, the characteristics of the disability are clearly related to learning difficulties in math. Visual processing, visual memory, and space awareness affect the acquiring of math skills, because they are significant parts of acquiring mathematical knowledge that includes conceptual knowledge and procedural knowledge (Kilpatrick et al., 2001).

Different researchers claim that difficulties in calculation and numeral sense reflect lacks in general cognitive functioning (Ardila & Rosselli, 2002; Geary, 1993; Shalev et al., 1997). Numeral sense includes the ability to identify quantities, estimate results and use action rules and different ways to represent numbers, which are mostly based on the number frame. All those allow the learner to perform control processes on different algorithmic operations (National Mathematics Advisory Panel‘s task group, 2008).

Mathematical Knowledge – Conceptual Knowledge and Procedural Knowledge

There are different approaches to explain the components of mathematical knowledge that children need to know. Skemp (1976) sees mathematical knowledge as mathematical understanding that is related to relational understanding, which relates to understanding mathematical concepts and their relations, and instrumental understanding, which relates to remembering rules and operating single algorithms. Fischbein (1987) defined three types of mathematical knowledge: a. Formal knowledge – includes the definitions, concepts and operations, structures and mathematical sentences that suit certain content; b. Algorithmic knowledge – includes the rules and processes that are used mainly for calculations; c. Intuitive knowledge – includes the ideas and beliefs about mathematical entities and their expressions to present mathematical concepts. The relations between the knowledge types allow better understanding of the total mathematical knowledge.

Some view the mathematical knowledge as divided into two types, procedural knowledge and conceptual knowledge. Procedural knowledge is knowledge of rules and algorithms to solve problems. This knowledge is a result of routine memorization and learning of the rules. When a student is required to perform calculations, in fact he is required procedural abilities. The calculation abilities include the speed of recalling mathematical facts and the accuracy level in which the calculations are performed (Hecht et al., 2001). The conceptual knowledge is a network knowledge of concepts that are interrelated and related to different pieces of knowledge (Hiebert & Lefevre, 1986). The relationships are the "meaning" to the mathematical symbols. The more complex the network of relationships between the internal representations themselves and between the internal representations outwards, and the more information the networks includes, the higher the student's understanding will be.

Difficulties in understanding can occur when there are "holes" in the knowledge "networks", or when the relationships between the different representations are not strong enough. The learning process is a dynamic one, in which each time knowledge relationships "re-organize". Every "re-organization" is better than its predecessor and depends on a net that already exists (Hiebert & Carpenter, 1992).

Hiebert and Lefevre (1986) claim that the combination between procedural knowledge and conceptual knowledge should be emphasized. Research show that there are differences between procedural knowledge and conceptual knowledge among children (Bayman & Mayer, 1983; Madison, 1995). A research of grades 1–3 found that in classes that emphasized conceptual knowledge, the pupils also acquired procedural knowledge, however, in classes where the emphasis was on procedural knowledge, the pupils had not acquired conceptual knowledge (Hiebert & Wearne, 1994).

Many comprehensive studies examined the difficulties of children acquiring mathematical knowledge and the common errors of their work. The results of these studies indicate a number of consistent findings: slow or inaccurate recall of basic facts, impulsiveness or lack of restrain, problems creating images of mathematical concepts, weak ability to produce numeral meaning of symbols or revealing rules, and difficulties in storing information in the working memory (Geary, 1993, 2005).

Learning from Errors

In many situations teaching must be based on the ideas that the children already have, even if they are incorrect, so that they would construct the mathematical understanding meaningfully and gradually (Campbell, 1997; Fennema et al., 1993; Hiebert & Wearne, 1993). A teacher that is familiar with common misconceptions of students can use them, in order to progress and deepen the mathematical understanding. The activities around errors bring awareness of the errors, identifying the errors, an understanding why the solution is wrong and the possible reason why they have made the error (Borasi, 1994). Children's misconceptions are in abundance. It is widely accepted that teaching while learning from mistakes, errors and misconceptions makes the learning of the mathematical contents meaningful (Booth et al., 2013; Borasi, 1996; Ding, 2007). Studying errors in learning math has great value beyond the therapeutic level. The goal of teaching is to develop the mathematical ability, and the challenge to teachers is to use the children's errors, in order to support their ongoing mathematical development.

Researches in mathematical education study misconceptions and their causes, and develop programs that combine the errors in the teaching-learning as an inherent part of the mathematical subject (Barkai & Zamir, 2005; Yerushalmi & Polingher, 2006). Borasi (1996) presents two main approaches to using errors in teaching-learning. First, the error is used as a leverage to promote learning. The teacher uses the mathematical error and encourages the student to use his mistake as an opportunity to learn math. Second, the goal is the teacher's analysis of the error. The teacher analyses the error in order to diagnose the student and improve his performance. First, the teacher identifies the error through a diagnosis that provides insights about what goes on inside the student's mind, and insights about his way of thinking. The treatment focuses on changing the student's misconception, so he may remove the error next time. The current research is based on the second approach, which is common among teaching children with learning disabilities (Gersten et al., 2009).

Didactically Adapted Teaching

The studies about special education in the past 20 years emphasize the change in the perception that teaching that is good for students with learning disabilities is similar to teaching that is good for regular students. This is because these students need different conditions and learning methods to those of their peers (Thurlow, 2000) and not to repeat work with computational procedures year after year (Schmidt et al., 2011). The guidelines about teaching students with learning disabilities are equality and individuality. "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (National Council of Teachers of Mathematics, 2000, p.12). That is, a student is entitled to an equal chance to fulfil his ability, so the teacher must create opportunities that will allow each student to fulfil their learning ability and social acceptance.

Much knowledge has accumulated in the literature about suitable teaching strategies for students with difficulties in math (National Council of Teachers of Mathematics, 2014). The most effective strategies found are: methodological explicit teaching, peer tutoring, visual representation and independent learning, which regards intuitive thinking, learning from errors, and creating a cognitive conflict as very important (Gersten et al., 2009; National Mathematics Advisory Panel's task group, 2008). Research show that choosing teaching strategies needs to be done according to the faulty functioning (Stecker & Fuchs, 2000). Students with learning disabilities have weak awareness of their cognitive processes, and they find it hard to describe them and explain how they solved exercises and problems. Therefore, the recommended treatment is explicit encouragement to use certain cognitive strategies that will improve the task performance (Schmidt et al., 2011). The therapeutic approach is focused on choosing learning strategies that are based on meta-cognitive principles (Baker & Brown, 1984; Wong, 1996). This approach is about raising awareness of the learning disability regarding cognitive processes and replacing ineffective strategies with effective ones. For example: a student who has difficulty reading numbers. The difficulty may stem from the perception of sequence and preserving it. Therefore, the suitable teaching strategy is building associative symbols together, in order to read numerals and using quantifying apparatus that lead to reading numbers based on their quantity meaning.

It is impossible to overcome and remove learning disabilities, therefore the role of special education is to teach strategies that will go around the weaknesses of the child. The Ministry of Education recommends cognitive adaptations: making the curriculum accessible, adapted instructions and guidelines, adapted assessments, change in the tempo or sequence of the activity, time allowance, reduction or addition of stimuli, deconstruction of tasks into components, and using multi-sensual simulations (Ministry of Education, 2016). Despite the multiple strategies, it is still unclear what teachers know about effective treatment of the difficulties of students with learning disabilities, so that they could succeed and improve their mathematical achievements (Heritage, 2007; McGraner et al., 2011).

The uniqueness of the current study is in the fact that it examined the level of analysis of errors made by children with learning disabilities, among preservice teachers of special education in their third year. In addition, the current study is unique in examining the mathematical pedagogical knowledge of preservice teachers that specialize in teaching students of special education. When these preservice teachers would have to teach different subjects, such as math, they will need to use didactical adaptations that will allow them to succeed teaching their class, promote their students, reduce the academic gap and reach the required achievements.

The goal of the study was to examine the knowledge of preservice teachers about assessing mathematical errors of students with learning disabilities, and using didactically adapted teaching strategies.

The Research Questions

The following questions were drawn from the goal of the study:

1. How do preservice teachers assess mathematical errors made by students with learning disabilities with regards to their functioning difficulties?
2. How do preservice teachers use mathematical errors made by students with learning disabilities as a basis for didactically adapted teaching strategy?

Method

The research method is qualitative – ethnographic. This method aims to understand the human activity, as part of understanding the entire process through following actions and experiences. The approach is naturalistic – it describes and generalizes three intervention programs of teaching students with learning disabilities by mathematical errors with didactically adapted teaching strategies.

Participants

Three pairs of female third year preservice teachers studying at the teacher training college, special education course, participated in the study. During the course "Adapted Teaching in Math" they have acquired mathematical-pedagogical-didactical knowledge for students with learning disabilities. At the end of the course, the preservice teachers have been asked to prepare an intervention program for a student with a learning disability who has a difficulty in math, and studying in a regular classroom.

Research Tool

The teaching based on mathematical errors of students with learning disabilities was examined by the intervention programs prepared and performed by the preservice teachers. These intervention programs were the formal documents for data gathering. The goal of assessing the programs was to examine the analysis of the arithmetical error and the application of the didactically adapted teaching, while planning a didactically adapted activity, performing it, and writing a reflective summary when it is finished.

Procedure

After studying the course "Adapted Teaching in Math", the preservice teachers prepared intervention programs that included semi-structured interviews and participant observations, which lasted about two non-consecutive lessons for a pupil known to them through the practicum.

During the first lesson the preservice teachers conducted an individual diagnosis for the pupil. At the end of the lesson they analyzed one error that refers to the faulty cognitive functioning, and prepared a didactically adapted activity. During the second lesson, they performed the intervention plan. At the last stage they have processed the data and wrote a reflective documentation. The three assignments presented in the current study were randomly chosen. Each intervention program has been mathematically content-analyzed.

Findings

In order to answer the research questions: How do preservice teachers assess mathematical errors made by pupils with learning disabilities with regards to their functioning difficulties? And: How do preservice teachers use mathematical errors made by pupils with learning disabilities as a basis for didactically adapted teaching strategy? Their intervention programs were analyzed through qualitative mathematical content analysis according to the model of Hiebert and Lefevre (1986), which divides the learner's mathematical knowledge into conceptual mathematical knowledge and procedural mathematical knowledge. The conceptual knowledge represents the understanding and the procedural knowledge represents the calculation skills.

General Description of the Intervention Program

The first intervention program was written for a 1st grade pupil, about the subject: addition and subtraction up to 10. The mathematical error was incorrect solving of the addition and subtraction exercises up to 10. The didactically adapted teaching strategy is a memory game.

The second intervention program was written for a 2nd grade pupil, about the subject: the meaning of addition and subtraction – solving addition and subtraction exercises up to 20. The pupil's mathematical error was incorrect solving of addition and subtraction exercises up to 20 with breaking a ten. Two teaching strategies were performed: one was a model that represents the metric structure, and the other was a lottery game that assists practice and exercise of the meaning of addition and subtraction.

The third intervention program was written for a 3rd grade pupil about the subject: one-stage and two-stage word problems. The mathematical error was incorrect solving of word problems. Two teaching strategies were performed: one was the use of navigation cards for direction and assistance according to the need, and the other, a simulation game of a "shop".

Conceptual Knowledge and Procedural Knowledge Based Teaching

In this study, the analysis is at the preservice teachers level: to what extent the preservice teachers present her conceptual knowledge and procedural knowledge through planning the intervention program, performing it and reflectively evaluating it. The knowledge assessment was done by specific categories: the sub-criteria of the intervention program, which are based on an individual program assessment index (Ministry of Education 2007). At this stage, representative examples for each of the sub-criteria were chosen, the conceptual knowledge represents understanding and procedural knowledge represents calculation skills. The scoring for frequency of mathematical knowledge was performed in the following manner: 0: There is no directedness to conceptual/procedural mathematical knowledge; 1: Apparent directedness to conceptual/procedural mathematical knowledge. Tables 1–3 present content analysis for the three intervention programs according to conceptual mathematical knowledge and procedural mathematical knowledge, through sub-criteria. The examples are authentic text of intervention programs.

Table 1. Content analysis for the first intervention program according to mathematical knowledge

	Sub-Criteria	conceptual mathematical knowledge		procedural mathematical knowledge	
		0-not apparent 1-apparent	Examples	0-not apparent 1-apparent	Examples
1	Locating the error			1	Wrong solving of the addition and subtraction exercises up to 10
2	Defining the mathematical difficulties	1	Perception problem - difficulty understanding the meaning of addition and subtraction		
3	The relation between the disability and the mathematical difficulties	1	Problem in thinking while recalling- data processing problem	1	Attention problem- the pupil does not pay attention to the operation symbol and the reference to the question
4	Strength points presentation	0		1	Omer can say that addition is "plus" and subtraction is "minus"
5	Presenting points to be strengthen	0		1	Omer will know the subtraction operation and the symbol
6	Choosing a didactically adapted teaching strategy for correcting the mathematical error	1	Through the game … will succeed in adding and subtracting quantities	1	Card game… Memory game… focus more on solving problems up to 10 and finding the right answer
7	Addressing the misconception during the activity	1	P: (mediation) "we have 3 circles and we took one off the three, like we have 3 candies and we ate one, how much is left?"	1	O: "in addition we add this plus… and in subtraction we deduct… and it's minus."
8	Achieving the goal of the intervention	1	To assist the pupil and make him understand the meaning of addition and subtraction and how to correctly	0	

	Sub-Criteria	conceptual mathematical knowledge		procedural mathematical knowledge	
			solve exercises		
9	Assessing the student's knowledge at the end of the activity	0		1	Two days after the activity, a worksheet was given to the student with addition and subtraction exercises, and improvement was apparent
	Total score	5		7	

Table 2. Content analysis for the second intervention program according to mathematical knowledge

	Sub-Criteria	conceptual mathematical knowledge		procedural mathematical knowledge	
		0-not apparent 1- apparent	Examples	0-not apparent 1- apparent	Examples
1	Locating the error	0		1	7+7=10, 3+9=15, 8+9=19, 13-5=6
2	Defining the mathematical difficulties	1	Ornit has difficulties with understanding the meaning of the number and its place value	0	
3	The relation between the disability and the mathematical difficulties	1	A fault in perceiving the relationship between a pattern and its items… making it hard to perceive number concept	0	
4	Strength points presentation	1	Quantity meaning of a number: estimation, quantity preservation, quantity understanding, quantity assessment	1	Completing the pattern forward, backwards, skipping up to 20
5	Presenting points to be strengthen	1	Multiplication and division: the meaning of multiplication and division	1	100 Table: completing an empty 100 Table
6	Choosing a didactically adapted teaching strategy for	1	Polystyrene model to illustrate ones and tens, and large and small beads	1	Addition and subtraction lottery

	Sub-Criteria	conceptual mathematical knowledge		procedural mathematical knowledge	
	correcting the mathematical error				
7	Addressing the misconception during the activity	1	Ornit looked at us, puzzled, and didn't know what to do	1	Number Table and Exercises cards, Ornit needs to solve the exercises
8	Achieving the goal of the intervention	1	The lesson was successful, the goals achieved, Ornit understood the meaning of addition and subtraction up to 20	0	
9	Assessing the student's knowledge at the end of the activity	1	At the end of the lessons we played a game, which encouraged Ornit to show me that she understood the lesson. We were glad… to see the improvement in her at the beginning and at the end of the lesson	0	
	Total score	8		5	

Table 3. Content analysis for the third intervention program according to mathematical knowledge

	Sub-Criteria	conceptual mathematical knowledge		procedural mathematical knowledge	
		0-not apparent 1-apparent	Examples	0-not apparent 1-apparent	Examples
1	Locating the error	1	Amir has 32 Shekels in the till. How many shekels does he need to receive in order to buy a game that costs 40 shekels? Danny quickly answered, and wrote: 32+40=72	0	
2	Defining the mathematical difficulties	1	Cannot visualize the problem, cannot tell the story presented before him, therefore	1	Difficulty connecting words to numbers, and identifying from the

	Sub-Criteria	conceptual mathematical knowledge		procedural mathematical knowledge	
		0-not apparent 1-apparent	Examples	0-not apparent 1-apparent	Examples
			does not understand which operation he needs to use		problem which mathematical operations he is required to perform
3	The relation between the disability and the mathematical difficulties	0		1	In this answer we noticed that he operates quickly, impulsively, and doesn't understand that there are words in the problem that can help him reach the correct operation
4	Strength points presentation	1	Danny understands and can solve exercises of changing and altering	1	Danny can solve addition and subtraction exercises vertically as well
5	Presenting points to be strengthen	0		1	Danny has difficulty identifying number operations in one-stage and two-stage word problems
6	Choosing a didactically adapted teaching strategy for correcting the mathematical error	1	Through imaging we help the pupil understand the story behind the problem, and more importantly, we make the story real by actually buying in the shop	1	The navigation cards will prevent quick answers without thinking and will ease the way to solution
7	Addressing the misconception during the activity	1	I would like a peeler for 5 shekels, 2 can openers at 2 shekels each, and a grater for 4 shekels… so right… so 4+4+5=13 shekels	1	I know... I will do vertically. 52+29=81, and now I have to subtract?
8	Achieving the goal of the intervention	0		1	The game made him do the exercises in an experiential way
9	Assessing the student's knowledge at the end of the activity	0		0	
	Total score	5		7	

To summarize the findings, Figure 1 presents the distribution of the mathematical knowledge according to the intervention programs.

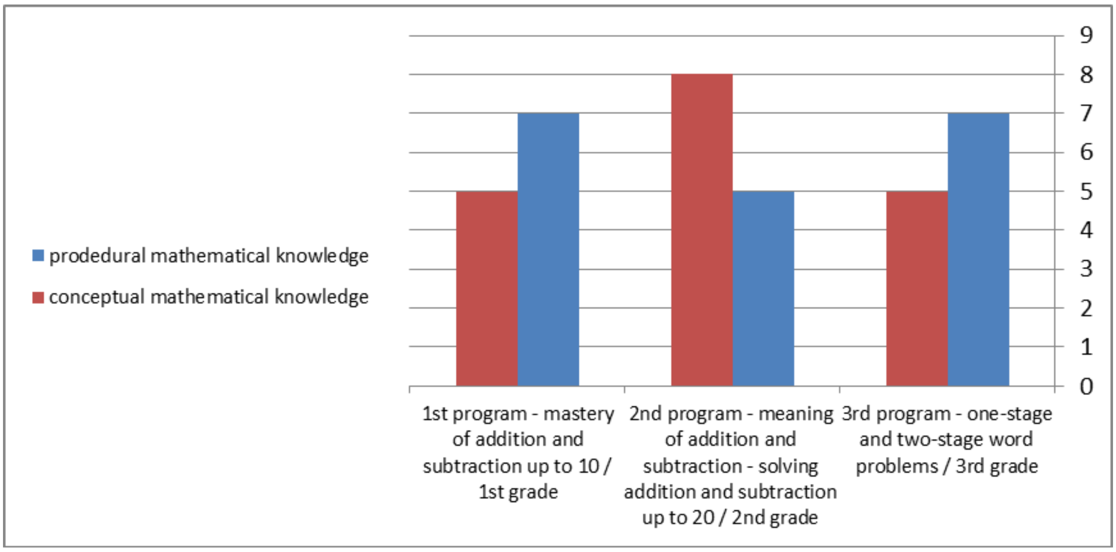

Figure 1. The distribution of the mathematical knowledge according to the intervention programs

The Tables 1–3 and Figure 1, show that the preservice teachers prepared intervention programs that incorporate conceptual mathematical knowledge and procedural mathematical knowledge. The 2nd intervention program emphasized the conceptual knowledge more, and the third – procedural knowledge. Table 4 presents the summary of frequency of mathematical knowledge and the appearance of conceptual mathematical knowledge and procedural mathematical knowledge according to the sub-criteria and intervention programs.

Table 4. Summary of frequency of mathematical knowledge according to the intervention programs

Sub-Criteria	1st intervention program		2nd intervention program		3rd intervention program		Summary of mathematical knowledge
	conceptual mathematical knowledge	procedural mathematical knowledge	conceptual mathematical knowledge	procedural mathematical knowledge	conceptual mathematical knowledge	procedural mathematical knowledge	
The relation between the disability and the mathematical difficulties	1	1	1			1	4
Strength points presentation	1	1	1	1	1	1	6
Presenting points to be strengthen	1	1	1	1		1	5
Choosing a didactically adapted teaching strategy for	1	1	1	1	1	1	6

correcting the mathematical error							
Addressing the misconception during the activity	1	1	1	1	1	1	6
Achieving the goal of the intervention	1		1			1	3
Assessing the student's knowledge at the end of the activity	1		1				2

The Table 4 shows that the intervention programs put strong emphasis on combining conceptual knowledge and procedural knowledge in presenting points of strength and points to be strengthened, in choosing a didactically adapted teaching strategy to correct the mathematical error and in dealing with the misconception during the activity. However, the intervention programs did not combine conceptual knowledge and procedural knowledge in locating the error, in presenting achieving the goals of the intervention and in assessing the pupil's knowledge at the end of the activity.

Discussion and Conclusions

The literature shows that few studies teach about effective math teaching for students with learning disabilities who have difficulties in math, due to their focus on specific subjects (National Mathematics Advisory Panel‘s task group 2008). The innovation of this study is that it examines the usage of pupils' errors as a teaching strategy, regardless of student's age, gender and learned subject. This way, conclusions can be drawn on a wider population. The goal of analyzing the intervention programs was to examine how the preservice teachers succeeded in connecting the conceptual knowledge to the procedural knowledge.

Assessment of Mathematical Errors Made by Pupils with Learning Disabilities with regards to their Functioning Difficulties

The study findings show that in all intervention programs, the preservice teachers defined the arithmetic mathematical difficulties of the pupils in the conceptual aspects, and in the 3rd program, procedural aspects as well. The preservice teachers used conceptual explanations, such as "problem in perception – difficulty in understanding the meaning of addition and subtraction" (1st program), "Ornit has difficulty with understanding the meaning of a number and its place value" (2nd program). In the 3rd program they used conceptual explanations, such as: "cannot tell the story presented before him, therefore doesn't understand which number operation he should use", and in addition, procedural explanations, such as: "difficulty in connecting the words and numbers, that is he has difficulty identifying from the problem which number operations he is required to perform".

These findings strengthen the importance of the distinguishing between mathematical functioning difficulties and general cognitive functioning difficulties (Augustiniak, Murphy & Phillips, 2006; Landerl et al., 2004; Mazzocco & Thompson, 2005). Geary (2005) and Strang and Rourke (1985) claim that students who have difficulties acquiring mathematical knowledge have a variety of learning disabilities. The definition of the mathematical difficulties must be done extensively and include conceptual and procedural aspects, in order to receive better insights about the arithmetic difficulties of students with learning disabilities.

However, the findings indicate that the preservice teachers presented the connections between the mathematical difficulties and the cognitive faults in different manners. In the 2nd program, the preservice teachers used a description of the student's functioning at a high level and conceptual knowledge, for example: "A fault in perceiving the relationship between a pattern and its items… making it hard to perceive number concept". At the 3rd program, the preservice teachers used a description of the student's functioning at a reasonable level and procedural knowledge, for example: "he operates quickly, impulsively, and doesn't understand that there are words in the problem that can help him reach the correct operation". In the 1st program – the functioning description was at a high level and a combination of conceptual and procedural knowledge was done. An example for conceptual knowledge: "Problem in thinking while recalling- data processing problem". An example of procedural knowledge: "Attention problem- the pupil does not pay attention to the operation symbol and the reference to the question".

The findings complete the findings of the literature that enforce the claim that difficulties in calculations and numeral sense reflect lacks in general cognitive functioning (Ardila & Rosselli, 2002; Geary, 1993; Shalev et al., 1997). Cognitive functioning, particularly mental, and memory affect acquisition of mathematical skills due to the fact that they are significant components of acquiring the mathematical knowledge that includes conceptual knowledge and procedural knowledge (Kilpatrick et al., 2001). The current study extends the meaning of didactical diagnosis in math, which examines alongside the mathematical difficulties, the cognitive mechanisms, not in order to explain the difficulties but to treat them; for example: in the first program, the preservice teachers tested the pupil on visual perception and short-term memory, in order to assist the pupil in dealing with these lacks.

The findings show that the preservice teachers conducted diagnoses for pupils about number sense in different ways. The preservice teachers who planned the first and second programs used procedural knowledge (for example, in the second program: 7+7=10, 3+9=15), as opposed to the preservice teachers who planned the third program, who used conceptual knowledge ("Amir has 32 shekels in the till. How many shekels does he need in order to buy a game that costs 40 shekels? 32+40=72"). Therefore, we can say that in all of the programs at the stage of the diagnosis, there was no combination of conceptual and procedural mathematical knowledge. These findings do not support the literature, which emphasizes the combination of conceptual and procedural mathematical knowledge (Hiebert & Lefevre, 1986). This is following researches that show the consequences of combined and separate learning of conceptual and procedural knowledge (Bayman & Mayer, 1983; Hiebert & Wearne, 1994; Madison, 1995). It is possible that in this study, the reason for this stems from little knowledge of effective combination between the knowledge types at the diagnostic stage, because this assumption is based on the claim that the knowledge is ambiguous about students with learning disabilities who have difficulties in math, regarding their definition and characteristics (Mark-Zagdon, 2011).

Also, findings show that the preservice teachers used parts of different existing diagnostic tests, which might be too narrow and specific. Therefore, there is a risk of missing central aspects of the comprehensive understanding of the disturbance and the deeper meaning of the student's misconception. These findings enforce the claim that there is still no agreed upon universal system of criteria to diagnose difficulties in mathematics. There are a number of useful practical indicators to diagnose the difficulty, such as lower than expected achievements (Geary, 2000). This is despite the great importance given in the literature to the use of uniform standardized tests in order to compare learning disabilities, check the effectivity of a long term treatment intervention and long term follow-up over the severity of the identified disabilities. This research sheds light on the complex challenging issue of didactical diagnosis in mathematics, which requires reliability and validity for the students with learning disability population of different ages in the education system.

To summarize, we can say that the current study's innovation is examining the assessment of the mathematical error of a pupil with learning disability not only with regards to the fault in the cognitive functioning and the difficulties in the mathematical functioning, but also with regards to the use of conceptual and procedural mathematical knowledge of the pupil and the teacher. The teaching-learning process is mutual, dynamic, active, aware and directed, and so it contributes to the teacher's understanding of the total mathematical knowledge of the pupil.

Using Mathematical Errors of Pupils with Learning Disabilities as a Basis for Didactically Adapted Teaching Strategy

The research findings show that in every intervention program the preservice teachers chose varied and significant didactically adapted teaching strategies to correct the mathematical errors. In the first program, the preservice teachers chose a teaching strategy of a memory game, which includes exercises with numbers and exercises with shapes (circles). Through the game, the pupil got to know the symbols of addition and subtraction and their meaning. In the second program, the preservice teachers chose two teaching strategies: one was a model that illustrates the place value, and the other was a lottery game that assists the practice of the meaning of addition and subtraction. In the third program, the preservice teachers chose two teaching strategies, one was navigation cards that are used to guide and direct when necessary, and the other - a shopping game in a "shop" that is used to simulate reality. Through the shopping, the pupil thought logically about the problems and solved correctly. The programs show, that all choices of strategies were a combination of conceptual and procedural aspects. Also, the preservice teachers and the pupils dealt with the misconceptions during the activity by combining conceptual and procedural knowledge.

These findings support the claim made by Kilpatrick et al., (2001), that teaching mathematics to students with and without learning disabilities needs to emphasize conceptual understanding, strategic ability, adaptive thinking, productivity and procedural fluency. These findings find support from Hiebert and Carpenter (1992), who claimed that teaching the subject material while relying on understanding mathematical concepts alongside the required procedures, will allow students thought flexibility and adapting existing knowledge structures to new ones. The student who has conceptual understanding can connect between the different resources and individual cases, to calculate needed arithmetic calculations (Schoenfeld, 1985).

Fennema and Franke (1992) claimed that the wider the teacher's knowledge of students' ways of thinking, typical answers and possible errors, the teaching has higher quality. The findings support their claim and show that the pedagogical mathematical knowledge of the preservice teachers of the pupils' difficulties and the typical errors, and their curricular knowledge of how to use the errors, supported their successful choice of didactically adapted teaching strategy.

These findings contribute and widen the meaning of adapting teaching strategies to students with learning disabilities who have difficulties in math. So far, the emphasis in adapted teaching was on the student's cognitive processes, for improving achievements (Baker and Brown 1984; Wong 1996). In this study, however, the preservice teachers supported meta-cognitive processes more than cognitive processes: they taught the pupils to replace inefficient strategies with efficient ones, and so made the learning of the mathematical content more meaningful for them.

Additionally, the findings of the study support and indicate the importance of the suitability between analyzing the student's error and choosing a didactically adapted teaching strategy, in order to achieve a more effective focus on the mathematical difficulties and the misconceptions, which may contribute to improving the treatment's effectivity and fulfillment of the student's abilities to the maximum (Stecker & Fuchs, 2000).

The findings complete other research findings that show that American preservice teachers are relatively strong in classroom teaching strategies, and weak in knowledge recall and problem analysis. However, German preservice teachers show the opposite, and The Taiwanese have balance

(Konig Blomeke, Paine, Schmidt and Hsieh 2011). The preservice teachers probably have to rely on a space conceptual and procedural mathematical knowledge in order to be able to combine it in their knowledge and skills.

In summary, the current research provides new information indicating effective didactically adapted teaching strategies for pupils with learning disabilities who have difficulties in math. These teaching strategies include mathematical conceptual and procedural knowledge in a balanced way while directing the student's meta-cognitive processes. This kind of teaching strategies may help the learner understand and succeed in math and fulfil his potential.

Practical Implications

The research findings have implication for classroom teaching. The errors made by students with learning disabilities who have difficulties in math are common ones, significant and deserve a deep pedagogical review in the regular and special education classes. The intervention programs positively affect the achievements of the pupil with learning disability. Therefore, it is important to nurture the awareness of the combination of conceptual and procedural knowledge both at the student's diagnostic stage and the actual teaching.

The research findings suggest that it is recommended to plan long-term comprehensive intervention programs, more than the ones examined in this study, in order to develop different kinds of mathematical knowledge that children should know. The knowledge includes conceptual and procedural knowledge (Hiebert & Lefevre, 1986), and according to Fischbein (1987), it is formal, algorithmic and intuitive knowledge.

In addition, the findings have implications for teacher training. It is very important for preservice teachers specializing in special education to experience planning different intervention programs during their training for students with learning disabilities who have difficulties in math. The Education system doesn't tend to nurture these students (Mark-Zagdon, 2011). These intervention programs, apart from being planned by the individual program assessment model (Ministry of Education, 2016), need to emphasize combination and balance between conceptual and procedural knowledge.

Errors based teaching is focused on changing the misconceptions of a student significantly. Therefore, the preservice teachers must be taught how to analyze the errors, while assessing the student's ways of thinking. Then they should be taught how to choose an adapted teaching strategy that is directed to the student's difficulties and his strong areas.

In order for preservice teachers to use their mathematical conceptual and procedural knowledge together, both at their learning and their teaching, more opportunities for acquiring them are needed; this while acquiring teaching strategies adapted to students' needs. The teaching strategies are most important, therefore it is recommended to plan them relying on the strong basis of mathematical knowledge that combines conceptual and procedural knowledge.

Research Limitations and Recommendations for Future Research

The intervention programs in this study included only two lessons. Therefore, caution should be taken when examining the results. Nevertheless, it is recommended to perform a diagnosis within the program after the intervention and examine the observed changes.

The current study used only three interventions programs. In future research it is recommended to expand the number of programs in order to enforce the findings and the conclusions. Also, not every child who has difficulties in math has a learning disability. This study is a milestone for future studies, which could examine intervention programs planned by preservice teachers and teachers for students who have difficulties in math and compare them to intervention programs for students with developmental dyscalculia or other learning disabilities.

The current study is based on the approach which focuses on the teacher's occupation with the error, a common approach in teaching students with learning disabilities (Gersten et al., 2009). According to the literature, combining errors during teaching students with learning disabilities has not been studied. Therefore it is recommended for future research to examine different types of intervention programs that combine common errors during teaching, in comparison to control group.

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