European Journal of Science and Mathematics Education, 2024, **12**(2), 276-296
ISSN: 2301-251X (Online)

Research Article

Enhancing mathematics education through collaborative digital material design: Lessons from a national project

Edith Lindenbauer [1*]
0000-0002-0140-4344

Eva-Maria Infanger [2]
0000-0002-6378-985X

Zsolt Lavicza [2]
0000-0002-3701-5068

[1] Department of Mathematics Education, University College of Education Upper Austria, Linz, AUSTRIA
[2] Department for STEM Education, Linz School of Education, Johannes Kepler University Linz, Linz, AUSTRIA
[*] Corresponding author: edith.lindenbauer@ph-ooe.at

Citation: Lindenbauer, E., Infanger, E.-M., & Lavicza, Z. (2024). Enhancing mathematics education through collaborative digital material design: Lessons from a national project. *European Journal of Science and Mathematics Education, 12*(2), 276-296. https://doi.org/10.30935/scimath/14323

ARTICLE INFO

Received: 5 Oct 2023

Accepted: 1 Feb 2024

ABSTRACT

This article offers insights into a national-scale project aimed at developing and disseminating digital learning materials for mathematics education in Austrian lower secondary schools. The design-phase and context of the project outline the noteworthy aspect of this project, namely the close collaboration of a diverse group of experts, including technology-experienced educators, GeoGebra developers, proficient GeoGebra users, and researchers specializing in technology's role in mathematics education. This approach reveals the various needs and perspectives of all stakeholders for the designing process. To meet these needs the project design is utilizing three different research-related ideas, the didactic tetrahedron, the instrumental approach, and the didactical functionalities provided by digital technologies. We will present the resulting and constantly readjusted workflow and how such collaborative efforts ensure the quality of materials from different perspectives, aligning with best practices in technology integration in mathematics education. The comparison of five carefully selected materials for different learning scenarios brings out various possible technology-added values that can be achieved through collaboration. Selected qualitative methods such as thematic analysis of learning diaries, evaluation of a qualitative questionnaire and analyzing notes from the project team leader during the ongoing project let us extract diverse lessons learned in form of opportunities and drawbacks (e.g., discussions with experts, missing knowledge about GeoGebra). This project exemplifies potential for collaborative material design to enhance mathematics education at a wide scale, offering valuable lessons for similar endeavors in field.

Keywords: digital material design, technology integration, student participation, teacher participation, mathematics education

INTRODUCTION

FLINK in math (a German acronym for 'supporting learners through interactive materials for a sustainable acquisition of mathematical skill', which means 'nimble in math') is a pilot project situated at the Center for Open Digital Education (CODE) at Johannes Kepler University (JKU) in Linz. It supports mathematics teachers in integrating digital devices in mathematics classes at a national scale and is a reaction to the Austrian ministry's device initiative that equipped most 5th and 6th grade students with digital devices (laptops or tablets with a digital pen and keyboard) starting in autumn 2021 (Bundesministerium für Bildung, Wissenschaft und Forschung [Federal Ministry of Education, Science and Research], 2018). It aims, in a first phase, at providing lower secondary education mathematics teachers with qualitative open and research-informed digital materials and is utilizing the dynamic mathematics software (DMS) GeoGebra. The developed digital materials should not only enable learners to practice their mathematical skills but above all, focus on enhancing their conceptual understanding. The choice of software fell on GeoGebra because it is open source, widely employed in Austrian schools, and enables us to work in close contact with the GeoGebra Development Center.

The FLINK project brings together various actors and thus their expertise: (technology-) experienced teachers and teacher-educators, GeoGebra developers, experienced GeoGebra users, and researchers in the field of technology in mathematics education are involved. So far (October 2023) 781 digital materials for grade 5 and grade 6 have been published (new materials are published regularly). The team continuously involves about 15 to 20 persons participating in the material design process. The materials are 'distributed' to Austrian lower secondary schools through announcements in social media, national conferences, newsletters, and in-service teacher trainings. Moreover, publishers have already included the idea in current schoolbooks using QR links. In this paper, we share our experiences on designing this project and digital materials for an integration on a national scale and lessons learned for other research groups working in the field of mathematical task design and technology integration. This work is guided by the research question, how to implement the project's structure and workflow to be able to design open, high-quality digital materials for lower secondary mathematics education, which should focus on learners' conceptual development and provide technology-added value.

One important factor of integrating technology in mathematics classes addressed here particularly is the change of the teachers' role making the task at hand more complex (e.g., Clark-Wilson et al., 2014; Jacinto & Carreira, 2023; Rocha, 2023). Clark-Wilson et al. (2020) highlight recent research focusing on this teachers' role. Also, Drijvers (2015, p. 147-148) summarizes three essential factors for a successful technology integration, all related to teachers' activities: The design, the role of the teacher, and the educational context. Besides the design of the integrated technology and its characteristics and affordances, the first factor includes the design of lesson plans and teaching as well as tasks and included student activities. Secondly, for being able to successfully orchestrate learning with technology, a teacher's role must evolve, which additionally leads to a need for professional development in this field–a key issue also highlighted by Clark-Wilson et al. (2020). Finally, educational context considers mathematical practices as well as pedagogical opportunities outlined by Pierce and Stacey (2010) in their notion of pedagogical maps (Drijvers, 2015). Summarized, this essential role of teachers in digitalization initiatives is emphasized in the FLINK project, as it needs more of support than only providing open digital resources. This paper focuses on the first factor for successful technology integration outlined by Drijvers (2015): The design of tasks and student activities, in our case more specifically the design of digital materials, but always with the overall context in view. That is why all relevant actors in the school partnership are included in the design process.

Starting with design-phase and context of the project, we will briefly outline our research-related ideas. The second part includes the description of the integration of this open-source project, its processes, and research-informed procedures. Then, the theoretical underpinnings for the project as well as the planned research follow. Afterwards, we outline important mathematics educational considerations for digital materials based on a selection of digital materials and their specifications. Finally, the collection of opportunities and drawbacks out of this project can give similar projects a head start in this field.

THE FLINK PROJECT

The principal aim of the FLINK project is to offer teachers open, high-quality, digital materials for supporting them in the integration of digital devices in lower secondary education. These materials should further support learners in acquiring sustainable mathematical skills through enhancing their conceptual development. For a better understanding of our project, we now describe our team, the workflow, and main characteristics and structure of designed materials so that others can profit from our experiences and procedures when integrating similar projects. In addition, this section should highlight how we ensure quality of designed materials by including experts from various fields as well as through our workflow including several cycles of review and (re-)design.

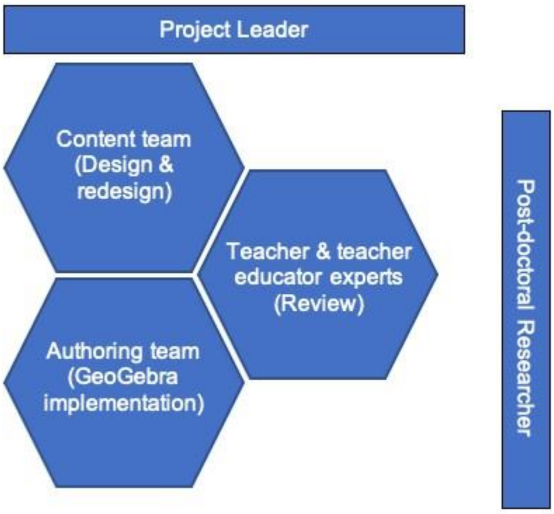


Figure 1. Project team (Source: Authors' own elaboration)

Project Team

The project, led by the head of the mathematics education department of the university, brings together experts in various fields (see **Figure 1**):

(i) starting with seven high-achieving students enrolled in our mathematics teacher training program (the so-called content team, participants henceforth labelled students or pre-service teachers),

(ii) five experienced teachers (regarding their experience in teaching generally and with GeoGebra) with at least eight years' experience in teaching in secondary education for each, who also work as

(iii) teacher educators (four of the five with at least 10 years' experience in working in pre- and in-service teacher training), and

(iv) a so-called 'authoring team': This team comprises seven experienced GeoGebra users (called authors) who actually implement the materials in GeoGebra and is led by a person (university degree in mathematics education as well as media technology and design) with six years' GeoGebra experience (including software development, managing the community team, managing the content team with a focus on creating high-quality GeoGebra materials and digital exam tasks).

For choosing the pre-service teachers, the four participating mathematics teacher educators pre-selected 18 high(er) achieving students (from 6th to 10th semester of their teaching program), seven of those accepted to be employed in this project on average 28 hours/week. The project is managed by a university assistant and teacher expert in the field of mathematics education. Furthermore, the project team works in close–personal and spatial–contact with the GeoGebra development center and is accompanied by a post-doctoral researcher in the field of mathematics education. Parallel, in the context of another project at JKU called MathSkill-Testing (https://www.jku.at/linz-school-of-education/forschung/mint-didaktik/mathskill-testing/), researchers and developers fashion different task formats for practicing skills that are also integrated in FLINK.

According to Koehler and Mishra's (2009) TPACK (technological, pedagogical, and content knowledge) model, teacher skills should encompass not only technological, pedagogical, and mathematical knowledge but especially skills on the respective interactions of these themes. TPACK integrates teachers' knowledge of challenges and changes in teaching when using technology, factors that make mathematical concepts easy or difficult to learn, ways to overcome learning difficulties by using technology, and the basis for a sound, meaningful teaching with technology. TPACK model thus covers a wide variety of factors relevant for integrating technologies successfully into teaching (Koehler & Mishra, 2009). In essence, this project profits from the various combining expertise–and proximity–of persons in the fields of mathematical knowledge for teaching, pedagogy, and technology for teaching mathematics as well as relationships among and between these fields, which constitutes all relevant components of TPACK (Koehler & Mishra, 2009) and thus, should ensure high quality of the designed digital materials.

Digital Materials: Basic Characteristics & Structure

A guiding objective of the material design is that each digital material provides technology-added value compared to paper-and-pencil tasks, which means exploiting functionalities offered by technology such as automated feedback, dynamic visualizations, task randomization, or more. For future materials, we also plan to examine the potential of technology for integrating more open-ended tasks, for instance, tasks focusing on modelling or problem-solving.

For structuring these materials, we follow the Austrian mathematics education curriculum for lower secondary education, consisting of four different content-related dimensions: working with

(i) numbers and measurements,

(ii) variables,

(iii) geometric figures and solids, and

(iv) models and statistics (Bundesministerium für Unterricht und Kunst [Federal Ministry of Education and Art], 2021).

For the structure of FLINK materials, the project team followed this categorization and started with dividing the topics of grade 5 curriculum (which corresponds to the first year of lower secondary education) into subtopics, each summarized into a GeoGebra book.

Following the didactical functionalities of technology further outlined later (see **Figure 5**), we decided to provide materials for developing concepts and practicing skills because the developed interactive materials are supposed to support students' learning of mathematics. A model used in German-speaking countries divides mathematics teaching situations into three phases for learning and two phases for evaluation:

(i) exploring, discovering, and inventing,

(ii) securing and systematizing,

(iii) practicing, connecting, and repeating (phases for students' learning),

(iv) diagnosing, and

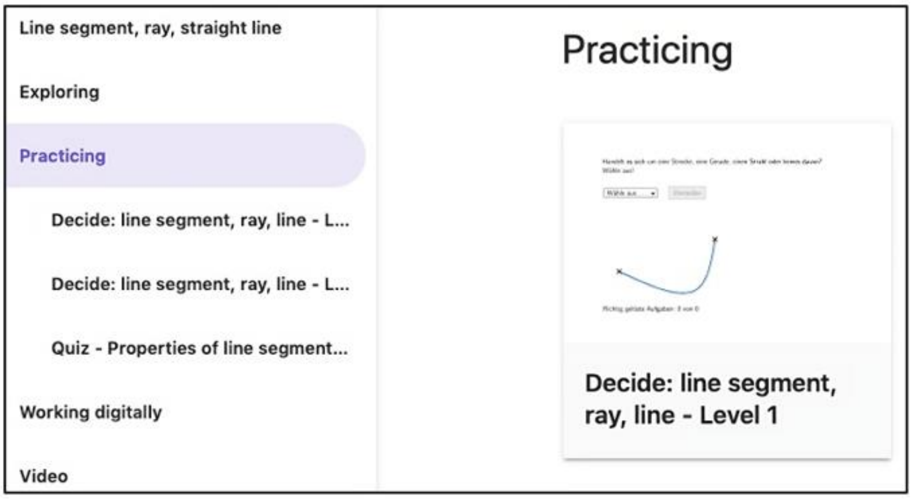
(v) assessing (situations for evaluating students) (Büchter & Leuders, 2009).

According to Büchter and Leuders (2009), materials and tasks implemented in mathematics classes should be designed based on their respective role in the teaching process regarding these phases. For our project and focus on learning mathematics, only the first three phases currently are relevant. We decided to synthesize the first and second phase under one theme (as mathematical activities and tasks for exploring and systematizing mathematical concepts are not always easy to distinguish), which results in two themes suitable to the didactical functionalities of technology: exploring mathematical concepts and practicing skills. Therefore, each subtopic consists of digital materials for *exploring* mathematical concepts, theorems, or algorithms ('exploring') and *practicing* skills, relating, and repeating mathematical topics ('practicing'). Additionally, some subtopics will provide videos summarizing the theory ('videos') as well as introductory tasks for learning how to utilize mathematical software applied in lower secondary education ('working digitally'). **Figure 2** visualizes this structure of one subtopic presented as a GeoGebra book.

Based on their learning goals, teachers can choose from this broad pool of digital materials provided on the FLINK platform and implement single digital materials, a book with subtopics or a whole chapter for self-directed learning suitable for classroom or homework.

Workflow

The workflow of the project team's duties is outlined in **Figure 3**. First, the students (content team members, usually working alone or in pairs) choose a particular subtopic of the curriculum (currently either grade 5 or grade 6) from a list prepared by the project leader. Afterwards, the work process is guided by a *checklist* and an accompanying *script* (Lindenbauer et al., 2021).



Line segment, ray, straight line

Exploring

Practicing

Decide: line segment, ray, line - L...

Decide: line segment, ray, line - L...

Quiz - Properties of line segment...

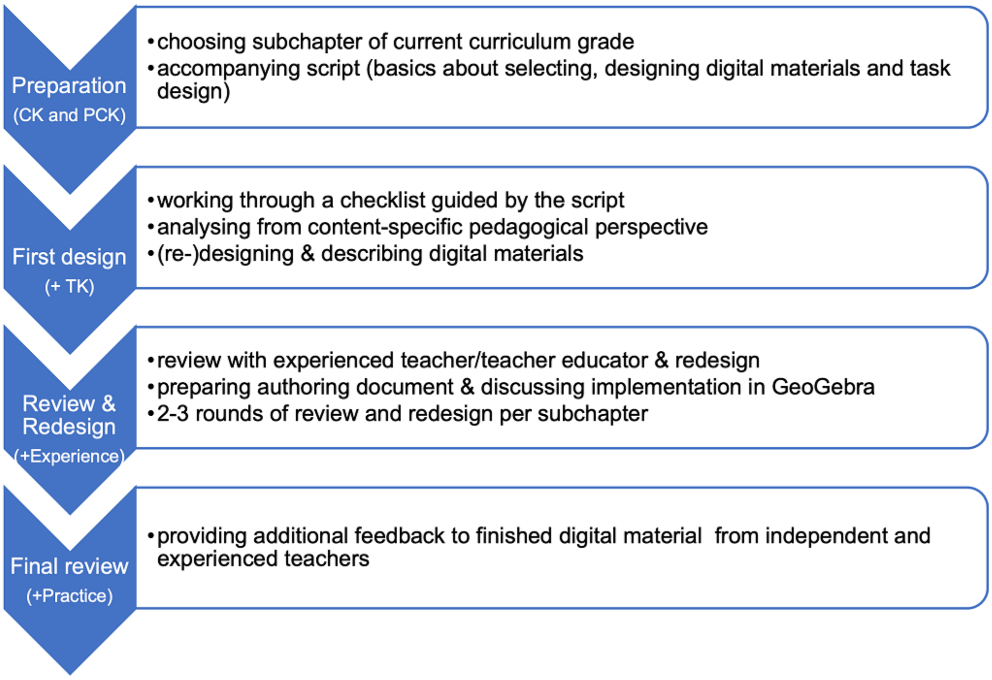
Working digitally

Video

Practicing

Decide: line segment, ray, line - Level 1

Figure 2. Exemplary GeoGebra book for one subtopic (translated version) (https://www.geogebra.org/m/bkajgrwu#chapter/688552)



- choosing subchapter of current curriculum grade
- accompanying script (basics about selecting, designing digital materials and task design)

- working through a checklist guided by the script
- analysing from content-specific pedagogical perspective
- (re-)designing & describing digital materials

- review with experienced teacher/teacher educator & redesign
- preparing authoring document & discussing implementation in GeoGebra
- 2-3 rounds of review and redesign per subchapter

- providing additional feedback to finished digital material from independent and experienced teachers

Figure 3. Workflow in FLINK project (Source: Authors' own elaboration)

Within the *script*, we summarized research-informed literature relevant for the design processes of those students working in the content team. Therefore, it contains a brief introduction in the project's aims and general information about the structure of materials as described above as well as language use (e.g., gender-neutral spelling and names, pupil-friendly language). The next section of the script provides an overview of the material design process and important considerations to be made in this context, which include:

(i) aspects to consider in a mathematics educational analysis of the chosen topic (e.g., curriculum, standards, literature review, visualization of concepts, typical errors, etc.),

(ii) detailed information on the structure of each GeoGebra book and selection of materials (e.g., license issues), and

(iii) mathematics educational background information on specific characteristics of mathematical knowledge (i.e., does a material cover a certain mathematical concept, a theorem, a procedure or algorithm, or is it about modelling or problem solving) and how to introduce it in teaching (Vollrath, 2001).

As the digital materials should exploit added digital value compared to traditional techniques, the script summarized information about the use of technology as outlined later (possible applications and methodical design). For supporting the students, the last part of the accompanying script contains recommendations and criteria for the design of digital materials, for example e-learning principles (Mayer, 2009) or guidelines for designing dynamic mathematics worksheets (Hohenwarter & Preiner, 2008).

After choosing the subtopic, a *checklist* structures students' first design processes; in essence, this form helps to summarize relevant information for material and task design. Firstly, the students should provide general information about the chosen topic (grade, subtopic). Secondly, they analyze their (sub-)topic from a content-specific pedagogical perspective as outlined in the script through studying the curriculum, schoolbooks, and additional mathematics educational literature in the field. Based on this analysis, learning goals for the subtopic should be formulated. In a third step, they either research a provided pool of already existing materials (the script offers some links to featured materials) and start to redesign them or they start to design new digital materials–for one subtopic usually several digital worksheets for 'Entdecken' (developing concepts) and 'Üben' (practicing skills). The students describe each designed material in the following way:

(i) the mathematical knowledge to be discovered or practiced (e.g., a concept, theorem, algorithm),

(ii) the learning goals for the specific material,

(iii) the presumed added value of technology integration,

(iv) a preliminary design and process description of the digital task,

(v) questions accompanying the digital materials, and–in case the students rely on already existing sources, and

(vi) reference and license information.

Finally, during their work students should also be aware of a consistent use of language and mathematical terms throughout the evolving materials.

In the third phase after finishing the checklist, students review their concepts with an experienced teacher and/or teacher educator from the project team, document the discussions, and additionally examine the order of materials within a subchapter. Afterwards, they integrate the feedback, redesign the digital materials, and prepare a so-called *authoring document* for the GeoGebra authors, which for each material suggests a detailed digital task design, a description of the underlying processes (e.g., what happens when clicking on a specific button), and accompanying question referring to the digital material. These suggestions will be discussed with and implemented by the authors. Subsequently, two to three rounds of review and redesign between students, teachers and teacher educators, and authors follow on average depending on evolving issues. For example, from a technical perspective authors cannot implement a design idea in GeoGebra and thus it must be redesigned, which also influences students' task design and requires a review of the content team with experts from a mathematics educational perspective. Furthermore, drafts of digital materials are often revised from a teaching perspective after first use and thus must be redesigned by the students. Due to these processes, students together with experts continuously evaluate digital content. According to Xie et al. (2017), this training of students lead to increased skills in the field of integrating technology in mathematics education and thus helps to ensure the quality of produced digital materials.

After publishing, the finished materials finally are reviewed again by experienced teachers independently from the project team following an open-ended questionnaire based on various quality criteria outlined in literature (e.g., Leacock & Nesbit, 2007; Trgalova & Jahn, 2013; Watson & Thompson, 2015). For organizing the workflow, the project team utilizes the software Trello (https://www.atlassian.com/software/trello). A Trello board allows the team to provide categories for structuring and describing tasks; in our case the subtopics (including digital materials, tasks, accompanying working and feedback notes) in their various stages of development (e.g., tasks for the content team, subtopics ready for 1st/2nd/3rd review, tasks for authoring team, material ready for final review, etc.). Summarized, this workflow should additionally enhance the quality of the designed digital materials.

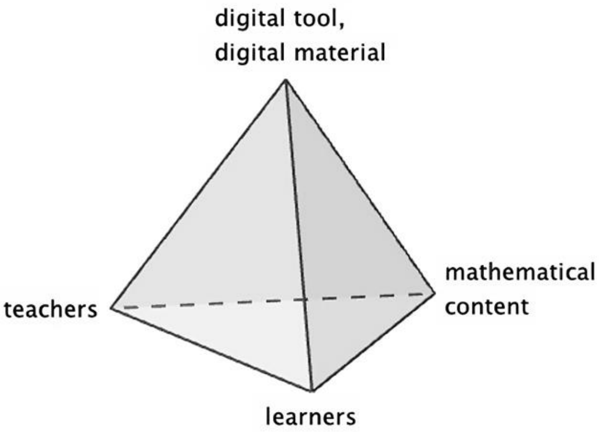


Figure 4. Didactic tetrahedron (adapted from Trgalová et al., 2018)

THEORETICAL BACKGROUND

We currently are in the project's design phase, where research in the context of FLINK is already planned and will be further conducted and reported in upcoming papers. The first part of this section deals with several theoretical perspectives that are relevant either for framing the project as a whole or for planned research about integrating digital materials in class. The first (didactical tetrahedron) should help to situate the project, and thus to understand its role, within the context of technology use in mathematics education. The others (instrumental and documentational approaches) constitute appropriate theoretical lenses for envisioning and researching the integration of the designed digital materials and lead to a more holistic view on the design process. Therefore, we provide a summary of these backgrounds and relate them to FLINK. The second part outlines research literature about technology integration that influence the concrete design of digital materials in FLINK project.

Theoretical Perspectives

Successfully implementing technology-based resources into teaching includes several actors and their interplays: teachers, students, technology, and mathematical content. These four components and their relations can be represented in a structured way via the *didactic tetrahedron* presented in **Figure 4** (Trgalova et al., 2018). Regarding mathematics teaching and learning, its four vertices represent the

(i) students who want to learn the mathematical content,

(ii) the mathematical content to be understood,

(iii) the teacher who wants to support learning processes, and

(iv) the digital tool or material with its mediating function between the other three agents, which in our context is represented by digital materials designed in FLINK project (Roth, 2019; Trgalova et al., 2018).

The base of the tetrahedron represents processes between learners, teachers, and the mathematical content, for example, how to design learning processes in a way that a mathematics lesson is as effective as possible considering reaching its goals. It represents the conventional teaching; however, technology adds another level of complexity. The rear surface can be interpreted in two ways: On the one hand it can represent a process in which teachers deal with mathematical contents and integrate digital tools or materials in his/her problem-solving process; on the other hand, it represents processes of teachers preparing mathematical content for lessons integrating digital tools or materials. The right-hand side of the didactical tetrahedron represents among others that students engage themselves independently with a specific mathematical content using digital tools, a process in which the material or tool ideally evolves, in the sense of instrumental genesis, into a personal instrument (see next paragraph). Finally, the left-hand side focuses on the interplay between teachers, learners, and digital tools/materials representing from teachers' perspectives how they can support students in the instrumentation and instrumentalization processes or how the tool mediates communication processes between learners and teachers (Roth, 2019). From the perspective of designing and creating digital materials in FLINK project, especially the rear side of the presented tetrahedron (teachers–mathematical content–digital tool/material) is relevant in the sense that the project members take teachers' perspectives when considering how to design digital materials discussing a mathematical content. In addition, the learners–mathematical content–digital materials area calls for a change of perspective, which enriches the design process.

The following two theoretical backgrounds are related and provide a fruitful view on learners' and teachers' implementation of digital materials. The instrumental approach offers a lens to examine the implementation of these materials from the learners' perspectives. An instrument is a combination of an artifact and *utilization* scheme–a psychological construct created by the subject when utilizing the artifact to execute a type of tasks. The process of becoming an instrument through interaction of the subject with the artifact is called *instrumental genesis* (Artigue, 2002; Trouche, 2004). The utilization schemes can have an *epistemic value* (help the subject to understand something, for example, gain insight about solutions of linear equations) or a *pragmatic value* (help the subject to do something, for example, use a calculator to solve a linear equation) (Artigue, 2002; Trouche, 2004; Verillon & Rabardel, 1995). Continuing the instrumental approach, *instrumental orchestration* further considers the didactic design and classroom activities for integrating technology into classroom practice. The instrumental approach is relevant for learners' perspectives on digital materials and thus, covers the relation between learner and digital tool of the didactical tetrahedron. When utilizing these materials in school, the digital materials developed in this project should evolve to individual instruments and the learners' corresponding utilization schemes should support them in understanding mathematical content (a third part of the didactical tetrahedron) and thus provide an epistemic value.

Similar to instrumentation theory, the *documentational approach* is relevant for our future project-related research about implementing these materials from teachers' perspectives because it can reveal a deeper insight into individual teacher's use of resources and resource system (Clark-Wilson et al., 2020). It is a theoretical evolution of instrumental genesis and involves a broad range of different–digital but also non-digital–resources, for example, Internet resources, textbooks, or interactive worksheets. A document is created from a resource or a set of resources during a process called *documentational genesis*, and it consists of resources and utilization schemes developed by the teacher. The documents evolving during documentational genesis are organized in a so-called documentation system, which comprises of all documents developed by a teacher. Furthermore, documentational approach focuses on teachers' knowledge and practices outside the classroom (e.g., planning, evaluating, writing) and therefore provides a theoretical framework to analyze teachers' professional development (Gueudet & Trouche, 2009; Gueudet et al., 2014).

From a documentational perspective it is interesting to research, if and how teachers integrate our digital materials into their lessons and their documentation system and what we can–regarding design–do, to support them in this process. Such research is planned within a next phase of this project. The documentational approach relates to the connection between teacher and digital tool of the didactical tetrahedron; furthermore, the other two parts (mathematical content and learners, see **Figure 4**) cover the mathematical and didactical components of resources. Finally, also the process of designing materials in FLINK project in which content-team members utilize various resources (e.g., schoolbooks, script, design guidelines, etc.) can be viewed under the lens of the documentational approach. The next section focuses on the specifics on technology use relevant for our project.

Technology Integration

The next framework highlights the underlying basic structure of our digital materials (for exploring and practicing) outlined before. For mathematics education, Drijvers et al. (2011) distinguish three didactical functionalities of digital technology: doing mathematics, practicing skills, and developing concepts (see **Figure 5**). From users' perspective, technology can serve as a tool for doing mathematics, for example, outsourcing the solving of an equation to a digital assistant thus being able to focus on an underlying problem. From the learners' perspective, technology can serve as an environment for practicing skills (e.g., a digital tool providing feedback to student's solutions) or for conceptual development. In the latter case, technology should support students in gaining understanding of mathematical concepts, for example, by connecting representations dynamically. These three functions are not mutually exclusive (Arcavi et al., 2017; Drijvers et al., 2011).

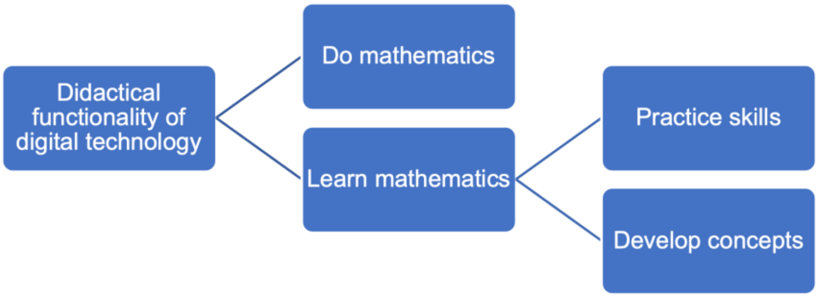


Figure 5. Didactical functionalities of digital technologies in mathematics education (adapted from Drijvers, 2018)

According to Drijvers (2018), features such as randomization of tasks or automated and immediate feedback support practicing of skills and are easier to realize than providing an environment that enables developing concepts, which is a challenging didactical functionality to exploit. The potential of technology particularly lies in utilizing digital materials to discover, develop, and explore ideas (Ball & Stacey, 2019). For this reason, we want to emphasize not only fostering procedural skills but also conceptual mathematical understanding.

The digital materials designed in our project relate to the Austrian mathematics curriculum of lower secondary education and should support pupils' learning. Therefore, we focus on materials for learning mathematics and thus create materials that either concentrate on developing concepts or practicing skills, although materials for practicing skills should also foster conceptual understanding. In terms of instrumentation theory, we essentially privilege the epistemic value of students' instrumentation schemes–the digital materials should help students to understand mathematical content.

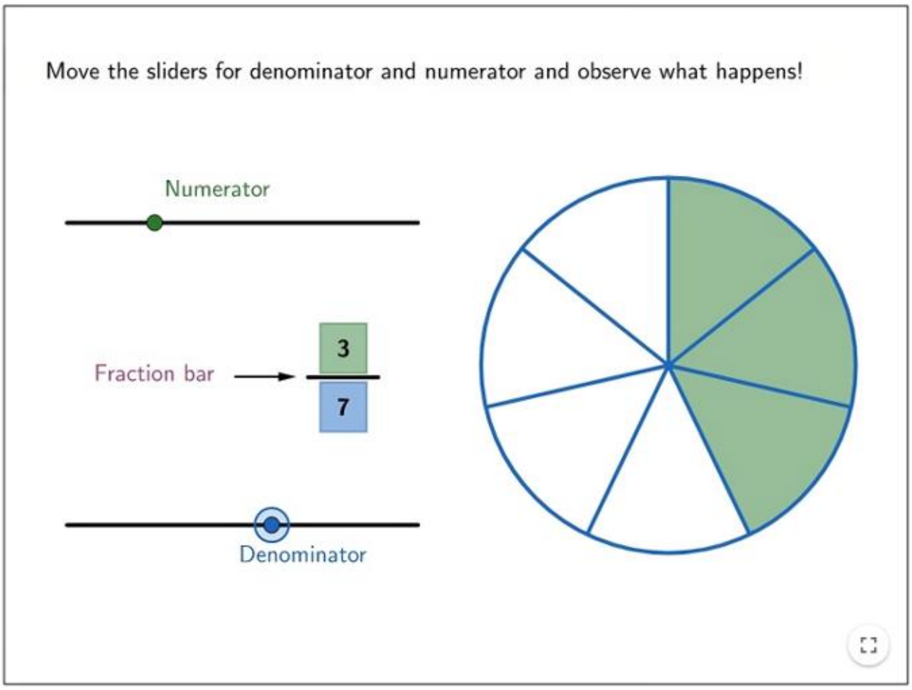
For FLINK project we utilize GeoGebra, an open-source software for educational purposes, because GeoGebra is widely employed in Austrian schools and it enables us to draw on expertise in this field, as GeoGebra is situated at JKU in Linz. DMS such as GeoGebra combines geometry, algebra, spreadsheets, calculus, and statistics and thus allows, for example, to investigate mathematical objects, to support conceptualization of mathematical concepts, or to address mathematical problems by providing different, dynamically linked representations (GeoGebra, 2022; Hohenwarter & Jones, 2007). Misfeldt (2011) describes the potential of DMS with regard to Duval's (2006) framework about semiotic registers by its feature to relate simultaneously different semiotic representations that provides a cognitively different approach compared to static representations in a paper-and-pencil environment. In addition, GeoGebra provides *dynamic control* of objects, described by Kieran and Yerushalmy (2004), as follows:

> "Dynamic control involves the direct manipulation of an object or a representation of a mathematical object … Dynamic control can be achieved by means of several devices, for example, slider graphs, sliders, dragging facilities, and so on" (p. 120).

The mentioned devices (sliders, dragging facilities) also available in GeoGebra enable users to explore and examine invariants of mathematical objects (Falcade et al., 2007). These characteristics of DMS are relevant for pursuing our goal at supporting teachers with digital materials that provide technology-added value.

Regarding technology-added value for our materials, literature describes an abundance of options for utilizing DMS in mathematics education, for experimenting, as means of communication, as heuristic or modelling tool (Roth, 2017). Furthermore, technology use can add value compared to traditional tools to developing concepts, visualizing dynamically (multiple) representations, practicing independently and more (e.g., Roth, 2017, 2019). For designing digital materials, we integrated information about these specifics of technology use and added value in our script written for the content-team members as mentioned before.

A further design decision for our digital materials relates to the right-hand side of the didactical tetrahedron (see **Figure 4**), which represents among others that students engage themselves independently with a specific mathematical content using digital tools (Roth, 2019). When designing such engagement, Roth (2017) distinguishes three levels of support for working with DMS:



Move the sliders for denominator and numerator and observe what happens!

Figure 6. Digital material: Denominator, numerator, fraction bar (https://www.geogebra.org/m/r4ryzr2q)

(i) a configuration is completely given, possibly elements can be shown or hidden, and students have pre-designed possibilities for variation (pre-constructed digital material),

(ii) a partial configuration is presented, or

(iii) an empty GeoGebra file is used.

Due to the age of the pupils first equipped with technology in Austria (grade 5) and their limited experience with GeoGebra, we focus on designing pre-constructed digital materials. In addition, introductory tasks in partially configured digital materials should support pupils in learning to utilize the software itself by making them acquainted with specific features or commands, currently especially in the context of geometric topics (e.g., https://www.geogebra.org/m/bkajgrwu#chapter/688553).

So far, we have outlined theoretical constructs relevant for the structure of FLINK materials and information about potentials of DMS for designing digital materials. The next section provides a description of main characteristics as well as concrete examples of digital materials.

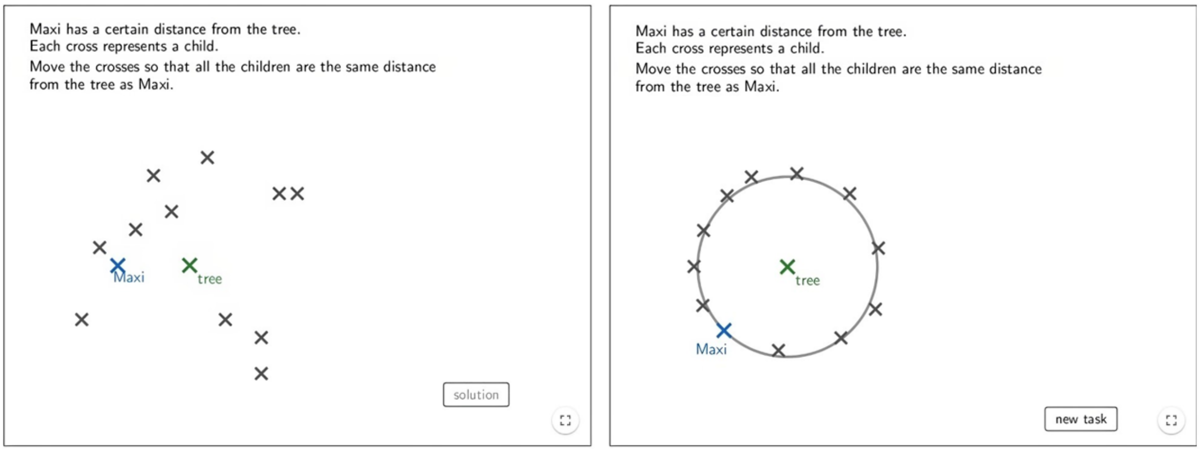
MATERIALS

As described before, digital materials developed in FLINK project are mainly assigned to two areas: digital tasks for *exploring* ('Entdecken') or *practicing* ('Üben'). For sharing our considerations and experiences about how to design materials with technology-added value, we now describe characteristics of these categories as well as exemplary materials in more detail. All currently published materials can be found at https://www.jku.at/flink-in-mathe/.

Exploring

This exemplary GeoGebra book discusses *fractions as part of a whole* (https://www.geogebra.org/m/pge8d4x3). It covers a first introduction into the concept of fraction and consists of six interactive digital worksheets for exploring this concept. With the first material, pupils should explore the concept of fraction as part of a whole or as any number of equal parts starting with unit fractions. The second material "denominator, numerator, fraction bar" (see **Figure 6**, translated into English) introduces and visualizes the terms denominator, numerator, and fraction bar.

This digital task visualizes the whole as a circle and parts of the whole as congruent, green colored sectors. Pupils should change the numerator (corresponding objects colored green) or the denominator of the fraction



Maxi has a certain distance from the tree.
Each cross represents a child.
Move the crosses so that all the children are the same distance from the tree as Maxi.

Maxi has a certain distance from the tree.
Each cross represents a child.
Move the crosses so that all the children are the same distance from the tree as Maxi.

Figure 7. Digital material: Maxi & tree (https://www.geogebra.org/m/pu36wutb)

(related objects colored blue) via sliders and observe how linked objects change. The sliders allow dynamic control over numerator and denominator. Depending on the numerical value of the denominator, pupils can vary the numerator from zero to the maximum value of the denominator. The authors defined the sliders in a way that varying the value of the denominator does not automatically change the numerator and thus the corresponding slider (except for representing fraction with value one). As outlined before, this material adds digital value by utilizing three dynamically linked representations for fractions: a slider, a numerical representation, and a visual representation in which related terms are colored similarly to highlight the related representations and thus considering the contiguity principle (Mayer, 2009). The following accompanying questions should guide the learners:

(i) Describe what happens when you increase (or decrease) the denominator?

(ii) Set numerator and denominator to the same number. What can you observe?

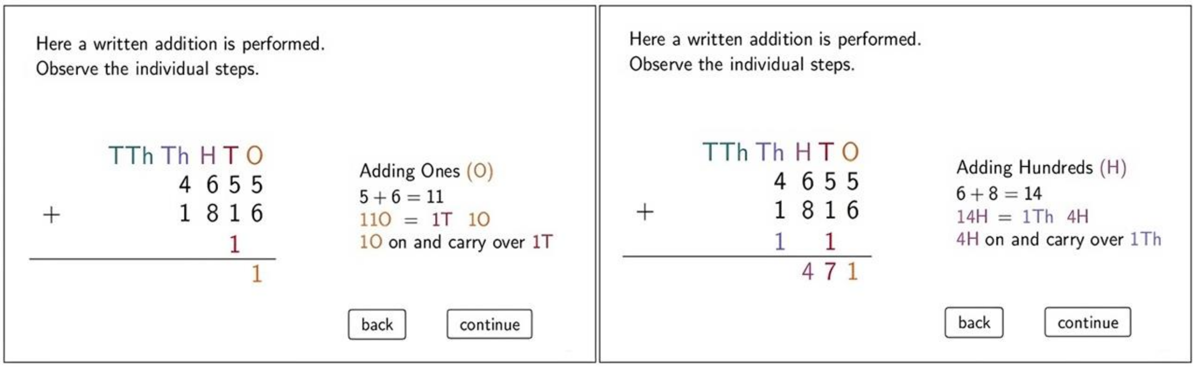
(iii) What happens if you change the denominator, but the numerator stays zero?

These questions relate to the digital material and should support pupils in connecting representation, discovering characteristics of the presented concepts, and to think about special cases. Through experimenting the learners should discover the effects of changing the numerator and denominator on the visualization and thus conclude to the mathematical meaning of these concepts. Utilizing DMS in this context aims at supporting learners in forming a base for understanding the fraction concept and in building the 'Grundvorstellung' (mental model) of a fraction as part of a whole.

Another GeoGebra book covers a geometric topic: *An introduction to circles* (https://www.geogebra.org/m/a4pppe7a). The part for exploring this new concept consists of three dynamic worksheets: circles in everyday life (for starting concept building with students' experiences), creating a circle, and characteristics of a circle (for learning relevant terms such as center, radius, or diameter).

Figure 7 presents the second material about creating a circle. On the left-hand side it visualizes several crosses: a green one represents a tree, a blue one a person called Maxi, and the grey ones illustrate various children. Learners should move the grey crosses in a way that all of them have the same distance from the 'tree' as Maxi. This material is an iconic representation of a corresponding activity that teachers could perform in class and aims at discovering a circle as a concept based on its defining characteristic: A shape consisting of all points in the plane with a certain given distance from a given point (the center). After at least moving one grey cross, pupils can push the button 'solution' and a figure corresponding to that on the right-hand side in **Figure 7** appears. Afterwards, the material provides the option for a new task. The accompanying question below the GeoGebra applet refers to the visualized task and asks for the common characteristic of all points on a circular line. From an educational perspective, this digital material enables independent discovery of a concept by focusing on its defining property and using dynamic iconic representations, it should support developing the concept of circle, and it provides dynamic control over presented objects.

So far, we have presented two digital materials from different content-related dimensions: geometry and numbers and measurements. Both materials focus on discovering and exploring a mathematical concept, but


Here a written addition is performed.
Observe the individual steps.

TTh Th H T O
4 6 5 5
+ 1 8 1 6
1
1

Adding Ones (O)
5 + 6 = 11
11O = 1T 1O
1O on and carry over 1T

Here a written addition is performed.
Observe the individual steps.

TTh Th H T O
4 6 5 5
+ 1 8 1 6
1 1
4 7 1

Adding Hundreds (H)
6 + 8 = 14
14H = 1Th 4H
4H on and carry over 1Th

Figure 8. Digital material: Written addition (https://www.geogebra.org/m/jcfu5rsj)

how to approach new algorithms or procedures that cannot be discovered (in the strict sense of the word)? According to Vollrath (2001), it does not suffice only to master a procedure–the goal is to understand it. In addition to applying algorithms to specific tasks, understanding also includes knowing what is achieved with a procedure, how it works, under what conditions it works, and why it works (Vollrath, 2001).

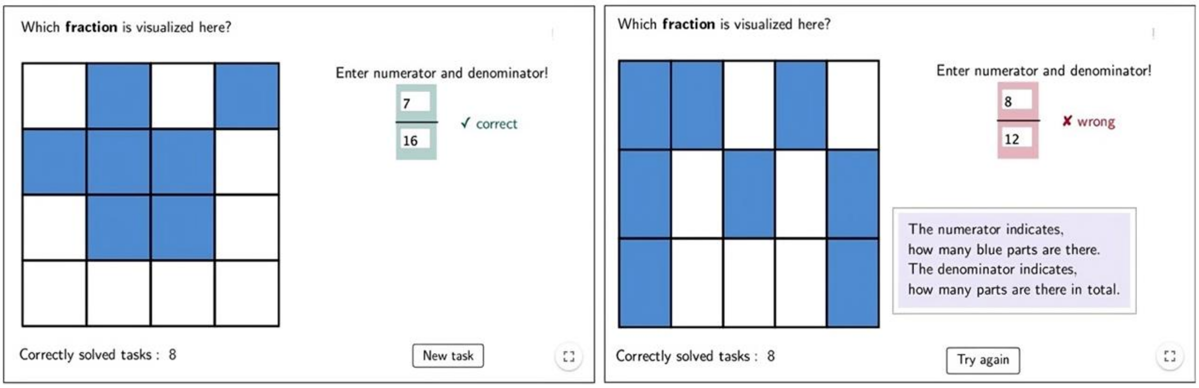
Figure 8 visualizes a digital worksheet about the standard algorithm for written addition of natural numbers that should explain how and why this algorithm works. The left-hand side of the digital material presents a written addition of two four-digit natural numbers. Above the first number, the value of each digit is highlighted via colored abbreviations of the place value (e.g., O stands for *ones*, T for *tens*). By pushing the button 'continue', the material reveals the standard algorithm step by step, and students should observe each step of the procedure. On the right-hand side of the material (see left rectangle of **Figure 8**), the current calculation is explained in terms of the decimal place value system. For example, five ones and six ones are 11 ones, which results in one ten and one 1. The one one is written in the final line, the one ten must be carried over to the tens (five and one) of both given numbers. The right rectangle in **Figure 8** presents a further step for adding hundreds: Six and eight hundred results in 14 hundred, which corresponds to one thousand and four hundred; four hundred are the results for the final line and one thousand is transferred for the next (and final) step. Step by step explanations and color coding for certain place values should support learners in learning and understanding this algorithm. Furthermore, the accompanying questions aim at reflecting the algorithm. Regarding the added value of technology, the material provides interactivity (back and continue buttons) that allows user control and thus enables learning at one's own pace, and it aims at communicating and reasoning specific mathematical content.

Based on these ideas, *exploring* sections of published GeoGebra books contain digital materials for discovering, exploring, and experimenting with new mathematical concepts, theorems, or algorithms. Accompanying questions refer to the digital tasks are intended to support learners' exploration and consolidation of new ideas and concepts. The next section outlines characteristics and specific examples of digital materials for practicing skills.

Practicing

The second part within each published GeoGebra books contains digital materials aiming at practicing skills and connecting mathematical ideas. In this first project phase, they mainly include closed mathematical task formats that are developed in the concurrent project MathSkill-Testing because closed tasks rather enable various possibilities of technology-added value compared to open-ended ones. These task formats involve input fields, multiple or single choice questions, drag and drop features, dropdown tasks, and assignment tasks. Furthermore, we designed digital materials with gamification.

One material is part of the above-mentioned collection that covers materials about *fractions as part of a whole.* The fourth applet 'Which fraction is visualized here?' is presented in **Figure 9**: Within a rectangle-shaped representative of number one, randomly several equally sized parts are shaded blue. Pupils should identify the fraction represented by the (non-)colored (both is correct) parts and put numerator and denominator into an input field. Through practicing with this material, pupils should deepen their understanding of the concept of fraction as part of a whole as well as the understanding of the meaning of denominator and numerator.



Which **fraction** is visualized here?

Enter numerator and denominator!

7 / 16 ✓ correct

Correctly solved tasks : 8

Which **fraction** is visualized here?

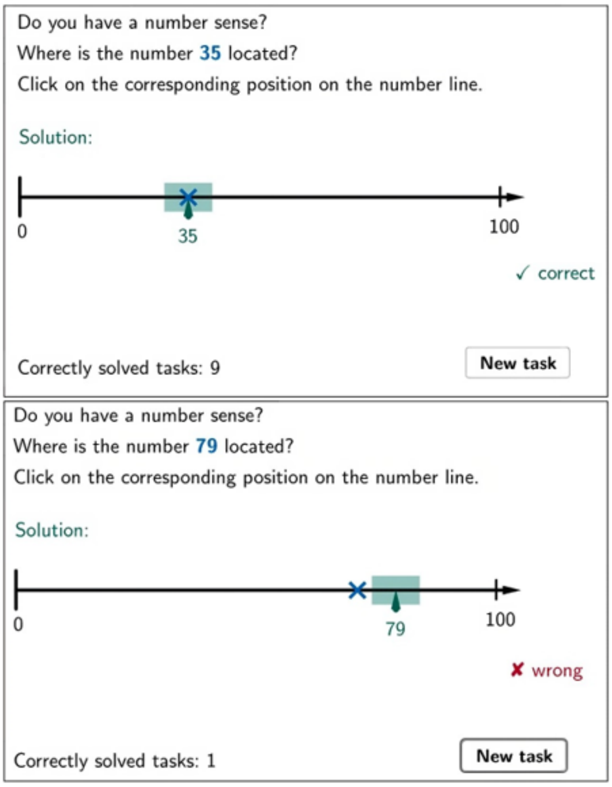
Enter numerator and denominator!

8 / 12 ✗ wrong

The numerator indicates,
how many blue parts are there.
The denominator indicates,
how many parts are there in total.

Correctly solved tasks : 8

Figure 9. Digital material: Which fraction is visualized here? (https://www.geogebra.org/m/befyvnma)



Do you have a number sense?
Where is the number **35** located?
Click on the corresponding position on the number line.

Solution:

✓ correct

Correctly solved tasks: 9

Do you have a number sense?
Where is the number **79** located?
Click on the corresponding position on the number line.

Solution:

✗ wrong

Correctly solved tasks: 1

Figure 10. Digital material: Number sense (https://www.geogebra.org/m/abxaff9k)

If the solution is correct (left of **Figure 9**), they receive feedback ‘correct’ and can choose a next task by pushing a button ‘new task’. In case the solution proves incorrect, learners get corresponding feedback ‘wrong’ and they can give it another try (‘try again’). In addition, the digital material provides a hint in case of a second incorrect answer about the meaning of numerator and denominator with respect to the presented visualization (**Figure 9** right side). Furthermore, a counter displays the number of already correctly solved tasks.

Figure 10 presents the task ‘number sense’: Given a random natural number between 0 and 100, students must estimate where to place it on a number line. They receive feedback (either a ‘correct’ or a ‘wrong’ and a green box on the number line for an approximately correct estimation). Then they can choose the next task by pushing the button ‘new task’. As previously, a counter displays the number of already correctly solved tasks.

Features of digital materials for practicing include immediate feedback on learners’ solutions on whether their answers are correct or not, displaying hints and solutions or solutions paths, providing new randomized tasks at pushing a button, and counters of correctly solved tasks. Task formats developed in project MathSkill-

Testing integrate research results on feedback in e-learning environments, for example, the following features suggested by Narciss and Huth (2006):

(i) such materials provide feedback only after learners have actually attempted to solve the task,

(ii) they give learners the possibility to try again after a first false response without providing more information on the solution process, and

(iii) they provide a more detailed hint after a second try.

Especially options concerning immediate automated feedback, hints, and solution (paths) options are considered to support pupils learning processes (Attali & van der Kleij, 2017; Fyfe & Rittle-Johnson, 2016). As mentioned before, these materials allow learners to practice independently and thus add digital value.

LESSONS LEARNED

After describing the project and materials, this section highlights various lessons learned and preliminary research results that are relevant for this project's procedures. We concluded these from data gathered within the first two months of FLINK project from learning diaries and a qualitative questionnaire from our content-team members within an exploratory qualitative approach. Furthermore, data included notes from the project team leader. The learning diaries were kept due to a parallel design-based research project for developing a teacher training course focusing on pre-service teachers' professional knowledge concerning technology integration (Lindenbauer et al., 2022) and included feedback on helpful factors as well as what additional kind of support they needed. For improving the procedures of material development, we additionally provided a brief questionnaire for the students of the content team addressing the processes and provided documents with questions inspired by SWOT (strengths, weaknesses, opportunities, threats) analysis such as: What features are helpful and should be kept? What is difficult and why? What could be improved? (Reinbacher, 2009). Data was analyzed following thematic coding procedures (Braun & Clarke, 2012) using the software MAXQDA. Emerging themes are outlined in this section and mainly cover organizational issues; additionally, some categories are related to mathematics educational aspects.

Supportive Features

As visualized in **Figure 4**, the processes of the project are represented by the rear side of the didactical tetrahedron connecting teachers, mathematical contents, and digital materials, which can be interpreted as the process of teachers' preparing mathematical content through integrating digital materials or tools. In our case, several actors take the teacher's role in the tetrahedron who work closely together: the content-team members (students designing the digital materials), the experts (teachers and teacher educators), and the authoring team members (implementing designs with GeoGebra) (see **Figure 1**). Organizational issues thus involve possible relationships between these persons.

All participating students emphasized the importance of discussions with experienced teachers and teacher educators, for example, Ida[1] commented after two weeks' experience:

> Ida: I think it's especially good that you get support from experienced teachers and other project staff right from the start.

Furthermore, students highlighted these meetings for specific mathematics educational issues:

> Julia: [Helpful are] mathematics educational discussions with teachers, [as] one becomes more aware of possible difficulties [of learners].

However, experiences revealed that early expert feedback on design ideas–that is teacher support from the beginning of the design process–is advisable; otherwise, designers and authors as well would invest more time and effort in creating drafts of digital materials than necessary because usually the material designs must be adjusted or even started anew after expert discussions.

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[1] All student names are pseudonyms to preserve anonymity.

Similarly, students reported positively regular contact possibilities with GeoGebra authors such as face-to-face discussions (compared to only formulating instructions for digital materials on the authoring documents as personal discussions are often done more quickly) as well as ongoing communication during design and implementation of the digital materials (e.g., for further inquiries or editing requests).

In addition, Ida and Laura exemplary highlighted following supportive features:

> Laura: … sitting in an office together with 'like-minded people', exchanging ideas with one another, being able to ask short questions at any time, sometimes a brief thought-provoking impulse from a colleague is enough.
>
> Ida: It is helpful to discuss different thoughts, perspectives, and experiences on various topics.

Concerning the participating actors, this and similar frequent comments draw our attention to how students supported each other: (i) by working in teams of two and (ii) by working in local proximity (i.e., sitting in the same office) that allowed for further content-related exchange, different student perspectives, and especially quick feedback among each other.

Besides the participating actors, students mentioned several resources and their features helpful for designing materials: The provided checklist and script, provided schoolbooks, literature about mathematics education and content from previous university courses in this field, and already existing digital materials. Students mentioned that the script created for this project is generally supportive; especially as it provides summarized information on mathematics educational topics and technology integration relevant in this field as well as references for design ideas. Accompanying to the script, the checklist guides students through the design process through 'small-step instructions and questions' (written comment of Christina in her first week). From the perspective of documentational approach, on the one hand it will be interesting to analyse if and how teachers will integrate the developed digital materials within their documentational system. On the other hand, the documentational approach can also be applied to the students who work in this project: At the beginning, they familiarized with the different relevant resources, especially the script, the checklist, and the authoring document for organizing their work. Additionally, the students reviewed relevant literature, schoolbooks, and already existing GeoGebra applets. Over the first weeks, they gained confidence in organizing these various resources by developing schemes for utilizing them and thus this process can be interpreted in a broader sense as documentational genesis for designing digital materials in this project.

Problems & Improvements

Basically, the project's structure and processes support the participating team members in developing digital materials for secondary school teachers in a research-informed way. During the first phase, several suggestions for improving these processes emerged–either from data or additional suggestions from the team members (mainly the project leader and content or authoring team members) based on their current participation and experiences in the project.

From the beginning, we already planned two to three review rounds with experts (teachers and/or teacher educators) and students highlighted the value of these reviews. However, data analysis revealed the need for a more structured implementation of these review processes as well as the necessity of discussing their ideas with more than one expert teacher or teacher educator in advance. For example, Ida commented in this context:

> Ida: It would be useful, if the checklists were discussed with more than one teacher before they are posted on Trello [i.e., before being implemented]. If considerations would be discussed in detail right at the beginning and their meaningfulness with respect to mathematics education as well, a lot of time could be saved in the work process. The authors would only have to implement digital materials, which then would in fact be published.

To ensure continuity, we thus integrated more internal review loops: momentarily, at least twice the same expert (discussing once the first concept and once an intermediate concept) and then–depending on experts' time resources–the same or another expert reviews the first final version of a GeoGebra book. In any case, at least two different experts review the materials and finally, the project leader must approve before online publication. This process results in total in at least four review discussions. This procedure ensures two discussions before digital implementation on mathematics educational issues and additionally considers at least three different opinions of experts, who usually focus on different issues depending on their specific experiences and expertise.

Another category covers collaboration with authors (i.e., GeoGebra experts). As Ida mentioned in the previous comment, the first project phase encompassed difficulties regarding communication and collaboration with authors. She–and similarly Christina–further noted:

> Ida: It would be good if we already had a contact person from the author team with whom we could discuss our ideas when creating the concepts of digital materials. This could save time and prevent implementation problems with the author team. It would also give us a better idea of what can be implemented by the team with GeoGebra.

> Christina: At the beginning, it would have been helpful for me to be briefly introduced …, which ideas could be implemented with GeoGebra by the authoring team, and which could be not, or what is particularly cumbersome to do and thus should better be avoided if possible.

In essence, first the content team members–and partly the experts–could not assess whether a concept could technically be implemented with GeoGebra or not. This led to repeated adjustment loops between students and experts (from a mathematics educational perspective) and students and authors (from a technical perspective). On the one hand, for improving discourses with GeoGebra authors we implemented weekly team meetings, and each team nominated a spokesperson who exchange latest information after their respective team meetings and pass on this information to their respective colleagues. On the other hand, the project leader encouraged ongoing regular discussions–even on daily basis if necessary–between students who designed a particular material and the author responsible for implementation for upcoming questions or feedback (e.g., feedback in case a design idea could not be implemented). In addition, this problem became less relevant over time as the students' knowledge in this field increased. A weekly common meeting between authors and designers did not prove effective; on the one hand the details of specific materials were not relevant for all participants, and on the other hand it did not take place often enough for an effective design and implementation process. For further fostering collaboration, we started a separate channel on Slack for all project team members, a specific workplace communication tool (https://slack.com/intl/de-at/).

Data analysis further revealed team members' problems with design considerations. Regarding the didactic tetrahedron (see **Figure 4**), these issues relate to the connection between teachers (in our case represented by all project team members creating the digital materials), digital materials, and the mathematical content as design considerations also depend on what is being presented. In the beginning, the leader of the authoring team provided a guideline for GeoGebra for communicating agreements on design within GeoGebra-based digital materials (e.g., specifications for color use; design of elements such as sliders, texts, or points; technical commands, dos, and do nots, etc.). As the digital materials evolved, however, various additional questions concerning the design arose that needed to be discussed for consistent appearance of and processes within materials. For instance, Flora commented:

> Flora: Sometimes it is not exactly clear how the applets should look like: For example, should there be a number field for each input field (such as on a calculator) for students working on a tablet? Or do all pupils have a keyboard as default? Should we insert a drawing field for secondary calculations? … It would be good if such specifications were verbalized clearly for authors and content team members.

Related issues deal with processes within digital materials for practicing, for example, how often pupils are allowed to try a task, or when and how digital materials should provide hints or solutions. Now, the content team members collect such questions and discuss them in the weekly meetings. They collect relevant notes on agreements concerning these issues within an online folder accessible for the project team. In case of closed tasks for practicing, they integrated digital task formats developed within the concurrent project MathSkill-Testing based on research results concerning feedback and computer-based assessment.

Finally, besides categories concerning mathematics educational issues, which we will not outline here in more detail (e.g., how to formulate hints for learners, how to assess difficulties of tasks), an important lesson learned concerns expressing learning goals. As the design issues discussed in the previous paragraph, this learned lesson also relate to the teacher–digital material–mathematical content side of the didactic tetrahedron in **Figure 4**. Frequently during the expert discussions with students, the issue arose that a digital material looked interesting and well-designed but from a mathematics educational perspective it missed a clear focus and thus the purpose was not tangible. Therefore, highlighting the importance of formulating explicit and precise learning objectives for the digital materials became a repeated and important issue–in the weekly meetings as well as in meetings with experts.

DISCUSSION & OUTLOOK

The implemented project FLINK aims at developing digital open-source materials for mathematics education and thus supporting teachers in Austrian lower secondary schools in the ongoing government's policy of equipping each learner with a digital device. These interactive materials should provide added value for teaching mathematics in the digital age and support students in their conceptual development. Creating such materials is a complex task that requires focus on various aspects, for example, mathematical exactness, content-specific pedagogical considerations, visualization, wording of tasks, layout, and presentation. In essence, these aspects represent quality issues for technology integration similar to those outlined by Trgalova and Jahn (2013). However, the location of the project at JKU in Linz encompasses the advantage of bringing together experts in various fields of mathematics education working in close collaboration with GeoGebra developers and experts.

This feature is also represented as supportive factor through data analyses: Students highlight the intense collaboration with teachers and teacher educators, with authors, and especially the possibilities of working closely with other students, which fosters communication and collaboration among them. Considering the above-mentioned quality issues relevant for digital materials, the students value the option to discuss material design with several experts who provide different perspectives and emphasize various aspects. Xie et al. (2017) summarize that evaluating digital content involves significant knowledge and skills in the field of technology integration in mathematics education. For this reason, one may conclude that including such experts in our project guarantees the quality of our materials, which confirms our approach and processes.

Interestingly, if one takes the perspective of the students as learners (in our case to learn how to design digital materials), three agents are relevant: learner (each student), peers (other students), and teachers (teachers, teacher educators, and authors). Black and Wiliam (2009) identify these as relevant agents in formative assessment (FA). Compared with the five key strategies of FA outlined by these authors, the FLINK processes actually address most of them:

(1) "Clarifying learning intentions and criteria for success" (Black & Wiliam, 2009, p. 8), which in our case is the sum of project requirements for students;

(2) "Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding" (p. 8), which is represented in our review rounds and manifests in the above-mentioned student collaboration;

(3) "Providing feedback that moves learners forward" (p. 8), in our case in the expert reviews;

(4) "Activating students as instructional resources for one another" (p. 8), something that is equally enabled through above-mentioned communication and collaboration among students; and finally

(5) "Activating students as owners of their own learning" (Black & Wiliam, 2009, p. 8).

This similarity to FA learning processes indicates that students can foster their own skills in the field of technology in mathematics education and that the project organization and characteristics are a suitable starting point for pre- and in-service teacher training–a design-based research already started and summarized below.

In essence, the project's workflow outlined before is suitable and enables the pursuit of our objectives. During the first month, we additionally altered some procedures based on our experiences and team members' feedback; mainly, we introduced more formally structured procedures regarding review loops, regular team meetings, a structural collaboration between authoring and content team, and a more regular and intense communication between collaborating authors and designers. From a mathematics educational perspective, the students value the information presented in the accompanying script and highlight the importance of intense literature review. In this context, one relevant feature in the design of a particular material is the focus on explicit and precise learning objectives. Summarized, the implemented workflow structures the time- and resource-consuming process of material design and creation and thus clearly enhances quality of digital materials regarding the requirements described in the previous chapters.

So far, we focus mainly on closed tasks for practicing skills. Next project phases will include considerations on how to integrate open-ended tasks (e.g., for modelling or problem-solving) in a digital environment providing technology-added value. For practicing skills individually, this will include researching possibilities to provide automated feedback (on randomized or open-ended tasks) and computer-based assessment. Also, how to integrate the materials fruitfully into FA (e.g., utilizing GeoGebra classroom) will be an upcoming topic.

Following steps in the project include planning empirical studies on the integration of digital materials in teaching, which focus on various aspects. Concerning the design of digital materials, we have started to examine quality aspects of digital materials for developing a suitable framework. In addition, the participation of pre-service mathematics teachers enables us to investigate the development of their professional knowledge on digitalization-related competencies and thus to draw conclusions for the design of courses (Lindenbauer et al., 2022). Our long-term research plans further focus on implementation into regular teaching from teachers' and learners' perspectives. One goal is to understand how teachers integrate these digital materials into their documentational system (documentational approach), and how they orchestrate teaching in their classes (instrumental orchestration). Our aim is to foster (pre-service) teachers' techno-mathematical fluency (Jacinto & Carreira, 2023). Another part should focus on learners' perspectives through the lens of instrumentation theory and additionally examine learners' mathematical thinking in a digital learning environment. In sum, research should provide an integrated view on how technology-based materials can be utilized as mediator between mathematical content, teachers, and learners in a supportive way.

Author contributions: EL, E-MI, & ZL: writing–review & editing; **EL & E-MI:** validation; **EL & ZL:** conceptualization; **& EL:** methodology, formal analysis & investigation, & writing–original draft preparation. All authors approved the final version of the article.

Funding: FLINK is a project of Johannes Kepler University funded through the Austrian Ministry for Education, Science, and Research. The authors did not receive funding for conducting this study.

Acknowledgements: The authors would like to thank Prof. Markus Hohenwarter (head of Center for Open Digital Education at JKU) and Christina Krenn, project leader of FLINK. The authors would also like to thank Alexander Huber from the project MathSkill-Testing for his support of the FLINK project regarding task formats for practicing skills.

Ethics declaration: The authors declared that they followed guidelines of the British Educational Research Association during the study. All participants volunteered to participate in the data collection. The authors further declared that, to preserve confidentiality and anonymity, they ensured secure data storage and changed names of participants to pseudonyms so that they cannot be identified in any publications.

Declaration of interest: The authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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