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Research Article

Collaboration and conflicts among university teachers in a lesson study

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ABSTRACT

In this article, we address the collaborative relationships among university teachers participating in a lesson study. Specifically, we examine teachers' changes in response to conflicts that arise during this professional development process. Data were collected through audio and video recordings and interviews during a lesson study conducted with mathematics teachers who taught differential and integral calculus to engineering students. The lesson study took place in the first semester of 2023, over thirteen sessions and included two research lessons. The results show that the group of teachers developed a deep understanding of what collaboration entails and how it should be conducted. Furthermore, the participants demonstrated mutual trust, and in the face of conflicts, their attitudes leaned toward negotiation to reach consensus on the actions to take.

Keywords: collaboration, conflicts, lesson study, professional development, higher education, calculus

INTRODUCTION

In higher education, collaboration is very common in research. However, when it comes to teaching, whether in lesson planning or in classroom teaching, the situation is quite different. Teachers tend to work individually, and this refers not to their individuality, which is indeed important, but to individualism (Hargreaves, 1998) as the sole way of working. As a result, the undertaking of collaborative professional development processes is often met with disapproval by teachers.

Lesson study is a reflective and collaborative professional development process that has gained increasing attention (Alshwaikh & Adler, 2017; Murata, 2011; Puchner & Taylor, 2006). It provides a conducive environment for teachers' professional development and for recognizing the benefits of working collaboratively. This collaboration does not simply involve informal conversation about students or specific classroom incidents but creates opportunities for in-depth discussion and reflection on past and future actions. Collaboration does not require agreement on all matters, but rather discussion and negotiation of different viewpoints. During a lesson study, collaboration often leads to moments of conflict and disagreement among the participants. Although many may view conflicts as negative, they may foster important moments of reflection and learning opportunities (Achinstein, 2002; Calleja & Formosa, 2020).

Research on the consequences of collaboration in the school environment is extensive (Hargreaves, 1998; Wood & Gray, 1991). Several studies explore the characteristics of collaboration in lesson study (Cajkler et al.,

2013) and how lesson study fosters collaboration (Puchner & Taylor, 2006; Richit et al., 2024). Previous studies also addressed critical incidents in the classroom (Choy, 2014; Hanuscin, 2013) and conflicts among participants in lesson studies with primary school teachers (Hourigan & Leavy, 2024; Quaresma & Ponte, 2021). Some studies researched the possibilities of conducting lesson study in higher education in preservice teacher education (e.g., Leavy & Hourigan, 2016; Martins et al., 2023). However, only a few studies included teachers in subject fields. For example, in the field of mathematics, that was the case of Druken et al. (2021) and Richit et al. (2024) who concluded that lesson study may provide a significant contribution to the professional development of higher university teachers.

This article seeks to expand the body of work on collaboration during lesson study, focusing specifically on conflicts with higher education teachers. To this end, we aim to answer the following questions:

1. What are the higher education teachers' attitudes and changes in response to conflicts during a lesson study?
2. How do the characteristics of collaboration manifest throughout the lesson study?

THEORETICAL FRAMEWORK

Collaboration

Individualism remains the most common model used by teachers in their teaching activities. It is "a welcome measure of privacy, a protection from external interference, which is often valued by them" (Hargreaves, 1998, p. 187). Individualism is often associated with mistrust due to the uncertainties of the work. However, Hargreaves (1998) suggests that individualism may stem from other factors, such as physical and administrative barriers (constrained individualism) that prevent alternative approaches. It may also arise from teachers believing that their time cannot be spent on collaboration tasks due to high demands and workloads (strategic individualism). Finally, it may also be a conscious choice (elective individualism), where teachers see it as the best strategy for efficiency and time management. Although individualism helps to avoid negative criticism of teaching, it also eliminates praise and valuable feedback regarding "merit, value, and competence" (Hargreaves, 1998, p. 187).

In the field of education, the idea of collaboration is not new, with studies on the subject dating back at least to the 1990s (Hargreaves, 1998; Wood & Gray, 1991). In many fields, terms such as "team" (Bertrand et al., 2006) or "professional learning community" (Grossman et al., 2001) express concepts closely related to collaboration. Vangrieken et al. (2015) provided a literature review defining terms such as group, team, and community of practice. They define collaboration as "the joint interaction of the group in all activities necessary to complete a shared task" (p. 23). However, this definition remains vague, as it closely resembles what Boavida and Ponte (2002) describe as simple cooperation, where tasks are divided, but there are significant differences in the participants' power and roles in undertaking the task. For Boavida and Ponte (2002), collaboration "is appropriate when the various participants work together, not in a hierarchical relationship, but on an equal footing, with mutual support to achieve goals that benefit everyone" (p. 3). Therefore, for these authors, it is essential that participants in a collaborative group do not act according to a hierarchy. Even if they hold different roles in other contexts, this should not be assumed in the collaborative group. In addition, it is important that the aim of the group benefits all participants, not necessarily in the same way, but by contributing to the work and satisfaction of all.

Collaboration "is not an end in itself, but a means to achieve certain aims" (Boavida & Ponte, 2002, p. 3). It carries characteristics such as being voluntary, having a common goal among participants, dialogue, shared responsibility for decisions and outcomes, and time and effort that are meaningful to all involved (Boavida & Ponte, 2002; Cook & Friend, 1991). Key aspects of collaboration include trust, dialogue, and negotiation (Achinstein, 2002; Boavida & Ponte, 2002). Trust allows participants to feel part of the group, with their contributions – whether ideas or questions–respected and valued. Dialogue is used to confront and build ideas, overcoming contradictions. Negotiation reminds participants that no idea is final, and negotiating perspectives is essential to achieve the proposed goals. Through dialogue and trust-based negotiation, conflicts naturally arise.

To understand the principles that promote collaboration in lesson studies, Richit et al. (2024) conducted a study with university teachers of mathematics and mathematics education. Sharing promoted an environment of listening and reflection, strengthening relationships and positively influencing participants' professional development. Similarly, Cajkler et al. (2013) discuss what teachers can learn from their participation in a lesson study. They found that participants in a secondary school lesson study reported that this professional development process fosters less teacher-centered approaches and creates a stronger sense of teaching community. These authors highlight characteristics of collaboration such as dialogue, the development of individual skills, and increased confidence in taking pedagogical risks during lessons. Puchner and Taylor (2006) describe two lesson studies in which participants created a climate of collaboration and inquiry. One case stands out for a teacher's "effort to transition from isolation to collaboration," illustrating this process and showing that the teacher enjoyed participating in the lesson study after realizing that her autonomy was not threatened. Richit et al. (2020) discuss how the prevalent isolation in teaching culture can be overcome and how professional collaboration can be promoted through the dynamics of lesson studies. The authors emphasize the promotion of characteristics such as respect, trust, dialogue, negotiation, and sharing among participants. In a lesson study reported by Quaresma and Ponte (2021), it was also noted that there was initially little communication among the teachers, but the lesson study provided a new dynamic for the group as they begun to carry out substantial common work in designing tasks and anticipating students' responses.

Conflicts and Critical Incidents

Conflicts may arise in collaboration. Hargreaves (1998) notes that in some organizations, "conflicts and disagreements are socially more significant to participants than the things they might eventually share" (p. 213). A conflict is defined by Achinstein (2002) as a social interaction process that "is both a situation and an ongoing process where views and behaviors diverge (or seem to diverge) or are considered somewhat incompatible" (p. 13). In other words, conflicts offer moments where beliefs and actions differ, providing opportunities to reflect on and understand different perspectives. For the author, people in disagreement can acknowledge conflicts and openly debate differing perspectives and practices to identify the best alternatives to meet the issue at hand. This may lead to the satisfaction of resolving differences and gaining deeper understanding of the matter. However, some individuals tend to avoid conflicts, which can lead to frustration and stress when they fail to reach consensus–such as when they do not reflect on other viewpoints or simply accept the views of others without reflection, merely to avoid contradictions.

Quaresma and Ponte (2021) address conflicts through critical incidents. According to the authors, critical incidents may arise from "conflicting expectations of participants and characteristics of activities, reflections on conflicting viewpoints among participants, as well as surprises and opportunities for reflection provided" (p. 95). Through two lesson studies, the authors describe how critical incidents–both positive and negative–strongly influenced relationships among participants. The authors note that conflicts occurred not only between participants but also within them, such as insecurity about their professional competence.

In a similar way, Calleja and Formosa (2020) use the term "discrepant event" to refer to situations where "the facilitator provides a new idea or evidence that contradicts the teachers' existing conceptions, thus offering them an opportunity to ponder, reflect, and critically evaluate ideas" (p. 386). The authors report on discrepant events involving an arts teacher during a lesson study and conclude that these discrepancies played a significant role in helping her move away from rigid practices, providing moments of reflection and continuous support through collaborative practice.

Rosales (2000) defines critical incidents as "specific situations where the normal teaching routine is disrupted, testing the teacher's ability to reflect, evaluate, and make decisions" (p. 18). Through various experiences, the author describes several conceptions of critical incidents in ongoing professional development, such as allowing teachers to identify significant situations, provoking debates on theoretical and pedagogical principles, and stimulating the teacher's decision-making ability.

A critical incident is induced by a conflict and can lead to new learning, as it allows individuals to identify aspects previously unknown to them. Therefore, collaboration among teachers does not exclude conflict but rather views it as important because it leads to critical reflection, promotes new perspectives on the topics addressed, and helps to "counteract the myopia" (Achinstein, 2002, p. 10) of teachers.

Table 1. Study participants

Name	Education	Teaching experience
Ana	PhD in science and technology education	17 years
John	PhD in mathematics	18 years
Laura	PhD in mathematics	18 years
Ronaldo	PhD in mathematics	6 years
Valentim	MSc in mathematics	16 years

Alshwaikh and Adler (2017) and Choy (2014) address critical incidents occurring in research lessons of lesson studies. There are also studies on critical incidents experienced by the authors themselves, such as Hourigan and Leavy (2024), who present critical incidents as they acted as facilitators in a lesson study that they conducted for the first time with in-service teachers. They discuss conflicts such as participants' resistance at the beginning of the professional development process, the time they spent, and their responses in reflection, concluding the need to adjust the lesson study for different audiences and considering the limitations of the facilitator's role.

In addition, Hanuscin (2013) discusses critical incidents experienced by the author and the other participants. The author addresses the case of a teacher, Jane, who was her student, and how critical incidents influenced the development of her pedagogical content knowledge. Data were collected over two years, during distinct events, with Jane as a student and later as a teacher during a summer course. The two worked on her perspective during critical events over this period. The conclusions show that the author perceives deficiencies in the courses attended by Jane and that one course is insufficient to prepare future teachers. Additionally, it is evident that critical incidents shaped Jane's knowledge.

In contrast, Quaresma and Ponte (2021) discuss critical incidents involving primary school teachers participating in lesson studies. The authors highlight critical moments such as discussing who would teach the research lesson and voicing difficulties and insecurities in interviews.

METHODOLOGY

Design, Data Collection, and Analysis

This research adopts a qualitative approach following the interpretive paradigm (Bogdan & Biklen, 1994; Stake, 2011), using a participant observation design (Jorgensen, 1989). The data for this study consists of a lesson study conducted between March and July 2023, along with semi-structured interviews conducted before and after (FI) this professional development process. Data collection involved observation, photographs of documents generated during meetings, with audio and video recordings of all meetings (S*x*) and research lessons. Additionally, Interviews were recorded in audio. Data analysis focuses on the activities carried out during the lesson study, identifying particularly relevant episodes representing critical incidents where conflicts occurred. Such episodes referred to situations in which as extended discussion took place, with conflicting points of view. The episodes were analyzed taking into consideration if different perspectives come or not to an agreement. We triangulated the data for the episodes with what the participants indicated in the interviews. We examined how the selected episodes influenced the collaborative process, either positively or negatively.

Participants and Context

Five teachers (named with pseudonyms) from a university in southern Brazil voluntarily participated in the lesson study (see **Table 1**). Initially, all instructors teaching differential and integral calculus 1 (calculus 1) were invited; however, one of them was unavailable for the entire semester and chose not to participate. Laura, John, and Valentim had worked together for over 12 years. Ana had worked with them for 10 years, while Ronaldo was at the institution for 1 year. The facilitator was Larissa, also a teacher at this university.

The lesson study was conducted face to face and was structured as described by Stigler and Hiebert (1999) (see **Table 2**). The semi-structured interviews were held face to face before and after the lesson study.

All meetings lasted one hour each week. In the first meeting, the first author, who acted as facilitator, introduced the lesson study and provided examples of investigation tasks. During this initial meeting, the

Table 2. Activities of the lesson study

Sessions	Lesson study steps
S1	What is lesson study; choose the topic
S2, ..., S11	Planning the lesson
RL1	Research lesson 1
S12	Post-lesson discussion 1 and reshaping the lesson
RL2	Research lesson 2
S13	Post-lesson discussion 2 and reflection

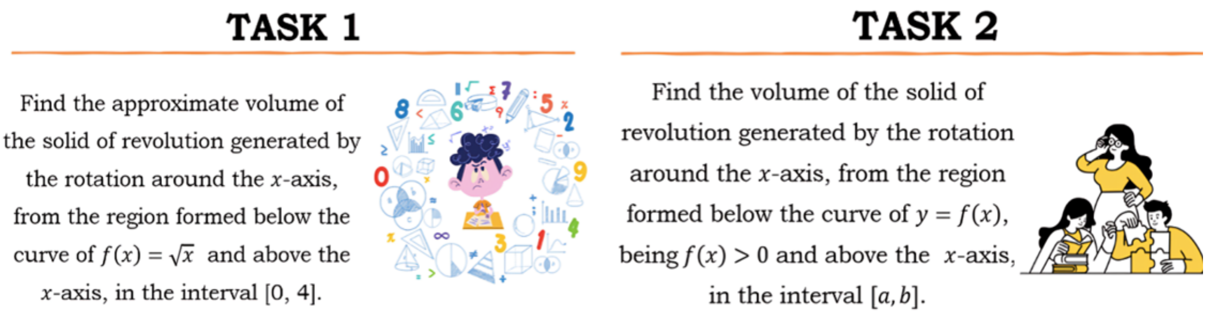

Figure 1. Research lesson tasks (photograph) (Source: Authors' own elaboration)

group also began a discussion on which topic to address. In the second meeting, due to the time spent planning the lesson, it was agreed that the topic would be integrals, specifically the volume of solids of revolution. From the second meeting to the eleventh, the participants worked on the detailed planning of the lesson, which included writing the task, anticipating students' questions and possible teacher responses, organizing a GeoGebra program to demonstrate the solids, and preparing the observation sheet for teachers, as well as discussing how to approach the topic with students. Finally, the task presented to students was divided into two parts (**Figure 1**).

The first research lesson was taught by Laura, and the class consisted of students from both Laura's and Ronaldo's computing engineering courses. The lesson began with the presentation of the first task, a student's autonomous work, followed by a whole-class discussion. The second task was then presented, and another whole-class discussion took place, but there was not enough time for a final synthesis. In the post-lesson reflection and lesson revision, the teachers identified issues such as the lack of a more enriched introduction and the inappropriate decision to combine the two classes. Once the lesson was revised, it was taught by John to his electrical engineering class and was conducted as planned. In the final session, the teachers discussed the benefits of having a single class and the students' difficulties with the tasks. They also considered how the investigation tasks might be better used if the students were not in their first semester, implying that greater maturity and mathematical background would be advantageous.

RESULTS

Episode 1 (Session 04)

The issue of group trust was evident as the teachers were analyzing a file created in GeoGebra. This episode occurred during the planning of possible student questions when Laura suggested that a student might ask if it is possible to rotate around an axis other than the x-axis. It was expected that, since a software was being used to show rotations around the x-axis, the same program could be used to illustrate the response to the student. Working with the example at hand, Laura proposed rotating the function $f(x) = \sqrt{x}$ around the y-axis by simply swapping the order of the variables in the parametrization created for rotation around the x-axis to see if the result would be as expected. However, the program ended up rotating the function $f(x) = x^2$ around the y-axis:

> Laura: It is wrong. Your parametrization went wrong somehow. Wrong [stands up and shows in the projection]. Because that's what it should do ... You have this curve, you rotate around here, it had to get the little hat in here, not the paraboloid [shows in the projection]. [...] She rotated something

here … she did something here … she just threw the paraboloid in, that's … because if you rotate it [the function] around here, there will be a paraboloid in y, in x. So, it's really not correct.

Valentim: Yes, because it isn't the change in the function, since when I rotate it around y we do that as well, when you do it by hand […] It is like … the inverse of f. Because the radius is not x anymore, the radius is this distance here [shows with his hand]. It's the distance from the curve until the y axis, the radius. And we took the radius as the distance from x up to the curve, which is different. Agreed? Do you have a pen there, Larissa? [goes to the board to write].

Laura: No, I agree and disagree. I didn't understand what you meant. I'm saying the software is wrong.

John: No. Ah, yea, not the software …

Laura: The rotation is wrong.

Valentim: The parametrization.

John: Yes. You can't leave the $f(t)$ there.

[…]

Valentim: For each point that is the radius [shows by hand in the blackboard]. When I rotate around the y-axis, that's not the radius. The radius is here now [draws on the board]. This distance is now the radius.

Laura: I agree. Do you agree that the figure is wrong?

Valentim: Yes.

Laura: Then …

Valentim: That's why you can't just switch x with y, you must change the parametrization.

Laura: Yes, that's what I'm saying. The way it is, it's not just switching x and y as I suggested. (S4)

Laura demonstrated confidence in arguing about a possible way to make the program generate the desired outcome. Even after the test did not produce the expected solution, she had no problem discussing the issue with the other participants, seeking to understand the error and how to solve it. She continued to argue about the variable swapping error, and the discussion continued:

Laura: So, you can't just switch x and y's places. You must change all the rest to be able to rotate y.

Larissa: You must change this [goes to the board].

John: No, that's the $f(t)$ there.

Larissa: It's not the $f(t)$, it's the inverse of f.

[…]

Larissa: You just need a new parametrization and leave it hidden… Fix this back [shows in computer].

[…]

John: And what's the new parametrization? We must understand the variations.

Laura: [Goes to the board]. So, what Larissa is saying is this … You change here to what it was [shows in the board] and put t up here.

Larissa: Yes, and you keep the parametrization in x.

Laura: Then you copy this [the parametrization] and put another one [glues] … It's this one, but instead of $f(t)$, it's going to be $f^{-1}(t)$.

John: And in this case that'll be x^2? Because the root [squared] of x …

Valentim: Yes.

John: … But it isn't, is it?

Laura: No, exactly. I also don't this, this would be the inverse of f. (S4)

At this point, two parallel discussions took place. While the others continued trying to fix the program according to the suggestion of Larissa using the inverse function, John and Laura, who disagreed with this approach, were working on writing the new parametrization in their notebook:

John: This won't be x^2.

Laura: No, it's the y^2 that you will rotate.

John: So, $y = x^2$. OK, that's here, no …

Laura: No, this … You have y equals to x^2 … y equals to the root [squared] of x … Let's just think of the area, calculate the area.

John: You don't need the integral.

Laura: No, but when you calculate here, you'll calculate the volume with π and so on, right? To calculate in y, you need to isolate y here. Then, x is equal to … Then you have the integral of $y^2 dy$. Because you're integrating on the other variable.

John: No, you're right, but … Rotate the two functions, the y^2 is the curve that's in y.

Laura: No, this curve that you're drawing is the x^2, the y^2 is this one. The function is the same x … $y = \sqrt{x}$ or $x = y^2$, in the first quadrant it looks the same. (S4)

The teachers negotiated their ideas openly, without fear of judgment for mistakes. The rest of the group also engaged in discussions, offering suggestions such as making the program automatically generate the inverse function to increase its efficiency. The session continued with the teachers successfully generating the image, calling Laura's attention as she continued her discussion with John:

Laura: Yes! [looking at the image] It's right!

Ronaldo: We just must limit the interval.

Valentim: When you cut … The inverse of f … Ah OK … You must write as a function of x.

Ronaldo: In y, you must go from zero up to f's last point, right?

Valentim: Yes. No, you can see it here, but in this line [in the software] you can see that they agree.

[…]

Laura: Where is it? What did you do?

Ronaldo: They inverted f. (S4)

All teachers were convinced that it was necessary to use the inverse function for the parametrization to meet the expectations, and they continued discussing several details to make the example more visible to the students. Laura concluded:

Laura: For the aim of the classes, that's what I wanted. That was it, our goal is to make it in the x-axis, also because of the area issue that you just explained there. Right? Then the question is, if the student asks me on y, then you can make the rotation and show "Look, this happens".

Larissa: Yeah.

Laura: Then fine. "But today the lesson goal is to study this one", then you go back to x. (S4)

In this episode, all teachers were open to giving opinions on how to solve the issue with the software to generate the correct image. After several attempts and parallel discussions, the participants successfully generated the expected image and concluded that the example was sufficient to address any potential student questions.

Episode 2 (Session 06 and Session 07)

The second critical incident in the lesson study revisits the issue of potential student confusion when rotating around the y-axis. When needed, the teachers always referred to the example of rotating the function $f(x) = \sqrt{x}$ around the x-axis, which was part of the task created. However, generally, the teachers referred to the function f, constrained by certain conditions, being rotated around either the x-axis or the y-axis. This led to the following conflict:

John: So, that question ... Doing it on GeoGebra, there ... To leave a hidden example with the y-axis is kind of confusing, you must be careful with it. Because ... Let's imagine we rotate, I don't know, the square root of x around the y-axis, what is the volume you're talking about? Because the given volume there, by the direct integral, the volume is going to be under the function until the x-y plan, right? The volume is not going to be in, what fits the solid, right? It is different from when you rotate there in x, for example the square root of x ... Then it is correct.

Laura: But then you must change the variable, you must integrate in y ...

John: Yes, OK, but what is the volume you're going to do?

Laura: It's the inside volume. You turn the function [points to the drawing] ... Then here, you're rotating this one, the result is going to be this thing here ... (S6)

We observe that Laura did not fully understand John's words and continued discussing the mathematical solution of the potential student question. When John changed the question but did not clearly reflect on what he wanted to clarify, she answered the question, but the previous issue remained unclear, as they were considering different situations. The discussion continued with the assumption that it was merely a matter of presenting an example to the student, without needing to go into detail about how the partition was done in this case. The negotiation of ideas did not seem to work, and no one conceded their viewpoint. John insisted, arguing about the student's perspective:

John: But the student might ask what the volume is ... What volume the student is calculating. Because if you simply ... The formula, it depends on how you use it. If you use it like this ...

[...]

Valentim: No, but I think it's best not to talk about those things, OK? Just draw the solid ... (S6)

John questioned again about which region the student would be using for the volume, but he referred to the formula, leading the other participants to focus on problem-solving procedures rather than on interpretation.

The conflict continued when Laura and Ronaldo expressed that they did not understand the problem John was referring to. Laura, from her perspective, tried to explain the situation to John. The teachers began sketching the situation in their notebooks. They attempted to adjust the function to aid in understanding what each participant wanted to conclude from the situation, specifically how to calculate the volume of the "function rotated around the y-axis." The discussions continued with John reiterating the possible perspective of the student:

> John: But the student might not see it yet. He can use … "Oh, it's not the same function here … To rotate this way or that way" it's the same …
>
> Laura: Yes … But this is what I'm talking about … The point is … At this moment, what are you going to say to the student?
>
> John: No … It's not what you are going to say. What is the student going to think? Are you going to control it?
>
> Ronaldo: But in the y-axis is not the same thing, is it? In y, it is the inverse function … (S6)

The communication among the teachers was intense but lacked clarity in the exchange of ideas. John tried to make the participants understand his concern about the student's perspective in the proposed situation, but he was unable to articulate his viewpoint in a way that the others could grasp. Consequently, the participants continued with an incorrect understanding of the issue and attempted to show John something that did not address his concerns.

The teachers continued to refer to the graph of the function on the paper, discussing its rotation, the radius to be considered, and the need to account for the variable y in the solution. They reviewed concepts about areas but still struggled to understand John's question. After more than half an hour of negotiating viewpoints, the participants began to converge on a common idea. This shift was initiated once again by John's question:

> John: Is it clear … For you … That … Is the volume the upper part you're going to calculate?
>
> […]
>
> Laura: What solid does this region form? [shows with the hands] This one. Oh, no! Now I want to rotate around the x-axis, in the interval $[a, b]$ and below the curve, so this region here that I'm going to rotate… Then the result is going to be the paraboloid down here.
>
> […]
>
> John: My question is: if the student is going to be induced by the area or not … Because if you have a positive function, the area is given by the lower region of the graph until the x-axis. (S6)

Laura and Valentim began to understand John's perspective, which emphasized that the lesson on areas could influence the students to think as he was thinking, as it would be more natural for them. The dialogue appeared to influence the other participants; they became thoughtful as they begun to understand John's question. Eventually, Laura consulted a calculus book, stating that she needed to solve a particular question. She leafed through it, read, and commented:

> Laura: That's right, look here [pulls the book to the center of the desk]. You're … Here … A region revolves around a line in the plane, right? The resulting solid is a solid of revolution. We say that the solid is generated by the region. So, you first define what the region you will rotate is … Then … OK,

> I'm going to rotate this region. So, I agree with you [John], because if you're rotating, for example, like this, the region I formed is the one below the curve ...
>
> John: That's it ...
>
> Laura: ... The solid you rotated is the outside part of the bowl. (S6)

Thus, the teachers finally began to understand that the situation causing the conflict was a problem with the definition of a solid of revolution:

> Laura: OK ... But then ... If you ... Oh, right! But now I want to rotate the other region, John's rationale is correct. Because you first define the region, and our region will be defined ...
>
> Ana: Because ... Because there's something else, this curve of the definition doesn't have to necessarily ...
>
> John: Be the axis (S6).

Regarding this conflict, it is important to highlight the various characteristics of collaboration that are evident. The negotiation of viewpoints, the dialogue aimed at confronting ideas, and the group members openness to questioning and understanding each other's positions are quite noticeable. Additionally, this conflict prompted the teachers to reflect and critically evaluate themselves. This is apparent as the meeting continued with the teachers analyzing their own discussion:

> John: What I meant was, like, this one below [area with the x-axis] is more intuitive for the student to calculate ... Because ... Look at the graphs that are here in the function [points to the book], where it is placed ... Exemplified ... The region is the part from the function till the axis ...
>
> Valentim: Mm-hmm. But I would say that it's a matter of interpretation ... Because I, of course, I would think differently.
>
> [...]
>
> Valentim: But I ... But this here was good for [taps his finger to his head meaning to think about the situation] ... To observe ourselves ... Because I would think, if the student asked, I would induce him/her ...
>
> Laura: I would answer it wrong.
>
> Valentim: No ... I would induce the student to say that I was thinking about the situation in another way ... (S7)

From that point, the teachers continued discussing that all, except for John, would attempt to guide the student according to the teacher's perspective, without considering what the student was actually thinking. This critical incident was revisited during the final interviews, where it was noted that the evaluation the teachers made of it was very positive:

> The discussion [session]. The one of the "rotation" and so. It took a long time for us to understand what was going on. So, it seemed as we were getting lost, but we weren't. It was, in fact, quite enlightening, this matter of, the student, he/she thinks differently from us, and we think we have all figured out. So, this [session], for me, was the best. In the sense that you have to re-think how to explain to the student. Cause, sometimes, the student didn't understand it because of the way you explained it, not that he/she didn't understand anything, it was the way you explained it. (Laura, FI)
>
> That session about the rotation around the y-axis ... That's finally ... It seems that there was an understanding among people, because apparently, someone would say something, and people

> would understand something else. And this was in a conversation between teachers, so I figure that it must happen a lot in class, the teacher is saying something, and the student understands something different, and the conversation goes on, each one with their own understanding of what was supposed to be the same thing. (Valentim, FI)

> You should not only look at the student but look at yourself. Reflect upon your knowledge and improve it. For example, that thing about what rotated when there was volume, that was not the function. Because in my class, what did I do? I gave them the formula and said: “Get the function and rotate”. I never paid attention to the fact that the region was supposed to rotate. Implicitly, we know it was this, but it rotated around the y-axis, it might be confusing, there are two regions. So, like, when I taught that lesson … That part over there … After applying the lesson study, I already focused on it […] So, also as a teacher, I think that session was, for me, the best of all. (Ronaldo, FI)

It is worth noting that the same critical incident is viewed from different perspectives. While Laura and Valentim emphasized the importance of reflecting on the difference between the perspective of a student unfamiliar with the content and their own, which is accustomed to dealing with it, highlighting the need to understand the student’s question rather than presuming to know their difficulty, Ronaldo went further. He discussed how this critical incident helped him improve his own knowledge and led him to reformulate his lessons.

Episode 3 (Session 11)

A third critical incident during the lesson study did not generate as open a debate. During the discussion, the participants did not reflect on divergent views, and by the end, Ronaldo decided not to challenge the ideas. This situation occurred in the session before the first teaching of the lesson. Laura was reviewing the beginning of the planned lesson and discussing the introduction of the task: she would define and show examples of solids of revolution based on a function using GeoGebra, and after these examples, the task for independent work would be proposed. John commented that the task was not conceived in this way:

> John: I think what we had in mind that day was to begin like this: “Look, today’s activity will be to calculate … To approximate the solid of revolution … Of this figure.” And then, after that, the next problem is to calculate it in a general function. That’s the problem. Then, let’s say, “Okay, but what is a solid of revolution?” If they already know what a solid of revolution is, fine, go straight to the problem. If they don’t know, you present the definition of a solid of revolution. After you present the definition, show it in GeoGebra, demonstrate it, and then they’ll try to do it. I think that’s what was thought.

> Laura: I thought it wasn’t like that. That the first part was already the introduction, with me working in GeoGebra showing what the solid was …

> John: But what if they already know?

> Laura: But some will know, not everyone …

> John: But he graphed it. The guy [student] did the graph of the square root of x and created the solid of revolution. No need to …

> Laura: For the whole class?

> John: Yes.

> Ronaldo: My students won’t know what it is, I think. I think we need to do it that way because my students …

> John: But it’s just a matter of the order in which you present it. I think what was planned was indeed in that order.

> Laura: But that's exactly it. I want to know what the order is because if we planned a lesson together, I need to try to follow that order. Not what I think is right. That's my question.
>
> [...]
>
> Laura: In my mind, this is how I thought of structuring the lesson. Talk about what a solid of revolution is and ask if they know what it is. Some might say yes, others no. Since there will be "no" answers, because in a class there will always be someone who doesn't know, it's unlikely that the whole class will know. I would then go to GeoGebra … Show those examples we saw, ask about the cylinder and so on, and then propose the first problem. Isn't it in that order? For me, that was the order …
>
> Valentim: Yeah.
>
> John: I'm saying that for me it wasn't that way. For me, it was to present the problem first, OK. "Oh, but I don't know what a solid of revolution is." Then you present that. (S11)

John did not appear willing to negotiate the situation, despite some diverging views and Ronaldo's warning about the difficulties his students might face. The participants continued to discuss but were hesitant to express their views or alter the lesson plan, which had already been revised several times. However, John persisted with his viewpoint, arguing that this could limit the students' work:

> John: I thought it was like that, but if you want to do it differently, it's just a matter of approach. Assuming they don't know what it is, without letting them, I don't know, imagine, try to draw, or do something …
>
> Valentim: You could even not show it in GeoGebra but explain what a solid of revolution is so that everyone at least thinks about the same thing. It's obtained like this–you rotate around the axis …
>
> John: And how will you present this? On the board, or will you …
>
> Valentim: And then let them imagine what it would be …
>
> Laura: But, OK, it's just about how to begin the lesson. (S11)

Laura then ceased negotiating the situation, accepted John's view, and continued the discussion on revising other aspects of the lesson plan. However, Ronaldo, still unsettled by the outcome of the discussion, resumed the conversation:

> Ronaldo: I'm … Now I'm not so sure. How exactly should we begin? I thought it should be with the solid first.
>
> Laura: No, but I understood John's idea …
>
> Ronaldo: I do too, but the other way might be better …
>
> Laura: But I think the way he mentioned is good. Because the idea was, as Larissa said, to first present the problem, "What's the problem? The problem is to solve this." Then I start with "What is this region?", "What is this solid?", and I begin to challenge them. If they don't know, then I go to GeoGebra.
>
> Ronaldo: Right, OK, then that's it. (S11)

The situation has several notable points. Ronaldo concluded the discussion by simply agreeing but expressed dissatisfaction with the outcome. Except for John, everyone agreed that it would be better to first explain the concept of a solid of revolution to all the students using GeoGebra. However, at the end, everyone concurred with John, and the lesson was conducted accordingly. As a result, the lesson did not get the intended beginning, as many students struggled to work on the problem due to their lack of understanding of the rotation of the defined region along with the function. In the final interview, Ronaldo revisited the episode, emphasizing that it was a collaborative effort involving five teachers, and thus it was necessary to accept the views of others:

> Larissa: At no point did you think, "Oh, maybe it should be something else, but I won't bring it up because I don't want to … Interfere with someone else's topic"?
>
> Ronaldo: Yeah… Just about the plan, maybe at times, yes. Because, well … When we were working on the plan, right? Sometimes I had an idea and then others had a different idea, right? And sometimes I'd suggest something, it wasn't accepted, and I'd just let it go. So … But I believe it might have worked differently. Like, for instance, in the first lesson, not showing the solids at the beginning, right? For example, many students didn't even know what it was. So, you begin with a problem, saying, "Calculate the volume of the solid of revolution and so on." But then we ask, what is this? So? I think an example should have been given first, shown and so on, and that wasn't done.
>
> Larissa: No, but showing it to everyone from the beginning?
>
> Ronaldo: Yes, yes. Showing what a solid is before calculating. Then, yes, I think it would have been more interesting. But then the other teachers thought it was better to go another way, so it was agreed upon … We were five, right? (EFLS01)

In this third critical incident, we see that Ronaldo chose not to escalate the conflict. Although he initially argued that a different approach would benefit his students, he did not pursue further negotiation of ideas and, along with the other teachers, decided to accept John's view. However, it is noteworthy that, during the discussion for the reteaching of the research lesson, all participants agreed that an introduction to the concept of a solid revolution using GeoGebra should be included.

DISCUSSION

Collaboration is an inherent part of the professional development process in a lesson study. This lesson study provided participants with opportunities for social interaction and dealing with conflicting situations that emerged during meetings. These situations were foundational for reflective thinking and changes in classroom practice.

Overall, the participants' perspectives on what constitutes collaboration, and its key characteristics align with the view of Boavida and Ponte (2002). However, when asked directly about collaboration, some participants offered definitions closer to Vangrieken et al. (2015), which are more aligned with Boavida and Ponte's (2002) notion of cooperation. During the initial interview, the participants highlighted that equality of status and common goals are important factors for collaboration. They also recognized that working collaboratively requires openness, trust, and negotiation among participants.

It is noteworthy that, except for Ronaldo, all participants had known each other for many years and had never planned lessons together, except for specific instances such as creating exams or problem sets. Unlike Puchner and Taylor (2006), who illustrated a teacher's transition from isolation to collaboration, this lesson study began with teachers accustomed to cooperation, though not in teaching contexts. Ronaldo mentioned that he was in a conducive environment for collaborative work in teaching but did not fully embrace it, continuing with a more individualistic approach (Hargreaves, 1998).

The first episode demonstrates that the group exhibited the characteristics of collaboration as described by Boavida and Ponte (2002). In this instance, the trust among participants, the lack of fear of criticism, and the open dialogue about Laura's question, and subsequently the other participants' contributions on how to illustrate a student's problem using GeoGebra, are highlighted. The fact that the participants volunteered to participate in the lesson study, with no external pressure, led them to not resist initiate and carry out the work.

The second episode brings to the forefront the major conflict of the lesson study, initially leading to incompatible positions as described by Achinstein (2002). However, negotiation and, primarily, dialogue were used to attempt to converge on a common understanding, leading to "surprises and opportunities for reflection" (Quaresma & Ponte, 2021, p. 95).

It is also important to note that John's question about the student's perspective had two major consequences for the participants. First, it highlighted the mathematical issue and the need for teachers, even with many years of experience, to be attentive to how they present concepts to students. This is evident in the final interview when Ronaldo mentioned that he already took care to adjust his practice that same semester. The second consequence was that participants reflected on their habit of envisioning situations so clearly that they struggled to understand students' fuzzy questions. This often leads to classroom dialogues where understanding is lacking, and students may accept the teacher's response without it addressing their question. As it occurred with Rosales (2000), this conflict created a discussion about theoretical and didactical principles, leading the teachers to reflect extensively on the situation, acknowledging the difficulty of remaining open to students' interventions and recognizing the tendency to lead students to their own thinking without seeking to understand the students' perspectives.

The third episode shows that collaboration and conflicts do not always conclude as expected. The conflict reemerged in John's argument about the introduction of the task. Laura highlighted a key aspect of collaboration, which is shared responsibility (Cook & Friend, 1991), in lesson planning, stating that despite her opposing view, the decision was not solely hers. After some arguments, the other participants, except Ronaldo, accepted John's view. Ronaldo attempted to revisit the discussion, but Laura ultimately defended John's idea, and Ronaldo, not wishing to continue the debate, accepted the view of his colleagues. Ronaldo also mentioned in the final interview the issue of shared responsibility rather than just his own view. The quick concession by participants had an additional factor: frequent changes in the lesson plan throughout the meetings. Thus, the teachers were reluctant to revise the plan again. However, after the first research lesson, they realized it was not the best decision and revised the lesson plan. This episode aligns with Achinstein (2002) regarding the attempt to solve a conflict from divergent views. Nevertheless, by revisiting the problem and discussing it openly, the teachers found a better solution.

Our work is quite different from that of Hanuscin (2013) regarding the time of data collection but is similar in the results. In both cases critical incidents helped to shape the thinking of participants. In our case, it helped to shape thinking regarding the mathematical view of situations and also regarding the didactical view and the perspective about students concerning the topic.

Finally, other characteristics of collaboration are observable, such as the participants being volunteers who invested significant time and effort in the lesson study. Therefore, as Richit et al. (2020) indicate, the lesson study may act as a catalyst for collaboration among participants. Additionally, it is evident that even minimal existing collaboration can positively influence the lesson study, making conflicts subject to open dialogue without fear of criticism from other participants, resulting in significant positive impacts on the teachers' development throughout the lesson study.

CONCLUSION

This research illustrates how university mathematics teachers approached collaboration in teaching through a lesson study. It highlights the main characteristics and challenges they encountered when working collaboratively. Collaboration was evident throughout the lesson study, particularly during the planning of the research lesson, which allowed for discussions and negotiations on how to design the lesson. There are also signs that demonstrate trust in post-lesson reflections, where participants openly discussed both strengths and weaknesses in the planning.

Participants' attitudes towards conflicts varied depending on the nature of the conflict and the goals involved. In some situations, persistence proved to be beneficial for understanding a problem, while in others, it led to discouragement in finding alternative solutions. Nonetheless, regardless of the situation, the observed changes were positive, both in classroom practice and in reflecting on inadequate choices, leading to better solutions. Furthermore, the study highlights how conflicts arising during a lesson study can be beneficial for the participants' professional development.

The first episode demonstrates how the group exhibited collaborative characteristics, particularly emphasizing the trust among participants during discussions. These discussions were conducted through open dialogue, with no fear of criticism or opposing views. The second episode underscores the importance of understanding the ideas others want to convey, reflected in the discussion about the real difficulty of understanding students' questions. This led teachers to reassess their teaching of certain concepts and modify their practice. The third episode shows that not everyone is always open to negotiations, whether in accepting others' ideas or in persisting with discussions. However, even if a conflict does not result in a satisfactory outcome for all participants, the failure of the decision prompted them to revisit the discussion and find a better way forward.

Thus, this work demonstrates how a lesson study may be indeed a collaborative process and how conflicts that arise contribute to the professional development of the participants. In addition, as there are few studies regarding lesson study in university subject courses (Druken et al., 2021; Richit et al., 2024), our study provides new knowledge about how lesson study may bring reflections about practice and support teachers' professional development based in situations of conflict. However, there are some limitations that must be noted, such as the small number of participants and the fact that they worked in the same environment for several years. In addition, different from what happens in most university settings, the participants could find an hour to work in the lesson study face to face during 13 weeks. Therefore, we leave as questions for further research: How difficult is to build collaborative relationships regarding teaching issues in universities? What are the characteristics of collaboration in a lesson study with university participants that do not know each other from a long time? In what conditions may conflicts be openly discussed to arrive at a consensus?

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