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Research Article

Cognitive structure of pre-service teachers on theorem and proof in Mayotte through the free word association test

Patrick Tchonang Youkap [1,2*]
0000-0002-4892-7126

Jean-Berky Nguala [1,2]
0009-0003-8913-9550

Stephen Rowland Baidoo [3]
0000-0003-4847-7572

[1] University of Mayotte, Iloni, Mayotte, FRANCE
[2] Laboratoire d'Informatique et de Mathématiques (LIM), Faculté des Sciences et Technologies, Université de la Réunion, Sainte-Clotilde, FRANCE
[3] OLA College of Education, University of Cape Coast, Cape Coast, GHANA
[*] Corresponding author: patricktchonang@yahoo.fr

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ABSTRACT

This article examines the cognitive structures of secondary school mathematics pre-service teachers (PSTs) in Mayotte regarding fundamental mathematical concepts such as theorem and proof. A deep understanding of these concepts is essential for effective teaching practice. To explore this, we employed the free word association test, a methodological tool designed to elicit spontaneous cognitive associations with specific concepts. The research is framed within the theoretical framework of concept image and concept definition as articulated by Vinner in 1991. Participants responded to the concept-stimulus theorem, which elicited nine distinct response categories, while the concept-stimulus proof yielded eight categories. The findings suggest that although PSTs demonstrate a basic familiarity with the notions of theorem and proof, significant gaps in their cognitive understanding persist. For example, there is a notable absence of association between theorem and its logical status as a statement, assertion, or proposition. Moreover, essential terms such as truth, deduction, and validity are not commonly linked to the concept of proof. In light of these findings, we recommend the integration of targeted training on the nature of mathematical statements and proofs within teacher education programs. Such training would aim to strengthen PSTs' conceptual understanding, equipping them to better support students in developing rigorous mathematical reasoning.

Keywords: theorem, proof, cognitive structure, free word association test, pre-service teacher, concept image

INTRODUCTION

A mathematical proof is a type of argumentation (a form of oral or written discourse conducted to reach a common conclusion about a statement whose validity is being debated (Durand-Guerrier et al., 2012) that is accepted by the mathematical community (Balacheff, 1988). Furthermore, a theorem is a true mathematical statement whose validity has been established by a proof. Proofs and theorems in mathematics are semantically close. Thus, a theorem can be seen as a triad made up of a statement, a proof (in this document, we will use the terminology proof to refer to mathematical proof) and a theory (Mariotti, 2002; Mariotti & Pedemonte, 2019). The construction of a theorem statement also requires the implementation of reasoning to validate it, as well as a clear understanding of its logical status (Durand-Guerrier & Arsac, 2003). In this study, we analyze the ways pre-service teachers (PSTs) organize their ideas and knowledge around the concepts of theorem and proof, by identifying the mental associations they make with these concepts (that is, “theorem” and “proof”).

Unlike theorems, proofs are not taught in France as a mathematical content. However, the secondary mathematics curricula require students to develop the practice of constructing mathematical proofs. Developing reasoning skills and introducing students to proof are fundamental objectives in mathematics education at the secondary level, targeting learners aged 13 to 18. Teachers must be able to introduce students to different types of reasoning (deductive reasoning, case disjunction, absurdity, etc.) to contribute to the latter’s’ development as societal and academic citizens. To achieve this, they need to have a rich cognitive structure (Segal, 1988) 2F on the concepts of “theorem” and “proof”. According to Segal (1988) cognitive structure refers to the organization of a person’s stored knowledge, including specific types such as prototypes, stereotypes, scripts and more general patterns that form the architecture of their thinking. Understanding how PSTs make semantic associations between proof, theorem and other mathematical concepts can provide valuable insights into teacher training needs.

This study follows the previous research that has explored various cognitive and didactic aspects of theorem and proof in an educational context (Balacheff, 1988; Duval, 1992; Fujita & Jones, 2014; Pedemonte, 2002; Tchonang et al., 2020). Research into the construction of proofs in mathematics has shown that students’ difficulties lie in understanding the specific requirements of a proof. In particular, their difficulties manifest in the transitions between deduction steps and within a deduction step itself (Duval, 1991, 1992). A proof task must be capable of evoking deduction, truth and validity in the teacher’s mind (Duval, 1992). According to Tchonang et al. (2019) students’ difficulties also stem from inconsistent knowledge of the elements of the theory (definitions and theorems) on geometric figures. Studies report that a lack of experience in writing proof as well as misunderstanding of mathematical concepts are responsible for the difficulties that many mathematics teachers face when writing valid proofs (Luna & Canoy Jr., 2021). These difficulties may have been left unchecked while teachers were under training. Thus, PSTs’ understanding of theorem and proof can directly influence their later pedagogical practice and the overall effectiveness of their teaching. According to Lesseig and Hine (2022), PSTs’ knowledge of proof is not static; it depends on their teaching and learning context. So, they need training to develop a deep understanding of proofs. A PST with a well-structured and coherent organization of these concepts is better equipped to present various types of mathematical reasoning, anticipate students’ difficulties in transitioning between deductive steps, and design tasks that effectively evoke deduction, truth, and validity. Conversely, as shown by the work of Sevgi Moralı and Filiz (2023), gaps or inadequate mental associations in a teacher’s cognitive structure may result in unclear explanations, difficulty in identifying students’ reasoning errors, or a tendency to overlook crucial aspects of mathematical proof. This influence is particularly significant in the French educational context, where proofs are not taught as a standalone mathematical content but are integrated into the broader mathematical practice that teachers cultivate in their students.

The previous work has shed light on various cognitive aspects of proof for both teachers and students. However, the ideas, knowledge and mental associations that pre-services teachers need to form around proof and theorem have yet to be fully explored. It is noted that a well assimilated concept is one that fits correctly into a family of earlier concepts to which they are semantically related. For instance, Bahar et al. (1999) argued that if two concepts are close in terms of distance, they have a close relationship; thus, when one concept is used as a stimulus, the other will be quickly evoked in an individual’s memory (Tünkler, 2021). Research in mathematics education into how teachers organize their ideas and knowledge around concepts such as functions, limits, continuity, probability, statistics, geometry and trigonometry has highlighted existing gaps in their understanding (Benibil & Erdoğan, 2016; Gökbaş & Erdoğan, 2016; Turan & Erdoğan, 2017). We anticipate that exploring the associations made in PSTs’ cognitive structures around the concepts of proof and theorem could allow us to analyze the links they make with other concepts. This would help to determine whether these concepts fit correctly into a network of semantically related concepts. The research question that guided this study is: How do secondary school mathematics PSTs in Mayotte, France organize their knowledge and mental associations regarding the concepts of proof and theorem? This question forms the main thread of our exploration.

THEORETICAL FRAMEWORK

Concept Image and Concept Definition

In the field of mathematics education, *cognitive structures* refer to the conceptual or mental frameworks that learners develop to organize, arrange, and derive meaning from information. These structures serve as the foundation for effective teaching and learning, as they provide a scaffold on which new knowledge can be built.

Tall and Vinner (1981) developed a theory of *concept images concept definition* that is used to describe the total cognitive structure associated with a mathematical concept. *Concept image* includes all the mental pictures and associated properties and processes. In other words, a *concept image* encompasses all mental representations, including visual, symbolic, and procedural elements, that are linked to a given concept. This structure evolves over time as learners encounter new stimuli and mature in their understanding. In contrast, the *concept definition* refers to the formal, precise description of a concept as established within mathematical discourse. A key challenge in mathematics education arises when the concept image conflicts with the concept definition, revealing cognitive dissonance. Such conflicts can limit learners' ability to fully grasp or generalize mathematical ideas, creating barriers to deeper understanding.

Once a concept has been acquired by a PST, he establishes associations with other elements already present in his memory. These associations are not limited to mental images, properties or processes linked to the concept. They also include words and expressions from the semantic and lexical field, which are activated in his cognitive structure. Consequently, we consider that a component of the concept-image of students or teachers can include the set of words that they spontaneously associate with a given concept. For example, in the case of axial symmetry, the word mirror may be part of a pupil's concept-image, because he intuitively associates it with this mathematical notion.

For a PST, the *concept image* of "proof" may encompass ideas such as "justification," "example," or "visual demonstration." For instance, a student might associate proof with procedures they have encountered, such as step-by-step reasoning in solving an equation or verifying specific numerical cases. However, the formal definition of proof demands a sequence of logical propositions grounded in axioms or previously established theorems. A concept map for "proof" could bridge these intuitive representations to more formal notions like "hypotheses," "deductive reasoning," and "conclusion," while also including examples of incorrect proofs to help identify and address common misconceptions. Similarly, the *concept image* of a "theorem" might evoke associations like "formula," "rule," or "important property." For example, a student might perceive Pythagoras' theorem merely as a rule that applies to the right triangles drawn in their exercise book. In contrast, the formal definition refers to a mathematical statement proven true within an axiomatic system. A concept map for "theorem" could link specific examples (e.g., Pythagoras and Thales) to broader notions such as "hypotheses," "formal statement," and "proof," helping students develop a more comprehensive and less context-dependent understanding of the concept.

Previous Studies about Theorems and Proofs

Evidence that PSTs struggle with the concepts of "theorem" and "proof" in building schema is accumulating. According to Stylianides (2007), students' initiation into proof writing depends on teachers' knowledge of proof. After investigating PSTs' understanding of this concept, the authors discovered that both elementary and secondary PSTs struggle with the role of the base step and the inductive step in mathematical induction. These difficulties are more pronounced among elementary PSTs. Additionally, there is persistent confusion regarding the scope of statements proved by induction, particularly concerning whether their truth set may include values outside their domain of discourse, which impacts the rigor of PSTs' reasoning. Ko and Rose (2022) explored the criteria used by pre-service secondary mathematics teachers (PSMTs) to evaluate the validity of self-constructed and student-generated arguments in algebra, geometry, and number theory. The teachers first created and analyzed their own proofs or counterexamples, then assessed six student-generated arguments for validity. The study found that PSMTs prioritized "clear details" in student work over their own and focused more on "verification" in their self-constructed proofs. The study highlights the differential weight given to these criteria and the impact of instructors on PSMTs' conceptions of proofs in proof-intensive courses.

The distinction between proof and demonstration (demonstration mean mathematical proof) is a pivotal concept in mathematics education. While a proof is fundamentally a social act aimed at convincing others using arguments deemed acceptable within a specific context, a demonstration adheres to the strict formal logic characteristic of mathematical tradition. However, teaching rarely involves making the rules of formal logic explicit, which complicates students' ability to validate reasoning. In analysis, demonstrations often rely on implicit rules for manipulating variables to handle quantified statements. These rules substitute for explicit logical references but can lead to errors, especially when dependencies between variables remain unarticulated (Durand-Guerrier & Arsac, 2003). Theorems, on the other hand, are expressed as universal or existential statements that differ across mathematical domains. In geometry, generality is often implicit, conveyed through the use of generic examples. In contrast, in analysis, formal dependencies such as "$\forall\varepsilon\ \exists\delta$" are explicitly stated. In educational contexts, theorems are frequently introduced as tools, with insufficient emphasis on the conditions underpinning their validity. This approach can foster misconceptions among students, both in their understanding of theorems and their application (Balacheff, 1987; Duval, 1992).

Studies That Have Incorporated Free Word Association Test

Within mathematics education, research on teaching mathematics emphasizes the need for integrated training among future teachers. For example, Tastepe (2023) studied the understanding of fractions among future math teachers. Using a word association test (WAT) with 32 participants, she found that their responses mainly centered on the meanings of fractions, particularly highlighting the concept of quotient, while showing little presence of related concepts like percentage. These findings suggest that PSTs demonstrate incomplete understanding of fraction as a concept and thus indicates a potential need for more integrated mathematical training.

Similarly, previous studies highlight the relevance of the WAT, introduced by Jung (1966) and its continuous adaptation in educational research. In mathematics education, this tool helps to explore the cognitive structures of both students and teachers, regarding various learning content. For example, Erdogan (2017) conducted a WAT on the concept of "limit," identifying 87 response words that fell into 18 categories, including "limitation, convergence, uncertainty, continuity, infinity, derivative, function, and left-right limits." Additionally, research by Gökbaş and Erdoğan (2016) on geometry revealed that representations were dominated by quadrilaterals and triangles, while solid shapes in space were largely absent. More recently, Ural (2020) highlighted gaps in understanding *integrals* among future math teachers, using the WAT.

Not only in mathematics education has the WAT been used to study PSTs' cognitive structures but also in science education. Research on teaching science, technology, engineering, and mathematics (STEM) also underscores the necessity for an interconnected approach to learning these subjects. For instance, Hacıoğlu et al. (2016) study examined the cognitive structures of PSTs in Turkey, regarding STEM concepts. Through the WAT and interviews with 192 participants, the study found that these future teachers often viewed STEM subjects independently, lacking clear connections between STEM concepts. They were also observed, struggling to distinguish between scientific concepts and science teaching. These findings father underscore the importance of integrating STEM education to foster deeper connections among subjects.

Further, Kostova and Radoynovska (2008) explored the cognitive structures that teachers and students build regarding the concepts of "living cell" and "biodiversity." Biology teachers strongly associate the concept of the 'living cell' with its structural components, but there are still insufficient links with biochemistry and cellular functions, indicating a need to improve curricula. Teachers' associations with 'biodiversity' are scientific, whereas pupils' associations are mainly emotional and aesthetic, reflecting fragmented teaching. These results highlight the need to clarify textbooks and strengthen curricula on sustainable development and biodiversity conservation. Similarly, a German study by Vlasák-Drücker et al., (2022) employed the WAT to uncover students' perceptions of insects, revealing a positive association with nature but also negative perceptions that necessitate educational initiatives to promote the ecological role of insects.

The application of the WAT extends to physics and chemistry as well. Türkkan (2017) studied future physics teachers' understanding of electric field by applying the WAT, finding association of key concepts such as power, charge, and vector. This study detailed a conceptual network of these ideas. In chemistry, Muningsih (2019) found that students often struggle to conceptualize buffer solutions, underscoring the need for more in-depth teaching approaches. Overall, the WAT has been used to provide a valuable framework for analyzing cognitive structures around various educational concepts, including proof and theorem in mathematics. By helping to describe how pre-service' teachers organize their knowledge in semantic networks, the WAT identifies meaningful associations between related concepts. In the remainder of this paper, we present the methodological approach we adopted for data collection and analysis. Next, we present the main results and, finally, discuss the findings with reference to literature and make some recommendations for the training of future secondary mathematics teachers.

METHOD

The aim of this exploratory research is to gain a better understanding of the cognitive structure of mathematics PSTs in Mayotte with regard to theorem and proof (Schwandt, 1996). We aim to do this by describing how they organize their ideas and knowledge around the concepts of proof and theorem, as well as the way in which these ideas interact with other concepts to form a network of concepts within their minds. We will then develop concept maps for each of the two concepts for these pre-services' teachers.

Schools in Mayotte

It is crucial to place this research in the specific context of the school system in Mayotte, which is characterized by unique cultural and linguistic diversity. This French department ranks among the lowest in terms of educational achievement in France (INSEE, 2023). The PSTs come from different departments of the country, which can influence their pedagogical approaches as well as their conceptions of theorems and proofs. By incorporating this contextual dimension, we aim to offer a nuanced understanding of the difficulties encountered by these future teachers. Furthermore, this study can serve as an example for other education systems facing similar challenges, thus enriching the field of mathematics didactics on an international scale.

Participants

The study involved 40 mathematics pre-services' teachers at the University of Mayotte, France, during the 2023-2024 academic year. All the participants hold a bachelor's degree in mathematics and were enrolled in a second-degree master's program[1]. It should be noted that the pre-service' teachers at the University of Mayotte are not systematically native to the island of Mayotte; some come from other regions of France. These pre-services' teachers spend two days a week in the classroom with a placement of responsibility, alternating between theoretical training at the university and practical training in the classroom. Participants were selected on a voluntary basis. The population was made up of 70% men and 30% women, with an even split between first- and second-year master's students.

Free Word Association Test

This tool is a psychological test, developed in 1905 by Carl Jung–a psychologist, and refined by others in the field. It is administered to assess underlying thought processes of individuals. It is used to study people's emotional responses and traits. The free word association test (FWAT) has a variety of uses and can be adapted to different fields such as psychology, therapeutics, research and education. Although less well known, FWAT as a tool was chosen for its ability to reveal associations and mental representations linked to specific concepts.

This study implemented the FWAT to study the cognitive structures of PSTs as they process information to generate ideas and connect concepts. The concepts of interest in this study are theorem and proof. In this version of the test, these two concepts (stimuli) are presented to the participants and asked to write down the words that come to mind within 30 seconds. One of the goals of mathematics education is to explore how individuals become familiar with mathematical knowledge, analyzing the cognitive processes involved in understanding concepts and solving problems. In this context, studying the cognitive structures related to

[1] The master's degree in teaching, education, and training for secondary education in mathematics

theorem and proof concepts, especially among pre-services' teachers, is essential for understanding how these future teachers build and perceive mathematical concepts.

Data Collection

The data was collected using the FWAT. The participants were presented with two stimuli concepts: "theorem" and "proof". For each stimulus, they were asked to provide related words that came to mind. **Table 1** depicts the collection sheet.

Table 1. FWAT data collection sheet for theorem and proof stimuli

Concept-stimulus: theorem	Concept-stimulus: proof
Theorem	Proof
Theorem	Proof
Theorem	Proof
Theorem	Proof
Theorem	Proof
Theorem	Proof
Theorem	Proof
Theorem	Proof

To reduce potential bias, each participant was given a sheet with a table of 10 cells to fill in. This prevented chain responses and limited the number of terms in a cell. The stimuli were presented one week apart between the two data collection points, with a time limit of 30 seconds for each response. Their responses, constituting detailed data on their respective cognitive structures, were then analyzed and grouped into categories according to their semantic proximity. By clarifying the FWAT methodology, this study meets the need to explore the cognitive structures associated with proof and theorem in mathematics and can identify areas of cognitive conflict.

Data Analysis

Participants' responses to the presented stimuli constituted the data for the study and were analyzed using a rigorous content analysis method. Our analytical process began with an initial coding phase conducted independently by two researchers, followed by the development of categories through a mixed approach that combines predetermined and emerging categories. Coding is an active process to classify data that belong to certain phenomena (Tracy, 2019). Therefore, participants' responses were read and re-read for this purpose. Each word produced was classified based on its semantic proximity to the established categories, using three specific criteria: synonymous relation, belonging to the same lexical field, and functional relationship. Three trained coders independently classified the responses, with inter-rater reliability assessed through Cohen's kappa coefficient (average score of 0.87). To clarify semantic ambiguities, 12 participants took part in follow-up interviews, allowing us to refine the classification and strengthen internal validity.

The results of this analysis were used to construct frequency tables for each stimulus, displaying the evoked categories and their occurrence rates, along with an additional table listing words corresponding to each category. The data were subsequently utilized to generate concept maps with frequency indicators for the notions of "theorem" and "proof," using the MindMeister software. In these maps, the concepts associated with each stimulus were selected as main themes, while the identified categories were represented as sub-themes. The response words linked to each category were organized according to their degree of semantic proximity, providing rich details for each sub-theme. An external expert in mathematics education checked the final concept maps and the semantic consistency of the categories. This process added to the study's external validity and ensured that the data correctly demonstrated the thought of the participants about the mathematical concepts under study.

Usefulness of the Study

This methodology has several advantages for research into mathematics and didactics. Firstly, it allows the cognitive structures of pre-services' teachers associated with a concept to be explored, highlighting their pre-existing conceptual structures and areas of cognitive conflict that may hinder their understanding of theorems and proofs. Secondly, it offers insights into teaching and learning methods, identifying key ideas in need of reinforcement or clarification. Finally, it paves the way for personalized teaching interventions, by adapting teaching strategies to the specific needs of learners. By broadening the scope of our study, we also hope to make significant contributions to understanding the teaching and learning of evidence in a variety of educational contexts, not just in Mayotte.

RESULTS

In this section, the students' answers are analyzed. First, the categories of words associated with the theorem are examined, followed by those associated with the proof. Next, concept maps corresponding to the theorem and the proof are drawn up. The data are presented in tables inspired by previous work (Erdogan, 2017; Türkkan, 2017).

Participants' Responses to the "Theorem" Stimulus Concept

Categories of response words obtained from the stimulus word: "theorem"

The analysis of the participants' responses to the stimulus word: "theorem" identified 46 words. These words were organized into 9 categories of words with which the participants associated the stimulus concept "theorem". **Table 2** presents these categories and their frequencies.

Table 2. PSTs' responses about the stimulus word: "Thereon"

Category		Frequency (f)	Percentage (%)
1	Theorem name	78	31.32
2	Reasoning	45	18.07
3	Theory and logic	38	15.26
4	Practical work	16	6.42
5	Geometry	16	6.42
6	Conditional statement	15	6.02
7	Mathematical method	15	6.02
8	Teaching and learning	14	5.62
9	Research and creation	12	4.81
Total		**249**	**100**

Table 2 indicates that the category: *"theorem name"* dominated participants' responses to the concept of theorem (approximately 31.3%)., The words most frequently mentioned in this category are "Pythagoras" and "Thales" (see **Table 3**). These are names of mathematicians who were named after their theorems. The next frequent word mentioned under this first category is "intermediate value" (12%), followed by "Al-Kashi", "Gauss", "Bézout" and "finite increase" with equal percentage (3%). **Table 3** shows the words mentioned in students cognitive structure associated with theorem in this category.

Table 3. Words mentioned under "theorem name" category related to the theorem stimulus

Category 1	Associations	Frequency (f)	Percentage (%)
	Pythagoras	27	34.61
	Thales	27	34.61
	Intermediate value	12	15.38
Theorem name	Al-Kashi	3	3.84
	Gauss	3	3.84
	Bezout	3	3.84
	Finite increase	3	3.84
Total		**78**	**100**

The second category is "reasoning" in mathematics, which accounted for 18.07% of the response words (see **Table 2**). In this category, the words most frequently mentioned included mathematical proof (33.33%) and "proof" (26.66%). These two concepts refer to the same idea in the classroom context. These concepts were followed by "reciprocal" and "condition" (see **Table 4**). The word "contraposed" is the least occurring word, accounting for 6.66% of the total words under "reasoning" category. These results suggest an association with the intellectual activity of manipulating information to validate a statement. **Table 4** shows the responses classified in the "reasoning" category and their frequency.

Table 4. Participants' response words in the "reasoning" category

Category 2	Associations	Frequency (f)	Percentage (%)
Reasoning	Mathematical proof	15	33.33
	Proof	12	26.66
	Reciprocal	9	20.00
	Condition	6	13.33
	Contraposed	3	6.66
Total		**45**	**100**

The third category is "theory and logic" (15.26% of total words, **Table 2**). The responses from participants refer to elements of mathematical theory or mathematical logic. The words associated with this category are (in order of occurrence): property, rule, implication, definition, conjecture, equality, equivalence, lemma, postulate and theory. We might also have expected the terms 'axiom' and 'corollary'. **Table 5** presents the results of this category.

Table 5. Participants' response words in the "theory and logic" category

Category 3	Associations	Frequency (f)	Percentage (%)
Theory and logic	Property	12	31.57
	Rule	4	10.52
	Involvement	4	10.52
	Definition	3	7.89
	Conjecture	3	7.89
	Equality	3	7.89
	Equivalence	3	7.89
	Lemma	3	7.89
	Postulate	2	5.26
	Theory	1	2.63
Total		**38**	**100**

The fourth category includes answers that refer to practical activities for applying the theorem. This category is referred to as "practical work", accounting for 6.32% of the total words. The words associated with this category include application, exercise, problems and calculation. We note that these are tasks that can be carried out during mathematics learning sessions, where the theorems can be applied. The distribution of the words in this category are shown in **Table 6**.

Table 6. Words in the "practical work" category in mathematics

Category 4	Associations	Frequency (f)	Percentage (%)
Practical work	Application	9	56.25
	Exercise	3	18.75
	Problem	3	18.75
	Calculation	1	6.25
Total		**16**	**100**

The fifth category of participants' responses to the stimulus concept of theorem is geometry (see **Table 2**), which refers to a branch or a field in mathematics. "Geometry" is most mentioned word (43.75% of words under this category), hence the name of the category "geometry". The other words mentioned in this category are "triangle" and "point". The concept of triangle occupies an important place in middle school mathematics teaching. We might also have expected words belonging to the class of quadrilaterals in this category. The responses in this category are shown in **Table 7**.

Table 7. Words in the "geometry" category for the concept-stimulus theorem

Category 5	Associations	Frequency (f)	Percentage (%)
Geometry	Geometry	7	43.75
	Triangle	6	37.5
	Points	3	18.75
Total		**16**	**100**

The next category is 'conditional statement' (6.02%) (**Table 8**). The results in this category include terms related to the components of an implication or deduction. The key terms associated with this category are 'hypothesis' and 'conclusion', which are also integral parts of a deductive reasoning process. It is noteworthy that theorems in secondary school textbooks are often presented in the conditional form.

Table 8. Participants' response words for the "conditional statement" category

Category 6	Associations	Frequency (f)	Percentage (%)
Conditional statement	Hypothesis	9	60
	Conclusion	6	40
Total		**15**	**100**

The words "justify," "check," "approach," "solve," and "find" are all associated with the category of *mathematical methods* (see **Table 9**) because each represents a key step in mathematical reasoning. "Justify" involves providing rigorous proof to support a solution or claim. "Check" refers to verifying the validity of a solution. "Approach" denotes the method or strategy used to tackle a problem. "Solve" is the act of finding a solution to a mathematical problem, while "find" refers to identifying a value or answer within a given context.

Table 9. Response words of participants in the "mathematical method" category

Category 7	Associations	Frequency (f)	Percentage (%)
Mathematical method	Justify	4	26.66
	Check	4	26.66
	Approach	3	20.00
	Solve	3	20.00
	Find	1	6.66
Total		**15**	**100**

Category 8 was named "teaching and learning" (**Table 2**). Participants gave answers relating to the teaching and learning process. The words mentioned under this category included: *lesson*, *course*, *revision*, *example*, *memory*, and *mastery* (**Table 10**). These responses suggest an association with elements of pedagogical system, to which the terms tutorials and revisions could also have been included. **Table 10** shows the distribution of the various response words in this category.

Table 10. Response words from participants in the "teaching and learning" category

Category 8	Associations	Frequency (f)	Percentage (%)
Teaching and learning	Lesson	3	21.42
	Example	3	21.42
	Control	3	21.42
	Memory	2	14.28
	Review	2	14.28
	Courses	1	7.14
Total		**14**	**100**

Category 9, *usefulness and discovery* (**Table 11**), in the WAT for the concept stimulus *theorem*, encompasses words reflecting the practical relevance and the process of uncovering the theorem. The words associated with this category include *utility*, *discovery*, *invented*, and *origin*. These terms highlight both the functional value of the theorem and its conceptual emergence. This category aligns with PSTs' cognitive connections between the theorem's applicability and its epistemological roots.

Table 11. Response words in the "research and creation" category

Category 9	Associations	Frequency (f)	Percentage (%)
Research and creation	Usefulness	3	25.00
	Discover	3	25.00
	Invented	3	25.00
	Origin	3	25.00
Total		**12**	**100**

Pre-service teachers' conceptual network linked to their cognitive structure about "theorem"

Figure 1 shows the conceptual network that explores the PSTs' cognitive structure concerning the concept of "theorem".

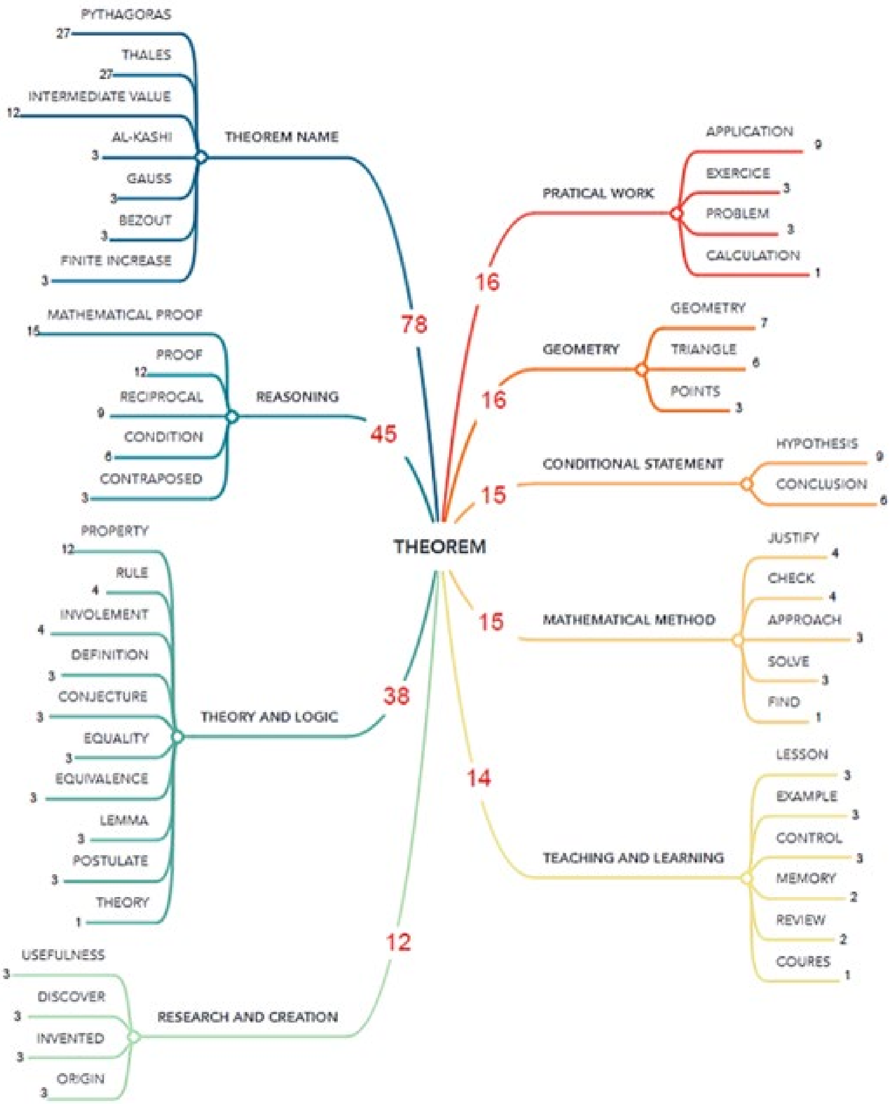

Figure 1. Conceptual network associated with the concept of theorem in the cognitive structure of pre-services' teachers (Source: Developed by authors using MindMeister website)

The term stimulus, introduced to the participants, occupies a central position in the diagram, surrounded by 9 categories of response-words, each represented in a separate-colored cell around the stimulus-concept. The 49 word-responses are also distributed in cells outside the white background, aligned according to their respective category. However, it should be noted that this conceptual diagram does not consider the word frequencies, as presented in the frequency tables. The predominant category seems to be "theorem naming".

Participants' Responses to the "Proof" Stimulus Concept

Categories of response words obtained from the stimulus word: "Proof"

The analysis of participants' responses identified 185 words or expressions that they associated with the stimulus concept "proof". These responses were grouped into 8 distinct categories. **Table 12** presents these categories as well as the words or expressions associated with them.

Table 12. Categories formed from participants' response words

Category		Frequency (f)	Percentage (%)
1	Validation methods	47	25.40
2	Mathematical concepts	42	22.70
3	Proof features	31	16.75
4	Reasoning tools	21	11.35
5	Mathematics statement	16	8.65
6	Argumentation	12	6.48
7	Historical and cultural context	12	6.48
8	Levels of complexity	4	2.16
Total		**185**	**100**

The first category consists of words that refer to validation, hence the name "validation methods" (see **Table 12** and **Table 13**). This category accounted for 25.4% of total response words. The dominant response word under this category is proof and accounted for 42.55% of the words that PSTs associated with "validation methods" (**Table 13**), suggesting a strong association with this category name. The other words mentioned in this category were reasoning, counterexample, writing, show that, proof by recurrence, proof by contraposition, and verify. **Table 13** illustrates the summary of the results.

Table 13. Participant responses in the "validation method" category

Category 1	Associations	Frequency (f)	Percentage (%)
Validation method	Proof	20	42.55
	Reasoning	7	14.89
	Counterexample	6	12.76
	Writing	4	8.51
	Show that	4	8.51
	Proof by recurrence	3	6.38
	Proof by contraposition	3	6.38
	Verify	3	6.38
Total		**47**	**100**

The concepts mentioned by the participants pertain to mathematical concepts, including elements of mathematical theory and logic. In the second category, labelled "mathematical concepts" (f = 42), the most frequently cited response is "example" (f = 15). Other concepts in this category include theorem, property, corollary, statement, and lemma. These responses indicate a strong association with theory, as proof relies on elements of the theoretical framework. **Table 14** presents the frequency of responses for the dominant category.

Table 14. Participants' response words for the "mathematical concepts" category

Category 2	Associations	Frequency (f)	Percentage (%)
Mathematical concepts	Example	15	35,71
	Affirmation	8	19.05
	Theorem	6	14.28
	Conjecture	4	9.52
	Lemma	4	9.52
	Property	4	9.52
	Corollary	1	2.38
Total		**42**	**100**

The third category is called "proof features" (f = 31). It includes participants' responses to the stimulus concept "proof", which refer to the characteristic properties of proof and its components, such as arguments. A key term strongly associated with this category is 'rigor', suggesting that for the participant the construction of a proof adheres to a certain level of rigor. This term is followed by other related terms such as "direct", "absurd", "step", "logical", "negation" and "accepted". **Table 15** shows the frequency of these response terms within the category.

Table 15. Participants' response words in the "proof features" category

Category 3	Associations	Frequency (f)	Percentage (%)
Proof features	Rigor	8	25.80
	Direct	7	22.58
	Absurd	6	19.35
	Step	4	12.9
	Logic	3	9.67
	Negation	3	9.67
	Admitted	3	9.67
Total		**31**	**100**

The fourth category of participants' responses relates to elements employed in the construction of mathematical reasoning, which we term "reasoning tools" (11.35%). This category includes terms such as calculation, hypothesis, diagram, class, level, and conclusion. These elements are crucial in the process of developing mathematical arguments and justifying proof. The frequency of these terms is presented in **Table 16**.

Table 16. Participants' Word responses for "reasoning tools" category

Category 4	Associations	Frequency (f)	Percentage (%)
Reasoning tools	Calculation	5	23.80
	Hypothesis	4	19.05
	Diagram	4	19.05
	Class	4	19.05
	Level	4	19.05
	Conclusion	4	19.05
	Calculation	5	23.80
Total		**21**	**100**

The terms "result," "property," "proposition," and "information" are associated with the category "mathematical statement" in **Table 17** as they represent fundamental components within the structure of a mathematical proof. A "result" refers to a conclusion derived from premises, a "property" denotes a characteristic or rule associated with a mathematical object, and a "proposition" is a statement that can be verified within a mathematical context. "Information," in this context, refers to the data or facts employed to support a proof. These terms, frequently used in mathematical reasoning, highlight the interconnection between mathematical statements and the construction of a proof.

Table 17. Word-responses from participants in the "mathematical statement" category

Category 5	Associations	Frequency (f)	Percentage (%)
Mathematical statement	Results	4	25.00
	Property	4	25.00
	Proposition	4	25.00
	Information	4	25.00
Total		**16**	**100**

The analysis of the participants' answers, as shown in **Table 18**, reveals the following associations: "necessity", "difficulties" and "beliefs". These terms reflect an association with the epistemic value of argumentation, which may be necessary to establish the truth of a result, difficult to construct, and dependent on beliefs about the validity of the results before full proof is provided.

Table 18. Participants' response words in the "argumentation" category

Category 6	Associations	Frequency (f)	Percentage (%)
Argumentation	Necessary	4	33.33
	Difficulty	4	33.33
	Belief	4	33.33
Total		**12**	**100**

The next category that was identified is "historical and cultural context" (see **Table 12**). Participants' responses to the concept of "proof", suggested words such as "mathematicians", "interest", "epoch", and "origin" under this category. Each of these words accounted for 25% of the total words suggested in this category (**Table 19**). This association underscores the importance of historical and cultural references in understanding and conceptualizing the notion of proof, suggesting a strong interconnection between mathematics and its socio-cultural context.

Table 19. Participants' response words for the "historical and cultural context" category

Category 7	Associations	Frequency (f)	Percentage (%)
Historical and cultural context	Mathematicians	4	25.00
	Interest	4	25.00
	Epoch	4	25.00
	Origin	4	25.00
Total		**12**	**100**

The words generated by the participants in response to the concept of "proof", classified as the "levels of complexity" which is the last category, illustrate the complexity inherent in proof tasks. **Table 20** shows the distribution of words that come under this category of responses.

Table 20. Participants' response words in the "levels of complexity" category

Category 8	Associations	Frequency (f)	Percentage (%)
Levels of complexity	Complex	2	50.00
	Simple	1	25.00
	Abstract	1	25.00
Total		**4**	**100**

Pre-service teachers' conceptual network linked to their cognitive structures regarding "proof"

Figure 2 presents the conceptual network that explores the PSTs' cognitive structures concerning the concept of "proof".

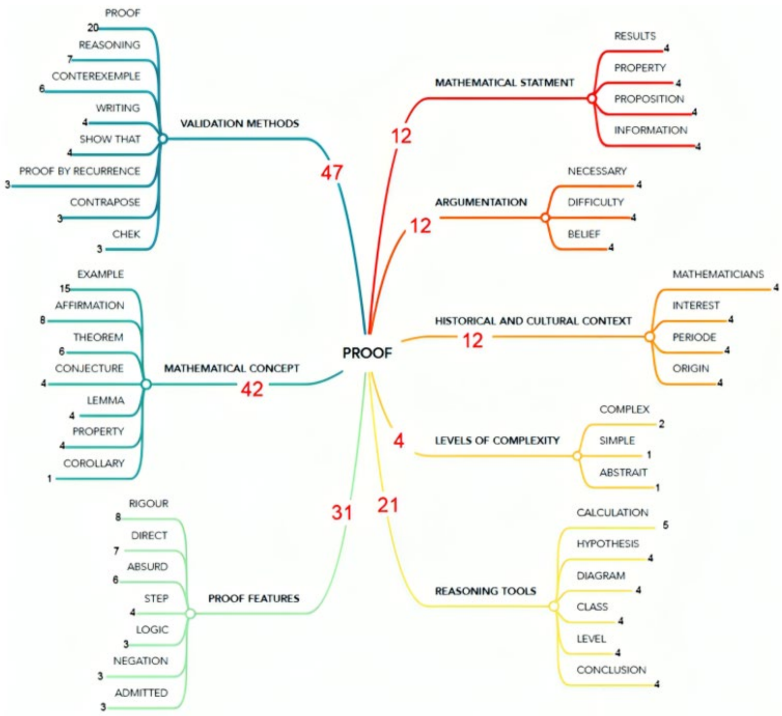

Figure 2. Conceptual network associated proof concept in the PSTs' cognitive structure (Source: Developed by authors using MindMeister website)

The conceptual structure (**Figure 2**) illustrates the cognitive configuration of PSTs in relation to the concept-stimulus "proof". The center of the diagram is occupied by the cell containing the "proof" concept-stimulus, around which there are eight categories of word-responses, each of which is represented by a separate-colored cell around the concept-stimulus. The word-answers are also distributed in cells with a white background, aligned according to their respective category. As can be seen in **Figure 1**, this conceptual diagram does not consider word frequencies.

DISCUSSION AND CONCLUSION

This study investigated the cognitive structure associated with the notions of "theorem" and "proof" among secondary school mathematics PSTs. Data were collected using the FWAT. The analysis identified nine response categories for the stimulus word *theorem* and eight for *proof*. The results suggest that while the participating PSTs appear to have developed some familiarity with these concepts, their responses reflect varying degrees of coherence and depth in their concept images.

The findings further suggest that the predominant response category for the stimulus theorem is the "theorem name." Notably, the most frequently cited theorems are Pythagoras and Thales, both of which are integral to secondary school geometry curricula. It is important to highlight that PSTs typically encounter these theorems during their secondary school years, when foundational geometric concepts are introduced and developed. Other responses within this category, such as "intermediate value theorem" and "finite increase theorem," also pertain to theorems commonly addressed in the secondary mathematics curriculum. This trend aligns with the expectation that PSTs would draw upon concepts embedded in their secondary education experiences. Such findings underscore a strong association between the concept of a theorem and its presentation during secondary school learning. Interestingly, the responses reveal an absence of references to theorems typically encountered in higher education mathematics. This could suggest that PSTs find it cognitively more accessible to relate the concept of a theorem to contexts and instructional experiences from their secondary schooling. From a *concept image* perspective, this observation may indicate that the PSTs' mental representations of the term "theorem" are shaped primarily by their early encounters with mathematical theorems, which were likely framed within explicit pedagogical contexts during their secondary education.

The analysis of participants' responses reveals that some of the terms used align with mathematical reasoning. In particular, the theorem is invoked as a warrant within argument, serving to legitimize the transition from given data to a conclusion. Additionally, the theorem is identified as a true mathematical statement that can be formally prove. A substantial proportion of the responses also reflect connections to practical mathematical activity, as well as to theoretical and logical dimensions of mathematics. These findings suggest that participants associate the notion of a theorem with two critical components of mathematical understanding: proof and theory (Mariotti, 2002). However, it is notable that terms such as "statement," "assertion," and "proposition," which signify the logical status of a theorem, are absent from the PSTs responses (Durand-Guerrier & Arsac, 2003; Njomgang, 2013). This omission points to potential gaps in the cognitive structure of PSTs regarding the concept of a theorem. From a concept image perspective, a theorem is fundamentally a statement, assertion, or proposition that possesses a truth value which can be rigorously established. The absence of these associations in participants' responses suggests a limited or fragmented concept image of a theorem, potentially hindering their ability to engage fully with its logical and epistemological underpinnings. These findings complement those of Sevgi Moralı and Filiz (2023), who highlighted gaps in the understanding of theorems and proofs among PSTs. The authors observed that PSTs struggled to properly assess theorems and proofs, suggesting a need for more targeted training to enhance their conceptual understanding of these key mathematical elements.

In relation to the concept of mathematical proof, the analysis of PSTs' responses indicates that the predominant category is validation methods. Within this category, the most frequently occurring term is "proof" a notion widely recognized and accepted within the mathematical community (Balacheff, 1988). Furthermore, additional response categories emerge that reference mathematical theory, often invoked as a form of justification in arguments, alongside categories that highlight specific attributes associated with mathematical proofs. However, it is notable that key concepts such as "argument", "deduction", "validity", and "truth" are conspicuously absent from the responses of the PSTs. The absence of references to the notions of "validity", "truth", and "argument" in PSTs' responses may reflect a limited concept image of proof, reducing it to a mere application of techniques rather than a rational process of justification. As Duval (1992) has shown, validating a proof is not solely based on its syntactic structure but also requires an understanding of its epistemological status. In classroom settings, this gap could lead teachers to assess students' proofs solely based on their adherence to formal procedures, without questioning their logical coherence or grounding in argumentative reasoning. A typical example illustrating these shortcomings can be found in PSTs' responses to the following question: *"Why is a proof necessary to establish the truth of a mathematical property?"* A frequent response is: *"Because it is a rule in mathematics."* Here, the teacher does not invoke the notions of truth or validity but rather refers to an institutional view of proof as an external constraint. A more sophisticated response would have been: *"A proof establishes the validity of a mathematical property by relying on axioms and accepted rules of logic".* This example highlights how PSTs tend to perceive proof as a formal exercise rather than an argumentative process, underscoring the need to strengthen these dimensions in teacher education programs. From a didactical perspective, we contend that these dimensions of proof should be given greater prominence in teacher education, with particular attention to their integration into practical activities. Doing so may help PSTs develop a richer *concept image* of proof, encompassing not only its validation function but also its epistemological and argumentative roles within mathematical discourse. These results align with those of Lesseig and Hine (2022). Their findings indicate that future teachers can develop a richer and more nuanced concept image of proof if an appropriately designed didactic training can be organized for them. This can incorporate both the argumentative and epistemological dimensions of proof. Further analysis reveals that the response words elicited from participants in reaction to theorem-related stimuli also appeared in their responses to mathematical proof-related tasks (e.g., *conjecture*, *lemma*, *property*). This finding highlights a significant overlap between the *concept images* associated with the notions of theorem and proof in the cognitive structure of the PSTs. Such results align with Mariotti's (2002) characterization of the theorem, underscoring its dual role as a deductive construct and a cognitive entity. Consequently, we infer that PSTs develop a tightly interconnected semantic network linking theorem and proof, suggesting that these concepts are not only functionally related but also semantically proximal within their cognitive structure. It is also notable that participants mentioned certain terms classified within categories that appear to have tenuous or irrelevant associations with the stimulus concepts *theorem* and *proof* in terms of semantic proximity (e.g., *lesson* or *course origin*). Specifically, for the stimulus concept *theorem*, terms falling into the category *utility and discovery* illustrate this phenomenon, while for the concept *proof*, terms associated with the category *historical and cultural context* serve as further examples. In this study, we refer to these phenomena as *incoherent associations*. Such associations point to potential conflicts within the participants' evolving *concept image* of *theorem* and *proof*. These conflicts suggest that the participants may lack a coherent concept image for these concepts, which could act as significant barriers to the development of their cognitive structures (or *mind-maps*) related to these mathematical concepts. This finding implies that PSTs require targeted support during their initial training to reconcile and refine their concept images. Addressing these incoherent associations through deliberate instructional interventions could be crucial in enabling them to develop a robust and interconnected understanding of *theorem* and *proof*.

This study examines the cognitive structures secondary school mathematics PSTs form around the concepts of theorem and proof, with particular attention to their *concept images* the mental representations and associations they develop. The findings illuminate the ways participants conceptualize these fundamental notions, offering insights into their underlying mental frameworks. Such insights are invaluable for designing more effective teacher training programs and instructional strategies. While proof is not systematically addressed in the French mathematics curriculum, the findings underscore the importance of equipping teachers with robust skills in this domain. The ability to engage with, comprehend, and construct mathematical proofs is central to fostering deep conceptual understanding in mathematics, both for teachers and their students.

To address this need, we propose several recommendations. As part of teacher education, it would be beneficial to engage PSTs in the analysis and correction of erroneous proofs to deepen their understanding of the notions of validity, truth, and argumentation. Next, structured mathematical debates should be implemented to strengthen their ability to distinguish between valid reasoning, which holds a positive epistemic value, and informal reasoning, which has a negative epistemic value (Duval, 1992). Furthermore, it is essential to enhance their knowledge of the theoretical status of mathematical statements by training them to identify and differentiate between various types of statements (axioms, theorems, conjectures) and the corresponding validation methods. In addition, initial teacher training should include a focus on mathematical statements, particularly theorems and definitions. This will support PSTs in constructing a coherent cognitive framework for the concept of a theorem in mathematics, encompassing their formulation, the various forms they assume in secondary school, and their function in proof activities. Secondly, practical sessions centered on mathematical proof should be incorporated into the teacher training curriculum. These sessions are expected to help teachers develop their practice by engaging with evidence, gaining an understanding of the nature of mathematical evidence, and the role this evidence plays in teaching. In turn, this will provide them with opportunities to explore the types of evidence that can be introduced to secondary school students, fostering a deeper understanding of how proofs are substantiated and communicated. These measures can bridge the gap between procedural fluency and the rich, interconnected *concept image* of proof required for effective teaching and learning in mathematics.

This study has certain limitations. While it could have been extended to include elementary mathematics PSTs, the study was limited to only secondary school mathematics PSTs in Mayotte (in France) educational context. Therefore, while the results are credible, overgeneralization to include all PSTs cannot be advised. Again, a comparative analysis with other regions would help determine whether these conceptual struggles are unique to the Mayotte setting or indicative of a broader trend. Although the study's focus was to investigate the cognitive structure of PSTs regarding their ideas about proof and theorem, it would have been relevant to investigate the extent of their understanding of these concepts and how they influence teachers' classroom practices. Finally, a future longitudinal study to track teachers over several years would provide deeper insights into the development of their concept image of theorem and proof throughout their training and professional experience.

In summary, this article underscores the critical importance of PSTs' understanding of the concepts of proof and theorem, as well as the challenges they encounter in both teaching and learning these mathematical ideas. Drawing from the theoretical framework of the concept image, it highlights how teachers' mental representations of mathematical concepts influence their pedagogical practices. The article further explores strategies to enhance teacher training, with an emphasis on developing a more profound and intuitive grasp of mathematical principles among students. It advocates for a more conceptual approach to teacher preparation that acknowledges the role of these images in shaping both teaching strategies and student understanding.

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