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Research Article

Assessing procedural and conceptual understanding on the limit concept: A study on first-year university students

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ABSTRACT

The limit concept is foundational in undergraduate mathematics. This study assessed procedural knowledge and conceptual understanding of the limit in 87 first-semester students from a major Greek university, enrolled in a calculus I course. Using a structured questionnaire, data were analyzed with quantitative methods (descriptive statistics) and qualitative methods (thematic analysis of student responses). Findings revealed a limited conceptual grasp and a reliance on computational techniques, supporting Skemp's (1976) distinction between instrumental and relational understanding. Students exhibited challenges in interpreting dynamic processes and showed persistent, rigid concept images that deviated from the formal definition. The study suggests practical adjustments for teaching the limit concept in undergraduate mathematics.

Keywords: limit, function, instrumental, relational, concept image, concept definition

INTRODUCTION

The concept of the limit is a foundational element in mathematics, valued both for its practical applications and its critical role as a prerequisite for more advanced topics in calculus (Nagle, 2013). Core concepts in mathematical analysis—including convergence, continuity, differentiation, and integration—are fundamentally rooted in the understanding of limits. Consequently, insufficient knowledge of the limit concept can hinder students' ability to engage with subsequent analytical ideas (Juter, 2006). Recent research confirms that these conceptual hurdles persist, often due to a disconnect between students' procedural fluency and their formal understanding of the limit's definition (Adiredja, 2014; Swinyard, 2011). A substantial body of research in mathematics education has examined students' procedural and conceptual understanding of limits, consistently revealing significant challenges and persistent misconceptions (Areaya & Sidelil, 2012; Denbel, 2014; Winarso & Toheri, 2017). This distinction is crucial, as students often master computational techniques without grasping the underlying logical structure (Jones, 2015). The present study aims to assess the extent of university students' procedural knowledge and conceptual understanding of the limit concept, and to systematically identify and analyze the misconceptions they hold regarding this essential mathematical idea. Current research emphasizes that students' struggle with limits is often linked to a lack of structural thinking (Swinyard & Larsen, 2012) and the persistent use of informal language in secondary education (Alcock & Simpson, 2017). Recent findings also suggest that university students often perceive calculus as a collection of disjointed procedures rather than a unified conceptual framework (Tall & Vinner, 1981).

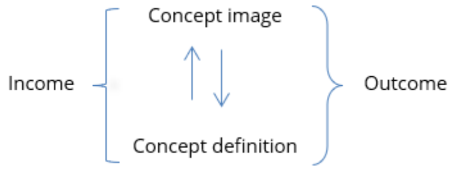

Figure 1. Model 1. Existence of interaction between concept image and concept definition (Vinner, 1991)

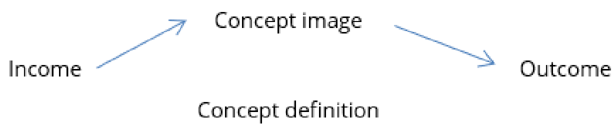

Figure 2. Model 2. Concept image is not used (purely formal abstraction) (Vinner, 1991)

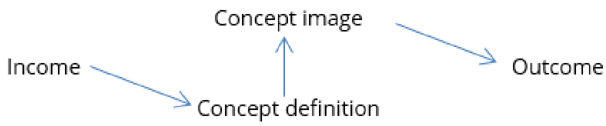

Figure 3. Model 3. Subtraction after intuitive processing (Vinner, 1991)

THEORETICAL FRAMEWORK

This study is grounded in the theoretical constructs of concept image and concept definition, as articulated by Tall and Vinner (1981). A *concept image* encompasses the total cognitive structure associated with a mathematical concept, including all mental representations, visualizations, intuitions, and experiential knowledge developed over time. In contrast, a *concept definition* refers to the precise, formal verbal definition agreed upon by the mathematical community.

Many everyday actions are carried out successfully without the explicit use of formal definitions; instead, individuals depend on intuitive perceptions to interpret and act upon the world around them. For example, we have the ability to perceive the concept of school, either as a place or as the activities that take place within it, without the necessity of a formal definition.

Students, as members of this broader cognitive environment, often transfer this behavior into their mathematical thinking. As a result, they may not perceive the necessity of applying formal definitions when solving problems. While such practices can sometimes yield correct outcomes, they may also lead to significant conceptual errors, especially when the informal image held by the student does not align with the rigor demanded by formal mathematics. A characteristic example of this observation is the root. In everyday practice, when we refer to a root, we mean the underground part of a tree or a plant, whereas in the world of mathematics we refer to the roots of an equation or the roots of numbers.

Vinner (1991) further elaborates on the dynamic relationship between these two constructs by introducing a two-cell model. In this framework, *cell 1 (C1)* corresponds to the concept definition, while *cell 2 (C2)* refers to the concept image. At the initial stages of learning, one or both of these cognitive components may be underdeveloped or empty. As students interact with instructional content, both cells are gradually populated. However, this development does not occur uniformly across learners; rather, it is shaped by multiple factors, including prior knowledge, learning experiences, and individual cognitive processing capacity.

This framework offers a useful lens through which to interpret students' mathematical thinking, particularly in identifying gaps or inconsistencies between formal definitions and intuitive understanding. The interaction—or lack thereof—between concept image and concept definition has significant implications for conceptual development and is often at the root of persistent misconceptions in mathematical learning.

According to Vinner (1991), there are four distinct cognitive models that describe the interaction between concept image and concept definition during problem-solving: **Figure 1** shows the model 1. **Figure 2** depicts Model 2. **Figure 3** shows Model 3.

Figure 4. Model 4. Absence of concept definition (Vinner, 1991)

Research related to the above issue shows that a large percentage of students do not use models 1, 2, and 3, but use model 4 in **Figure 4**.

These theoretical perspectives constitute the conceptual foundation of the present study, which aims to investigate how first-year university students understand the concept of the limit of a function. By employing the notions of concept image and concept definition, along with the distinction between procedural and conceptual understanding, this study seeks to identify students' misconceptions, analyze their reasoning strategies, and assess the extent to which their understanding aligns with formal mathematical rigor.

INSTRUCTIONAL GUIDELINES FOR TEACHING LIMITS IN HIGH SCHOOL AND UNIVERSITY AND STUDENTS' MISCONCEPTIONS REGARDING THE CONCEPT

The ε-δ definition of the limit is not taught in Greek secondary schools (lyceum). As a result, the teaching of limits at this level necessarily relies on intuitive approaches to the concept. For example, it is suggested to construct the graph of the function $f(x) = xsin\frac{1}{x}$, and through it students can intuitively realize that the existence of a limit does not necessarily imply the monotonicity of the function. It is also suggested to construct the graph of the function $f(x) = sin\frac{1}{x}$, so that students can understand that when a limit does not exist, this does not necessarily mean that the one-sided limits exist and are unequal.

Indeed, the official mathematics teaching guidelines for the final year of lyceum emphasize that instruction should focus on an intuitive introduction to limits, highlighting the use of graphical representations of appropriate functions to provide students with a clear conceptual image and to prevent common misconceptions that, according to the literature, tend to arise in students' understanding of the notion.

At the university level, the teaching of limits begins with the presentation of the ε-δ definition, followed by examples of limit calculations based on this definition. In some mathematics departments in Greece, as in the department of mathematics of the Aristotle University of Thessaloniki and in its counterpart in Athens, the instruction of sequences precedes that of function limits, so that students first encounter the concept of limit in the context of sequences. It is noteworthy that no systematic bridging exists between the intuitive treatment of the concept at the secondary school level and the formal ε-δ definition introduced at university.

The literature reports numerous misconceptions identified both among high school students and university undergraduates. Several researchers have highlighted such difficulties (Cottrill et al., 1996), which primarily concern students' conceptual understanding of the limit.

Williams (1991) observes that students often struggle with the concept of limit, raising questions such as whether a function can in fact attain its limit, whether the limit itself should be interpreted as a boundary, whether limits should be regarded as processes in motion or as fixed objects, and whether the idea of limit is inherently linked to movement.

In the same study, Williams highlights that most students who complete a calculus course retain only a preliminary, non-rigorous understanding of limits, while relatively few manage to achieve full mastery of the formal definition. Cottrill et al. (1996) likewise stress the persistence of these difficulties, pointing out students' limited success in developing a deeper grasp of this fundamental mathematical idea. Although such preliminary levels of comprehension may appear adequate for some learners, Tall and Vinner (1981) caution that informal conceptions of the limit can give rise to serious misconceptions and may obstruct students' future progress in mathematics. Recent studies (Swinyard, 2011) confirm that these same misconceptions—specifically the struggle to view the limit as a static object rather than a dynamic process—remain prevalent among modern university students.

THE RESEARCH

Description of Participants

This study was conducted with undergraduate students from the department of mathematics within the faculty of sciences at a Greek university. A total of 87 participants completed the calculus I course at the time of data collection, which included instruction on fundamental topics such as limits, convergence, continuity, and power series. In addition to the core mathematical content, the study also accounted for the following demographic variables as control parameters:

(1) year of study and

(2) gender.

To ensure the validity and reliability of the findings, a mixed-methods approach was employed for data analysis. Quantitative analysis involved descriptive statistics to determine the frequency and percentage of correct, incorrect, and partially correct responses for each of the seven items. Qualitative analysis focused on the thematic analysis of students' written justifications, particularly for questions 3, 4, 6, and 7, to identify recurring misconceptions and align them with the cognitive models of Vinner (1991).

METHOD

The research was conducted through a written questionnaire, administered after the conclusion of the calculus I course and prior to the final examinations. The instrument comprised seven structured questions, each designed with a specific investigative aim related to students' procedural knowledge, conceptual understanding, or both, regarding the concept of the limit.

Below is a detailed overview of the questionnaire items and their respective pedagogical intentions in detail:

1. Calculate, if it exists, the limit: $\lim\limits_{x \to 0} \frac{e^x - 1}{\ln(x+1)}$. Question 1 is designed to assess students' procedural knowledge. It is important to note that upon entering university, most students have already been exposed to a range of procedures and techniques for evaluating limits. Among these, L'Hôpital's rule is the most commonly applied, often functioning as the primary strategy used for solving indeterminate forms. This item examines whether students rely exclusively on such procedural tools or demonstrate flexibility in applying alternative techniques, such as the use of the conjugate expression, factorization, and the application of the squeeze theorem.
2. Calculate, if it exists, the limit: $\lim\limits_{x \to +\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$. Question 2 aims to examine students' ability to evaluate a limit that cannot be solved using L'Hôpital's rule**.** This is a particularly significant focus within the context of Greek mathematics education, where the application of L'Hôpital's rule is often employed uncritically and treated as a universal method. As a result, students may fail to engage with or develop alternative approaches to limit evaluation, such as algebraic manipulation, rationalization, or asymptotic reasoning. This item thus serves to reveal the extent of students' procedural flexibility and conceptual adaptability in limit computation.
3. Given a function $f: A \to \mathcal{R}$ such that $\lim\limits_{x \to 1} f(x) = 3$. Check the truth of the following sentences, justifying your answer each time.
   a. The value of f for *x* = *1* equals 3.
   b. As x approaches 1, f tends to take the value 3 without being able to equal 3.
   c. The number 1 belongs to the domain A.

   Question 3 is intended to elicit and categorize students' concept images and concept definitions associated with the limit of a function**.** By requiring students to evaluate the truth value of specific statements and justify their reasoning, this item offers insights into the nature of their underlying conceptual frameworks. It allows for the identification of potential discrepancies between formal definitions and students' personal interpretations, thereby shedding light on common misconceptions and the degree of alignment between procedural understanding and conceptual meaning.

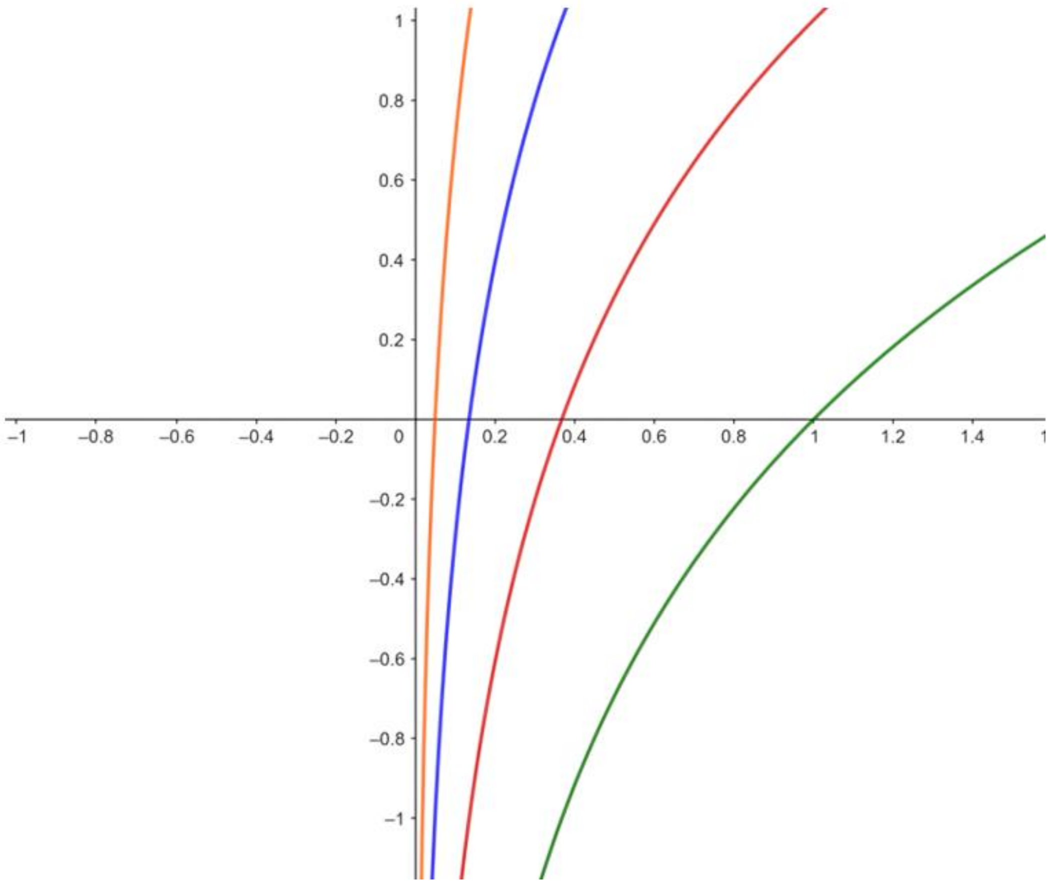

Figure 5. Graphical representations of the functions $f_0(x), f_1(x)\,, f_2(\mathrm{x}), \text{and } f_3(\mathrm{x})$ (Source: Created by the authors)

4. Given a function $f\colon \mathcal{R} \to \mathcal{R}$ such that $\lim_{x\to 1} f(x) = 3$. Write what the above equality means. Question 4 shares the same objective as question 3, namely, to investigate students' concept images and concept definitions concerning the limit of a function**.** However, it differs methodologically by providing an open-ended format: instead of evaluating predetermined statements, students are asked to articulate, in their own words, the meaning of a given limit expression. This structure enables participants to express their thinking without external scaffolding, thereby offering a more authentic representation of their internalized conceptualizations and intuitive understanding.
5. Write the ε-δ definition of the finite limit of function of a real variable. Question 5 is designed to assess students' ability to accurately recall and reproduce the formal ε-δ definition of the limit of a real-valued function. Mastery of this definition is essential for a rigorous understanding of calculus, and this item aims to reveal the extent to which students can transition from intuitive or procedural interpretations of limits to their formal mathematical characterization.
6. Let's assume that from today onwards you will save money in an infinite process, with the following rule: Today you save 1 euro, tomorrow ½ of a euro, the day after tomorrow ¼ euro, and so on. Will there be a day when the money you have saved in total will exceed or be equal to 2 euro? Justify your answer.
7. The sequence of function is given: $f_n(x) = lnx + n, n = 0, 1, 2, \ldots$ and $x > 0$. In the following Cartesian coordinate system there are the graphical representations of the functions $f_0(x), f_1(x)\,, f_2(\mathrm{x}), \text{and } f_3(\mathrm{x})$ (**Figure 5**).

Consider whether there exists $n_0 \epsilon \mathbb{N}$ such that the point of intersection of a graph of $f_{n_0}(x)$ with the $x'x$ axis is the minimum possible distance from the origin of the axes, if it exists. Justify your answer.

Question 6 and question 7 serve a dual purpose. First, they require students to engage in elementary mathematization, as both questions involve infinite processes presented through different representational

Table 1. Performance on question 1 and question 2

Question	Correct (%)	Incorrect (%)
1. Calculate, if it exists the limit: $\lim_{x\to 0}\frac{e^x-1}{\ln(x+1)}$	95	5
2. Calculate, if it exists the limit: $\lim_{x\to +\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}$	76	24

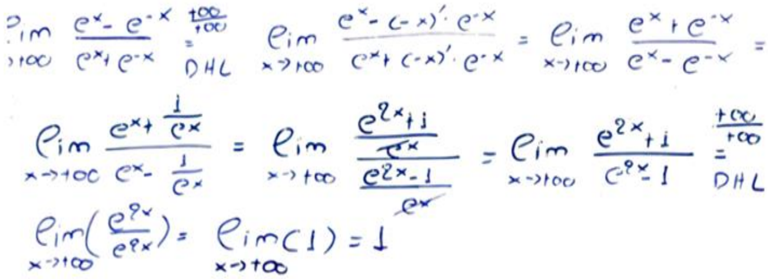

Figure 6. Student's answer to question 2 (Source: Created by the authors)

Table 2. Performance on question 3

Question		Correct (%)	Incorrect (%)
3. Given a function $f: A \to \mathcal{R}$ such that $\lim_{x\to 1} f(x) = 3$. Check the truth of the following sentences, justifying your answer each time.	a. The value of f for $x = 1$ equals 3.	60	40
	b. As x approaches 1, f tends to take the value 3 without being able to equal 3.	46	54
	c. The number 1 belongs to the domain A.	61	49

formats. Question 6 necessitates familiarity with the concept of a geometric series, while question 7 requires the ability to construct and interpret elementary function graphs within a coordinate system. Ultimately, these two items are designed to probe the depth of students' conceptual understanding of limits, particularly in contexts that combine formal reasoning with intuitive, visual, and applied mathematical thinking.

RESULTS

The first two questions aim to assess students' ability to compute limits. In question 1, the limit is evaluated using L'Hôpital's rule, whereas in the second case, this rule is not applicable, and a different technique is required. It is worth noting that incoming university students have typically been taught limit computation techniques with particular emphasis on the application of L'Hôpital's rule. In this context, the results are largely as expected. Students' procedural knowledge and their ability to apply it are clearly evident, as shown in **Table 1**.

It is noteworthy that in the case of the second limit, the success rate is significantly lower (76%), indicating that students demonstrate limited proficiency in applying limit computation techniques that do not involve L'Hôpital's rule. This suggests a strong dependence on this particular method, while alternative approaches appear to be less well understood or utilized, e.g., conjugate expression, factorization, and the squeeze theorem.

The increased failure rate in question 2, compared to question 1, indicates the strong dependence of students on the use of L'Hôpital's rule. A very characteristic example is the following case, in which the student applies L'Hôpital's rule incorrectly (since he does not initially verify the existence of the limit of the quotient of the derivatives) and then proceeds to calculate the limit using simple algebraic manipulations, even though it is evident that this procedure could have been applied from the outset (**Figure 6**).

Question 3 consists of three sub-items, each aiming to provide an initial insight into students' conceptual understanding of the limit concept. It is important to note that these students have already been taught the formal *ε*-*δ* definition of a limit and have studied related topics such as sequence and series convergence. The phrasing used in the sub-items—such as "arbitrarily close" or "tends to"—draws from terminology commonly employed in upper secondary education, where the *ε*-*δ* definition is not included in the curriculum. According to national instructional guidelines, the teaching of limits at the high school level is based primarily on an intuitive approach (**Table 2**).

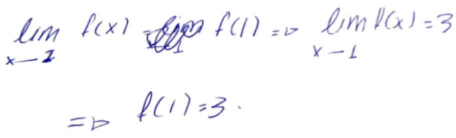

Figure 7. Student's answer to question 3a (Source: Created by the authors)

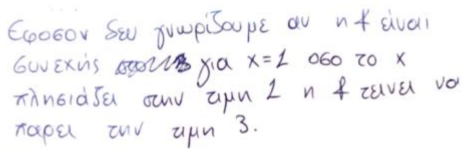

Figure 8. Student's answer to question 3b (Source: Created by the authors)

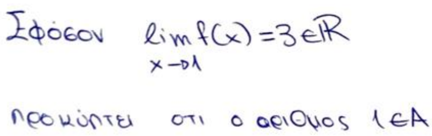

Figure 9. Student's answer to question 3c (Source: Created by the authors)

It is readily observable that the percentages of correct responses decrease significantly compared to the first two questions. As factors related to conceptual understanding of the limit, such as the understanding of the role of quantifiers in mathematical logic, the role of the absolute value, and the proper interaction between the concept image and the concept definition, begin to play a more prominent role in arriving at a correct answer, success rates decline accordingly. Moreover, common misconceptions regarding the notion of limit begin to emerge—misconceptions that are well-documented in the relevant literature. The qualitative analysis of these responses suggests that many students operate under model 4 (absence of concept definition), where their reasoning is guided exclusively by a fragmented concept image, or model 3, where intuitive processing overrides any formal mathematical deduction (Vinner, 1991).The most significant of these, as reported in the literature, refer to: equating the limit with the function value, assuming that the existence of a limit implies that the function actually reaches the limiting value and confusing limit with monotonic approach (Cornu 1991).

With regard to item 3a, a widely documented misconception in the literature concerns the conflation of the limit of a function with its value at a point (Adhikari, 2020; Denbel, 2014). Among the incorrect responses to this item, approximately 80% of students demonstrated this specific misunderstanding, indicating a strong tendency to assume that $\lim_{x \to c} f(x) = f(c)$ must always hold, regardless of the underlying conditions.

A representative example of misconception is illustrated in **Figure 7**, that is, in question 3a the student responds as shown in **Figure 7**.

In question 3b, where nearly 54% of students provided incorrect answers, two characteristic patterns of reasoning become apparent. First, there is evidence of the commonly reported view that a function approaches its limit asymptotically without ever attaining it (Denbel, 2014). Second, a tendency is observed among students to associate the limit at a point with the actual value of the function at that point, essentially interpreting the notion through continuity. An illustrative example of this type of response is presented in **Figure 8**.

The student considers the statement in question 3b to be correct and explains: 'Since we do not know whether the function *f* is continuous at *x* = *1*, as *x* approaches the value 1, f tends to take the value 3.

In question 3c, nearly half of the students provided incorrect answers, reflecting the belief that it only makes sense to investigate the limit of a function at a point when that point is included in its domain. This type of reasoning, identified in the literature (Denbel, 2014), can be considered a fundamental misconception, as it suggests that students struggle with grasping the concept of limit even at an introductory, intuitive level, for instance, attempting to interpret limits through the graphical representation of a function.

As shown in **Figure 9**, the student responds: "Since $\lim_{x \to 1} f(x) = 3$. it follows that the number $1 \in A$."

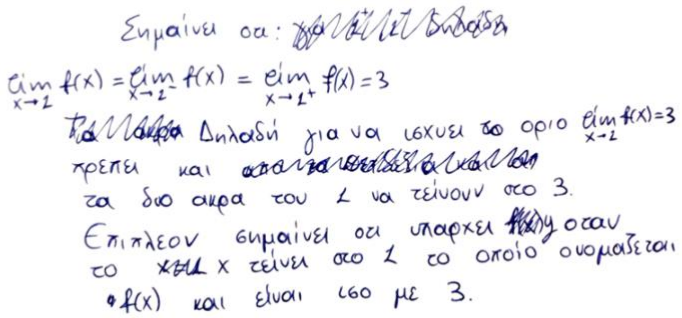

Figure 10. Student's answer to question 4: First case (Source: Created by the authors)

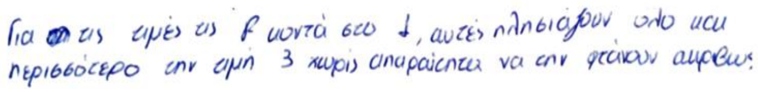

Figure 11. Student's answer to question 4: Second case (Source: Created by the authors)

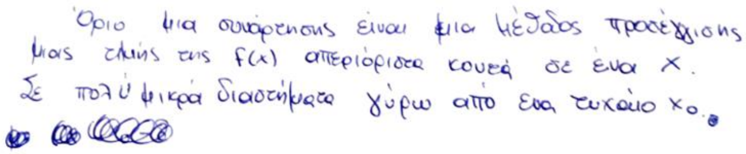

Figure 12. Student's answer to question 5: First case (Source: Created by the authors)

Regarding item 4, and based on the results of the study, the proportion of responses that can be considered mathematically adequate does not exceed 40%. It is worth emphasizing that the participants in this study were students who had been formally taught the *ε*-*δ* definition of the limit. Nevertheless, not a single response explicitly referenced this definition. The students' answers to this item clearly revealed that their concept images remain primarily grounded in the intuitive approach taught at the secondary education level. Furthermore, their concept image appears to have been only minimally influenced—if at all—by their university-level instruction (**Figure 10**).

In this case, the student states that

> the equality $\lim_{x\to 1} f(x) = 3$ means that $\lim_{x\to 1} f(x) = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 3$. That is, for the limit to hold $\lim_{x\to 1} f(x) = 3$, both one-sided at 1 must tends to 3. Moreover, this means that there exists a value *y* when *x* tends to 1, which is denoted as *f(x)*, and it tends to 3.

The student's misunderstanding of the limit concept is both evident and illustrative of the broader set of incorrect responses. It appears that the prior, intuitive knowledge of limits—developed during secondary education—continues to shape reasoning, often outweighing the influence of subsequent exposure to the formal *ε*-*δ* definition at the university level.

According to the response of another student, we read that the given equality means that "for values of *f* close to 1, they approach the value 3 more and more closely without ever reaching it exactly" (**Figure 11**).

As widely reported in the literature, university students face notable challenges in reproducing the formal *ε*-*δ* definition of a limit (Cornu, 1991; Cottrill et al., 1996; Fernández-Plaza et al., 2013). In the present study, as evidenced by student responses to item 5, the percentage of incorrect answers reaches 73%. The *ε*-*δ* definition comprises a sequence of logical quantifiers, each serving a specific and nontrivial role. When a student's conceptual understanding of these roles is underdeveloped, the likelihood of incorrect reproduction of the definition increases substantially. There are also instances in which the formal definition of the concept is entirely absent from a student's cognitive framework, and only a *concept image*—likely shaped by earlier instruction—remains. Such is the case in this particular example (**Figure 12**).

As shown in **Figure 13**, the student writes: "The limit of a function is a method of approaching a value of *f(x)* infinitely close to some *x*. Within very small intervals around an arbitrary x_0."

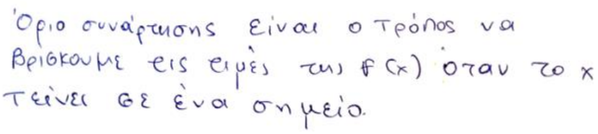

Figure 13. Student's answer to question 5: Second case (Source: Created by the authors)

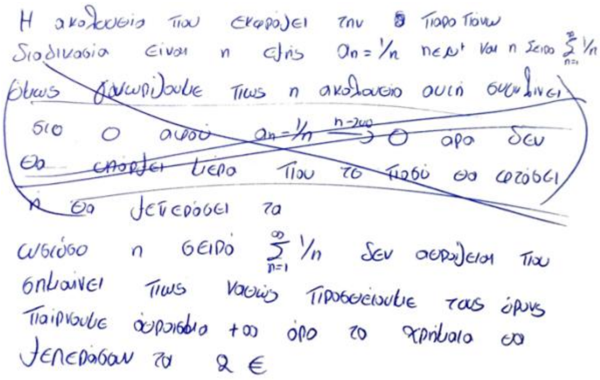

Figure 14. Student's answer to question 6 (Source: Created by the authors)

In **Figure 14,** another student writes: "the limit of a function is the way to find the values of *f(x)* when *x* tends to a point." A closer look at the answers shows that, even though the task explicitly required the formal definition of the limit, students tended to rely on personal mental images of the concept. These images, however, were often inaccurate. The majority of responses followed this same pattern, which highlights that university teaching had little impact on reshaping their understanding. As a result, students were still unable to produce the formal definition of the limit when asked.

It is particularly significant that this definition concerns a foundational mathematical concept upon which all major ideas in Analysis—such as the derivative, the integral, and continuity—are built. From this perspective, conceptual understanding, and thus the ability to accurately reproduce the formal definition, is not merely desirable but essential for deeper engagement with the core constructs of Mathematical Analysis.

Question 6 serves a dual purpose**.** First, it aims to assess students' ability to mathematize a problem situation. The results of this study suggest that students experience significant difficulty in this regard. Out of 87 scripts, 34 were left unanswered, possibly due to the absence of the relevant mathematization skills. This difficulty has also been well documented in the international literature, with some studies reporting even higher failure rates (Brahmia 2014; Jupri & Drijvers 2016). Among the remaining 53 responses, only 11 demonstrated mathematically correct reasoning. The rest revealed prominent misconceptions held by the students, largely confirming the findings of existing research on students' struggles with mathematical modeling and abstraction.

It is also worth noting that, in several cases, students provided responses that are mathematically or conceptually incoherent—responses that fail to align with the internal logic of the problem or the broader mathematical reality. Nevertheless, such answers were accepted as valid by the students themselves, reflecting a concerning disconnect between reasoning and validation. A particularly illustrative example is the following. This student's answer is as follows:

> The sequence that represents the above process is the following: $\alpha_n = \frac{1}{n}$ and the series $\sum_{n=1}^{\infty}\frac{1}{n}$. However, the sequence $\sum_{n=1}^{\infty}\frac{1}{n}$ does not converge, which means that as we keep adding the terms, the sum grows $+\infty$, therefore, the total amount of money will eventually exceed 2 euros.

While the student manages the formal aspects of mathematization reasonably well, his later reasoning reveals challenges in understanding the convergence of sequences and series.

Question 7 involved four distinct mathematical domains: the graphical representation of functions, sequences of functions, convergence, and mathematical modeling. Given this multifaceted nature, the item was of increased complexity. As anticipated, the majority of student responses were incorrect or omitted

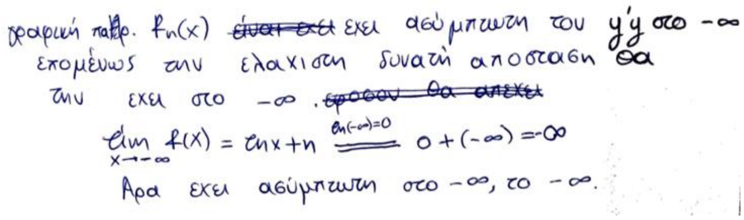

Figure 15. Student's answer to question 7 (Source: Created by the authors)

entirely. Specifically, 79% of the students did not provide any response to the question, which may be indicative of either a lack of familiarity with the interconnected concepts or an inability to engage in mathematization of the problem. Among the remaining 21% of responses, only 9 submissions (approximately 11%) were mathematically correct, highlighting the significant challenge posed by this item. These findings underscore the broader difficulties students encounter when required to coordinate multiple representations and formalize reasoning involving limit processes and convergence within a modeling context.

A representative example of the difficulties described is found in the following student response, which demonstrates a complete lack of mathematical coherence (**Figure 15**). The answer fails to establish any consistent logical progression or accurate mathematical reasoning, indicating that the student may not have been able to access or coordinate any of the required conceptual tools (e.g., function representation, convergence, and modeling). Such instances reflect not only a superficial understanding of the concepts involved but also the absence of meaningful mathematization, as discussed in related literature (Brahmia, 2014).

"The graph of *f(x)* has the asymptote *y′y* at $-\infty$. Therefore, the minimal possible distance is attained at $-\infty$. Hence, the function has an asymptote at $-\infty$. the $-\infty$."

The student's answer reveals a blend of misunderstandings that span several areas: asymptotes, the idea of shortest distance, the use of mathematization, limits, and the way mathematical processes are interpreted through graphs of functions. This mixture of difficulties shows how easily intuitive reasoning can conflict with the formal framework of mathematical analysis.

DISCUSSION

The findings of this study reveal a wide range of significant difficulties faced by university students, not only in their understanding of the limit concept, but also in related processes such as mathematization, limit computation, and reproduction of formal definitions. Although students demonstrated strong procedural fluency in computing limits using L'Hôpital's rule (question 1), their performance declined markedly in tasks requiring alternative techniques (question 2). This outcome was expected, given that in the final year of Greek secondary education, L'Hôpital's rule receives considerable emphasis. These results indicate a reliance on rote procedures and a narrow set of strategies for calculating limits, a trend also reported in the literature (Sebsibe et al., 2019). More recent studies emphasize that this procedural over-reliance often prevents students from developing the structural sense necessary for advanced calculus.

Items 3 and item 4, which focused more directly on probing conceptual understanding, revealed a range of persistent misconceptions. Responses to question 3a confirmed the widely documented misunderstanding that the limit of a function at a point must coincide with the function's value at that point (Adhikari, 2020; Denbel et al., 2014). Similarly, in question 3b, many students expressed the belief that a limit represents a value that can never be attained—an idea rooted in intuitive concept images rather than in the formal definition (Tall & Vinner, 1981).

Question 4, which required an open-ended articulation of the concept of limit, showed that only 40% of the responses were mathematically adequate. Despite the fact that all students had been explicitly taught the *ε-δ* definition at university, none referred to it in their answers. This finding echoes earlier research (Cottrill et al., 1996; Fernández-Plaza et al., 2013), which highlights the difficulty students face in internalizing and using the formal definition of limit, even after explicit instruction.

The results of question 6 and question 7 point to further challenges related to mathematical modeling and mathematization. In question 6, which asked students to model a convergent geometric process, over 40% of the responses were left blank. Among the responses provided, only 11 were fully mathematically correct, highlighting both the cognitive complexity of the task and students' general difficulty in translating informal problems into formal mathematical representations (Brahmia, 2014). Similarly, in question 7—which involved interpreting graphs of function sequences and concepts of convergence—only 11% of students submitted fully correct answers. This illustrates the significant difficulty students have when dealing with multi-level problems involving function families and convergence.

Taken together, these results show that while some students possess isolated procedural skills, their conceptual frameworks are often fragmented or based on incomplete mental models. The gap between concept image and formal concept definition, as described by Tall and Vinner (1981), is evident across multiple questions. The findings suggest that university instruction often fails to transition students from model 4 (intuitive/image-based) to model 1 (integrated interaction), leaving them stuck in a state where their formal definition cell remains inactive during problem-solving. This highlights the need for more targeted pedagogical interventions that explicitly bridge intuitive reasoning with formal mathematical rigor, as suggested by recent literature (Swinyard, 2011). Recent findings suggest that university students often perceive calculus as a collection of disjointed procedures rather than a unified conceptual framework (Tall & Vinner, 1981). This is further complicated by the fact that many students fail to see the necessity of the formal definition in non-routine problems (Brijlall, 2021).

CONCLUSION

This study highlights the multifaceted difficulties students face in understanding the concept of the limit of a function. Although many students demonstrate procedural fluency, particularly when applying well-known algorithms such as L'Hôpital's rule, their challenges become apparent when they are required to employ alternative techniques or demonstrate deeper conceptual understanding.

The data reveal persistent misconceptions, such as equating the limit with the function's value at a point or interpreting the limit as a static notion. These misconceptions illustrate the gap between students' concept images and formal definitions. The findings also confirm the difficulty in reproducing the formal ε-δ definition of a limit, despite explicit instruction, as well as the challenges students face in engaging in mathematical modeling tasks.

These results underscore the need to address not only procedural proficiency but also the development of strong conceptual frameworks. University-level instruction must go beyond routine techniques and incorporate strategies that explicitly bridge formal definitions with intuitive understanding. As emphasized in recent literature (Eichler & Viedtk, 2021; Swinyard, 2011), addressing these conceptual hurdles requires a shift from purely procedural instruction toward tasks that promote relational understanding. Additionally, instructional practices that foster modeling, visualization, and the discussion of misconceptions can serve as valuable tools for restructuring students' conceptual grasp of limits.

Future research could focus on long-term interventions that track the evolution of students' concept images throughout their academic studies. Moreover, digital tools and visual representations are increasingly proposed as essential means to bridge the gap between intuitive images and formal definitions in modern calculus classrooms (Pinto & Scheiner, 2018). Such investigations could provide further insight into how targeted instructional approaches can effectively bridge the gap between intuition and mathematical formalism.

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