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Review Article

A systematic review on visualization in mathematical problem-solving in secondary schools

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ABSTRACT

In the field of mathematics education, the importance of visualization cannot be overstated, especially in mathematical problem-solving. The review's objective is to investigate visualization in mathematical problem-solving in secondary schools. Its status, regional contribution, subject domains, and outcomes should be the focus and characteristics of visualization and research methods. The PRISMA method was used to conduct this review study, which resulted in the identification of 19 empirical studies from 1,135 studies based on the Web of Science and Scopus in 2006–2024. As a result, 2024 revealed trends in empirical studies on visualization in secondary school mathematics problem-solving. The country with the highest number of studies was the USA The most frequently used visualization was drawing and GeoGebra. Individual differences in the studies included grade, age, cognitive style, disabilities, and gifted students. While effective research mainly focused on attitudes, self-efficacy, and satisfaction. Problem-solving strategies, the VIMATE framework, Van Hiele, modelling performance, and visualization methods were the commonly adopted pedagogical approaches. Geometry and word problem-solving are the most common content used for visualization. Using quantitative research methods and knowledge tests for data collection. The findings have substantial ramifications for the design and implementation processes. of visualization in secondary school mathematical problem-solving for 21st century skills.

Keywords: mathematical problem-solving, PRISMA, secondary school, visualization

INTRODUCTION

Conceptual understanding and problem-solving skills are essential for success in mathematics, one of the core subjects in education. In the context of learning in secondary schools, many students have difficulty solving math problems. Mathematical problem-solving is a critical mathematical process in the mathematics curricula of education systems worldwide, and it is essential for students to develop the skills necessary to solve problems effectively and an essential part of well-rounded 21st century education (Mwei, 2017; Sharma et al., 2022). Students who believe that problem-solving should be a short process. They could address it by memorizing rules and believing that problem-solving is difficult (Ozturk & Guven, 2016). Ozturk and Guven (2016) stated that belief affects problem-solving, along with personal factors such as life experiences. Forming and developing students' problem-solving abilities is one of the crucial tasks in teaching mathematics, especially in elementary schools (Tong et al., 2020). For example, if teachers wanted to train students to solve problems at level 4 to evaluate students' problem-solving ability, a proposed scale of 0–4 could be used. Level 4 is the highest grade in the evaluation of students' problem-solving skills. The support process would continue with the problem-solving level in increasing situations in an experimental sample primary school in Vietnam. People with strong problem-solving skills use visualization processes (Sung, 2017).

Visualization is an effective tool in the process of acquiring knowledge in the field of mathematics because it can help students build connections between concepts and real-world representations. Several studies have shown that visual strategies can improve conceptual understanding, spatial thinking, and problem-solving skills. In addition, visualization can also support students' various learning styles and increase their engagement in the learning process. Therefore, it is crucial to explore how visualization is used effectively in solving mathematics problems in secondary schools. It helps learners develop a deeper understanding and restrain interpretation. Visualization is no longer limited to illustrative purposes; it is also recognized as a critical component of reasoning, problem-solving, and proving. Visualization allows students to internalize concepts and establish correlations between them (Arcavi, 2003). Visualization helps create an image of the information, which supports retention and comprehension (Sung, 2017). Visualization in mathematics includes graphical representations, diagrams, drawings, models, and illustrations that help students understand abstract concepts more concretely.

Although many studies have discussed the significance of visualization in mathematics learning, there has been no systematic review that explicitly evaluates the effect of visualization on mathematics problem-solving at the secondary school level. Mathematics learning is important for the use of visualization in general, such as in mathematics education, without considering how visualization affects problem-solving abilities in secondary students. This statement the results are consistent with the findings of Schoenherr and Schukajlow (2024) of the study that the secondary school level uses the most visualization in mathematics education. Students at the secondary school level face different challenges, especially those related to more complex and abstract mathematical concepts. Therefore, more in-depth and systematic research is needed to investigate visualization in mathematics problem-solving in secondary schools.

RESEARCH QUESTIONS

Empirical studies on visualization and mathematical problem-solving were reviewed in secondary schools published over the past eighteen years to answer. The following research questions will be addressed:

RQ1: What are the current trends in visualization research for secondary school mathematical problem-solving, and what are the emerging trends in this field?

RQ2: Which countries or regions have made the most significant contributions to studies on visualization in secondary school mathematics problem-solving?

RQ3: What type of characteristic visualization is most often used in solving problems in secondary schools?

RQ4: Which type of secondary school level is based on characteristic visualization in mathematical problem-solving?

RQ5: What are the key characteristics of visualization in secondary school mathematical problem-solving, including their individual differences, mathematical and non-mathematical capabilities and skills, affected relations, pedagogical design, topic mathematics-related, and types of problems in mathematical problem-solving?

RQ6: What primary research methods do authors use when studying visualization in secondary school mathematical problem-solving?

METHOD

The present systematic literature review gathered primary literature by searching the WoS and Scopus databases. Identifying keywords was the first step in finding relevant journal articles across the two search engines after constructing research questions. This ensured that all articles meeting our criteria were included in the dataset. The review was conducted using the suitable keywords "visualization" that were obtained from the titles visualization and mathematical problem-solving.

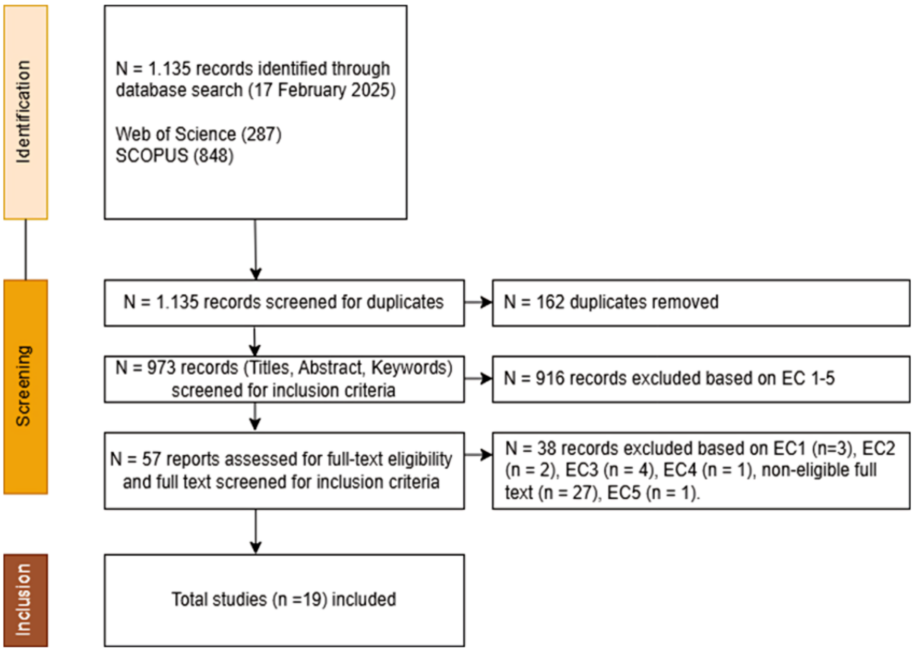

Figure 1. PRISMA protocol flow chart [Adapted from Hidayat and Wardat (2024)]

Article Search Strategies

The keyword terms were combined using the Boolean operators "OR" and "AND" to create a search string for the identification process, which was part of the literature search plan. **Figure 1** summarizes the literature search plan: ("visualization") AND ("mathematical problem-solving").

Inclusion Criteria and Exclusion Criteria

The combination of the words "visualization in mathematical problem-solving" was used to search for relevant studies in each of these databases. Then, a search was conducted in selected databases and journals, and the abstracts were read to apply the four inclusion criteria:

(1) studies focusing on secondary school students,

(2) articles published in reputable journals,

(3) studies with empirical data, and

(4) studies in English.

To ensure relevance to the interdisciplinary nature of visualization in mathematical problem-solving and the role of visualization in mathematical problem-solving The full text of the articles was read for learning activities. The initial articles were then filtered based on five exclusion criteria:

(1) articles that only discuss elementary schools or colleges,

(2) articles published in non-reputable journals/proceedings,

(3) studies without empirical data, and

(4) studies in non-English.

Figure 1 illustrates the sequential methodology employed in this review. Initially, searching databases and journals yielded a total of 1,135 articles which consists of Scopus data of 848 and Web of Science (WoS) of 287. This study draws its dataset exclusively from Scopus and WoS, the two most authoritative and comprehensive bibliographic databases, to ensure data quality, scientific rigor, and the inclusion of only literature relevant to the visualization in mathematical problem-solving domain. The selection criteria, which include relevance and duplicates, were used to cross-reference 162 articles, 162 of which were deemed non-compliant. Further examination of titles and abstracts identifies 916 articles that were not centered on visualization in secondary school mathematical problem-solving. After thoroughly reviewing the full texts, 38 articles were eliminated, leaving 19 that met the criteria. To ensure the accuracy and consistency of the selected studies, two researchers conducted a thorough manual screening process and reached a consensus through team discussions and review by one expert. This collaborative approach ensures a robust and credible synthesis of the research findings for this review.

Data Coding and Analysis

To analyze the data, we developed a set of coding schemes based on the common strategies used in the review study. These schemes helped us address the research questions of our study. The coding categories and analysis methods are as follows:

1. Years of distribution of studies: Analyzing the annual publication volume can determine the level of attention and changes in attention regarding this research.
2. Contributing countries/regions: Analyzing the statistics of contributing regions using maps can identify the active areas and areas in need of improvement for this research.
3. Types of visualization in mathematical problem-solving: The number of types involved in secondary school use was labeled, and the number of studies was tallied accordingly.
4. Types of secondary schools: The number of types is based on the level of ICSED (international grade secondary school, upper secondary, and lower secondary).
5. Outcomes domain: The selected studies were coded based on their contribution to
   (a) individual difference student development,
   (b) capabilities of students (mathematics and non-mathematics skills),
   (c) affective development,
   (d) the facilities of pedagogies and teaching strategies,
   (e) types of the topic in mathematics development, and
   (f) types of problem-solving (i.e., word problem and complex problem).
6. Research methods: Quantitative and qualitative studies. We classified and counted those using mixed methods as well.

RESULTS AND DISCUSSION

This section consists of the analyses of research findings on visualization in mathematical problem-solving in secondary schools from databases with 19 empirical studies. The findings are analyzed under six main themes: number of studies published, contributing countries/regions of the studies, types of visualization, types of secondary schools, the reviewed studies involved outcome domains, and research method.

Number of Studies Published on Current Visualization Mathematical Problem-Solving

Figure 2 shows the distribution of the selected studies by year of publication. In general, there was an increasing trend in the number of empirical studies related to visualization in secondary school mathematical problem-solving published along the timeline. Published throughout the period. The first time this study appeared was in 2006, but it did not experience a significant increase each year until 2012. But decreased from 2013 to 2016, there was no research. In 2017 and 2018, there was an increase from the previous 2 studies in each year, but it decreased again until 2019, with no research. 2020 to 2023, consistent at 1 study each year. But in 2024, it became a trend and increased rapidly in 6 studies.

Based on the results of the systematic literature review, it was found that the first research related to visualization of mathematical problem-solving at the secondary school level began in 2006 (Garderen, 2006). In the study, visualization introduced spatial visualization and spatial imagery for children with learning disabilities. This shows that large visualizations can help students with disabilities, and then, over more and more years, they develop visualizations as technology grows more sophisticated. An example is the emergence of GeoGebra software and computer-based technology. Not only to help students in deepening mathematical concepts, visualization is also present in helping students in making it easier to answer exams through computers using images, tables, and also other forms of visualization. A notable number of studies still make use of technology tools (Schoenherr & Schukajlow, 2024). Knowledge visualization frameworks, such

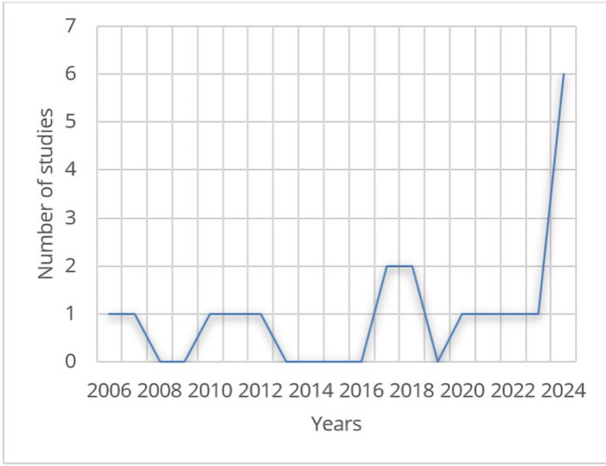

Figure 2. Number of visualizations in secondary school mathematical problem-solving studies published from 2006 to 2024 (Generated by Authors, 2025)

as interactive visualization, have been identified as tools to improve knowledge transfer and learning engagement among students in primary schools (Yusoff et al., 2013). Over the past two decades, the term visualization has been used in various ways in the research literature. There is a long tradition of research on learning with visualizations in mathematics education, as described in Presmeg's (2006). The review aimed to understand the benefits and challenges of learning with visualizations in an educational setting, among other topics. Visualizations can be powerful tools for thinking and learning mathematics because they help students understand abstract concepts and make connections between mathematical ideas (Arcavi, 2003). Visualization is the ability to create, interpret, use, and reflect upon pictures, images, and diagrams in our minds, on paper, or with technological tools. It is also the process of using these images to depict and communicate information, think about and develop previously unknown ideas, and advance understanding.

The year 2024 is a trend, with the growth of increasingly sophisticated technology visualization in mathematical problem-solving also continuing to grow, namely starting to appear related to mathematics content, drawing, instructional design, and also technology-based multimedia for tests (Alihodžić et al., 2024; Fetaji et al., 2024; Shin & Park, 2024; Wilkie, 2024). The gap from previous research is that there is no emerging virtual reality and augmented reality on visualizing problem-solving skills in secondary schools.

Contributing Countries/Regions of the Studies

The findings concerning geographic distribution revealed the contributions of researchers from disparate countries to research on visualization in secondary school mathematical problem-solving. The analysis was conducted separately based on the following: the number of publications. The authors came from 14 countries, with the United States being the most prominent. **Figure 3** shows the distribution of authors by country and continent.

Figure 3 and **Table 1** show the countries/regions that contributed to empirical investigations of visualization in secondary school mathematical problem-solving from 2006 to 2024. As was expected, the USA, where the concept of visualization mathematical problem-solving in secondary school was conceived and first studied on the topic, was followed by Germany (3 studies), Australia, North Macedonia, and Indonesia (2 studies), respectively. The countries that contributed the least to this research were Malaysia, Switzerland, Ukraine, Turkey, Costa Rica, Spain, Singapore, and Colombia (1 study each), respectively.

As previously mentioned, a growing number of countries have started researching and implementing visualization in secondary school mathematical problem-solving. During the period examined, the USA and Germany (see **Table 1**) contributed the most to The study and practice of the field. The findings indicate the growing importance and vitality of the field in different regions. It is aligned with (Cevikbas, 2024) that the most prominent clusters are in mathematics education in the USA, Germany, Australia, etc.

A significant country in the contribution of visualization research is the USA, followed by Germany. These two countries do have a major contribution to the development of mathematics education in particular. The

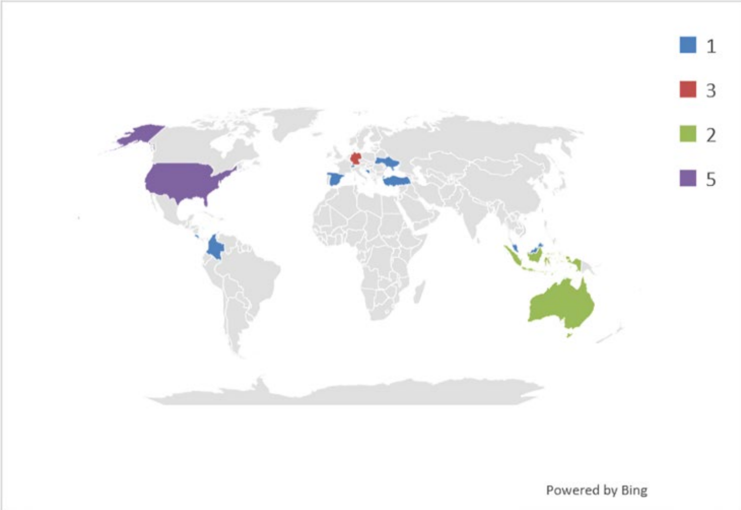

Figure 3. The map showing the distribution of research on the visualization of secondary school mathematical problem-solving (Generated by Authors, 2025)

Table 1. Countries contributing to the visualization in the secondary school mathematical problem-solving study from 2006 to 2024

Countries/region	Frequency (N)	Percentage (%)
USA	5	21.73
Germany	3	13.04
Australia	2	8.69
North Macedonia	2	8.69
Indonesia	2	8.69
Bosnia and Herzegovina	1	4.34
Malaysia	1	4.34
Switzerland	1	4.34
Ukraine	1	4.34
Turkey	1	4.34
Costa Rica	1	4.34
Spain	1	4.34
Singapore	1	4.34
Colombia	1	4.34

USA and Germany are already concerned with problem-solving skills because it is one of the abilities that must be achieved in the secondary school education system in their countries. This proves that their curriculum supports the development of children's problem-solving by using various visualization tools. The USA is an advanced country in terms of technology, so it is undeniable that there are always contributions from researchers related to visualization. In the USA standards, the curriculum concepts refer to the core ideas, concepts, and principles of mathematics; thus, it means highlighting the need for learners to provide reasonable explanations and understand the essence of mathematics (Xu & Zhu, 2025).

The German secondary school mathematics curriculum also states that within dimension and form in the content domain of the curriculum, students' abilities in spatial orientation and spatial visualization and drawing are also required. And also, problem-solving is included in the learning process in the German secondary school curriculum with content-based student competencies of planning and describing approaches to problem-solving; using algorithms to solve standard mathematical problems and evaluating answers in terms of practicality; examining different ways to solve problems; using problem-solving strategies; and applying different forms of representation (e.g., graphs, sketches, and equations) when solving problems.

Types of Characteristic Visualization in Secondary School Mathematics Problem-Solving

Table 2 represents the distribution of types of characteristic visualization in secondary school mathematical problem-solving studies based on tools, learning method, ability, usage, and assessment.

Table 2. Characteristics of visualization in secondary school mathematical problem-solving

Characteristic visualization	Types	Frequency (N)
Tools	Drawing	4
	GeoGebra	3
	Advanced graphic calculator	1
Learning methods	Online mathematics games: 1. Games	1
	Augmented worked example (animation)	1
	Technology-assisted instruction	1
	Van Hiele theory	1
	Visualization method	1
Ability	Visual spatial representation	1
	Spatial visualization	2
Usage	Visualization of mathematical content	1
	Generalization	1
Assessment	Computer based multimedia testing	1

Overall, the most widely used types of visualization are tools, which are drawing, GeoGebra, and advanced graphic calculators. Drawing is the tool for visualization of mathematical problem-solving most predominant in secondary school (n = 4, 50%) in the 4 studies, respectively. Drawings belong to the lower resemblance visualization type (abstract visualizations) (Schoenherr et al., 2024). Those studies are in line with research (9). The students were invited to create drawings for each task. Group work and teacher-led discussions shaped their beliefs about the role of visual representations in solving word problems. Followed by GeoGebra (n = 3, 37.5%). Fewer studies (n = 1, 12.5%) used advanced graphic calculators. The second characteristic is the learning method that uses visualization of problem-solving; there are online mathematics games, augmented worked examples with animation, technology-assisted instruction, Van Hiele theory, and a visualization method (n = 1), respectively (Konyalioglu et al., 2012; Lee et al., 2022; Scheiter et al., 2010; Shin & Park, 2024; Utomo et al., 2018). The third visualization characteristic is based on ability. There are two abilities that emphasize visualization of mathematical problem-solving, including visual spatial representation (n = 1) (Blatto-Vallee et al., 2007) and spatial visualization (n = 2) (Garderen, 2006; Harris et al., 2021). The fourth visualization characteristic based on the use of visualization in problem-solving is visualization of mathematical content and generalization (n = 1), respectively (Alihodžić et al., 2024; Mora et al., 2024). Finally, the characteristics of visualization of mathematical problem-solving on the assessment computer-based multimedia testing (n = 1) (Ehrhart & Lindner, 2023).

One of the earlier classifications of visualization was defined by Presmeg (1986a, 1986b), who mentioned five different types of visualization: Concrete imagery is a mental picture. Kinesthetic imagery involves physical movement, such as walking several vectors head to tail with fingers. Dynamic imagery involves moving or transforming the image itself. Memory images involve formulas. Pattern imagery involves pure relationships stripped of concrete details.

According Presmeg (1986a), They emphasized two types of visual representations: schematic and pictorial. Using schematic representations was associated with success in solving mathematical problems, while using pictorial representations was negatively correlated with success. Schematic representations were also significantly correlated with one measure of spatial ability. Therefore, their research helps clarify the relationship between visual imagery, spatial ability, and mathematical problem-solving (Hegarty & Kozhevnikov, 1999). In this research, visualization is more emphasized in forms that help solve problems.

According to Finesilver (2022) and Schoenherr and Schukajlow (2024), types of visualization are divided into two, namely physical resemblance and structural resemblance. In the review of research results, it was found that drawing is a type of visualization that is very widely used for secondary school students. Drawing falls into the category of physical resemblance with low resemblance and schematic. In an empirical study on drawing experiments on third-grade elementary school students, The experimental group was significantly more likely to agree that creating an appropriate drawing of a word problem would make solving it easier (Csíkos et al., 2012). According to Rellensmann et al. (2017), drawing strategies are widely used as a powerful tool for promoting students' learning and problem-solving skills.

Table 3. Types of secondary school level

Category	Characteristic visualization	Types	Frequency (N)
Lower secondary school	Tools	Drawing	3
		GeoGebra	2
		Advanced graphic calculator	1
	Learning methods	Online mathematics games: 1. Games	1
		Augmented worked example (animation)	1
		Technology-assisted instruction	1
	Ability	Spatial visualization	2
	Usage	Generalization	1
	Assessment	Computer based multimedia testing	1
Upper secondary school	Tools	Drawing	1
		GeoGebra	1
	Learning methods	Van Hiele theory	1
		Visualization method	1
	Usage	Visualization of mathematical content	1
	Assessment	Computer based multimedia testing	1
Both lower and upper	Ability	Visual-spatial representation	1

Types of Secondary School in the Reviewed Studies Based on Characteristic Visualization of Mathematical Problem-Solving

We classified the selected studies into two groups based on the types of visualization used in secondary school mathematical problem-solving. **Table 3** shows the distribution of studies with various types of integration (i.e., upper secondary and lower secondary). This indicates the characteristic visualization of mathematical problem-solving of secondary schools.

Most participants in studies used lower secondary school (n = 13, 68.4%), and all characteristics of visualization of mathematical problem-solving are applied in lower secondary school, such as tools, learning method, usage, assessment, ability, and also assessment. Drawing tools dominated in terms of visualization and were followed by GeoGebra. It is different with upper secondary school only (n = 5, 26.3%), and only three characteristics of mathematical problem-solving visualization are used, such as tools, learning methods, and also usage. The same tools were used between drawing and GeoGebra (n = 1), but both mathematical problem-solving visualization tools were used.

It was found that the secondary school level that uses visualization a lot in mathematical problem-solving is the secondary school with an average age of 13–15 years. Visualization can be used universally and supports learning at different levels of education (Van Meter & Garner, 2005). Research indicates that visualization is a challenge for learners of all ages, but the specific challenges vary (Gunčaga & Žilková, 2019). These theories in the field of cognitive processes are significantly influenced by visualization (Gunčaga & Žilková, 2019). Jean Piaget's theory of cognitive development suggests that children progress through four distinct phases of learning. Based on the cognitive level according to Jean Piaget, The formal operational stage category includes individuals who are 11 years old (Babakr et al., 2019). This is also in accordance with Bruner's theory of three types of representations: enactive, iconic, and symbolic. Learners can understand material, organize it in new ways, and apply their knowledge to new situations. Bruner viewed learning as a process through which students acquire concepts and problem-solving abilities (Stapleton & Stefaniak, 2019).

The Reviewed Studies Involved Outcomes Domain

We classified the selected studies' outcome domains into six groups that are based on

(1) individual differences,

(2) research on students' capabilities in mathematics and non-mathematics,

(3) affect-related research,

(4) pedagogical design,

(5) research on the subject topic of mathematics, and

(6) types of problem-solving.

Research on Individual Differences in Students' Visualization in Secondary School Mathematical Problem-Solving

The visualization in secondary school mathematical problem-solving contributes to the development of students. There are many kinds of individual differences in students using visualization in secondary school mathematical problem-solving. Based on individual differences divided by

(1) Age: Are there any studies that explain age? (Alihodžić et al., 2024),

(2) Grade: Many studies explain not only one grade but many grades,

(3) Most studies explain the gender (Alghadari et al., 2020; Alihodžić et al., 2024; Blatto-Vallee et al., 2007; Wilkie, 2024). And one study explains the demographics of students,

(4) Cognitive style, and

(5) Disabilities (3 studies), gifted students (1 study), and type progressors (i.e., fast, intermediate, and slow) (Lee et al., 2022).

This finding aligns with the findings of Schoenherr et al. (2024) that the efficacy of visualization techniques may be contingent on the educational level of the learners, specifically whether they are at the primary or secondary stage, and the extent of their prior knowledge.

Based on the visualization characteristics that have been found, there are several kinds of important points in individual differences. such as grade, gender, age, cognitive style, gifted students, disabilities, and type progressors. This would be consistent with studies showing that as people practice new skills, individual differences reflect the integration of the new components into the developing associativity (Xu et al., 2021). For example, gender (Alghadari et al., 2020; Alihodžić et al., 2024a; Blatto-Vallee et al., 2007; Harris et al., 2021; Wilkie, 2024), age (Blatto-Vallee et al., 2007), grade (Alihodžić et al., 2024), cognitive style (Utomo et al., 2018), gifted students (Garderen, 2006), disabilities (Blatto-Vallee et al., 2007; Garderen, 2006; Shin & Park, 2024), and type progressors (Lee et al., 2022). The gap in the research found is that there are still many kinds of individual differences that have not been discussed, such as socioeconomic status, intelligence, and also individual abilities possessed by secondary school students. Paz-Baruch and Spektor-Levy (2024) suggest that future studies further examine the impact of gender and SES on their mathematical problem-solving abilities and consider how these factors may interact with other variables to influence problem-solving outcomes.

Research on Students' Mathematics and Non-Mathematics Skills

To identify the various mathematical skills involved in visualizing secondary school mathematical problem-solving, we reviewed the related research questions, data collection methods, and results. Since one study may involve more than two variables, each study calculates the measured variables separately. There are many mathematical variables related to visualization in mathematical problem-solving. Students' achievement (n = 4): one focuses only on geometry, and one studies the average achievement. Reasoning ability and verbal (n = 2). Spatial visualization ability (n = 2). Mathematical equivalence, mathematical concepts, mathematical visualization, higher-order thinking skills, mathematical drawing, spatial ability, mathematical capacity, visual-spatial representation, spatial orientation, visual abilities, generalization skills, creative thinking, metacognitive skills, problem-solving, and cognitive skills, with each having (n = 1). There are many non-mathematical variables related to visualization in mathematical problem-solving, like students' research skills and reading skills (n = 1), respectively. According to the empirical study (Özcan & Doğan, 2018), both early math and reading skills predicted mathematical problem-solving ability. Students applied five themes in problem-solving: logical thinking, computational skills, problem-solving strategies, justifications, and representations (Al Farra et al., 2022). The most significant challenge in mathematical problem-solving is comprehending the problem itself (Amalina & Vidákovich, 2023).

Based on mathematical and non-mathematical capabilities and skills, it was found that there are some mathematical abilities, such as students' mathematical achievement, reasoning ability, verbal and spatial visualization ability, mathematical equivalence, mathematical concepts, mathematical visualization, higher-order thinking skills, mathematical drawing, spatial ability, mathematical capacity, visual-spatial representation, creative thinking, metacognitive skills, and cognitive skills. There are 2 non-mathematical variables related to visualization in mathematical problem-solving, like students' research skills and reading comprehension. Spatial skills may be important for holding and manipulating information in the mind during mental computations, as well as for organizing information during problem-solving that requires simultaneous processing (Booth & Thomas, 1999). Based on the results of structural equation modeling (Likhanov et al., 2024) Spatial visualization contributes to word problem-solving, and analogical reasoning contributes to the process of solving word problems. Thus, spatial ability is important for solving math word problems. Aligning with results from Munoz-Rubke et al. (2021) mechanical problem-solving and reasoning are potentially viable approaches to enhancing spatial thinking. The empirical studies (Galman, 2019) found that spatial visualization ability can be applied to solving mathematical problems, and that the use of visual images is significantly correlated with spatial visualization measures.

Research on Affect-Related Visualization in Mathematical Problem-Solving

The selected studies explored indicators in the affective domain of student attitude, self-efficacy, and satisfaction (n = 1), respectively. The findings suggest the need for research into the development of methods and practices to further empirically elaborate on and validate the role played by visualization in secondary school mathematical problem-solving in improving educational outcomes in terms of enjoyment, self-resilience, and self-esteem. Previous research has identified the need for more empirical studies to demonstrate the role of visualization in mathematical problem-solving and its impact on affective outcomes.

The affective research conducted on visualization of mathematical problem-solving in middle school is attitude, mathematical self-efficacy, and satisfaction (Alghadari et al., 2020; Alihodžić et al., 2024; Ehrhart & Lindner, 2023). The result from Alihodžić et al. (2024) considering individual needs and student attitudes, strategies for promoting visualization in the classroom should be implemented. Students' ability to solve problems accurately impacts their self-efficacy and geometry achievement (Alghadari et al., 2020). The new findings obtained in terms of pedagogical design are problem-solving strategies, the VIMATE framework, Van Hiele, learning approaches and activities, modeling performance, and visualization methods. Many learning methods are used to improve students' problem-solving skills, one of which is the problem-solving learning strategy. The problem-solving strategy emphasizes how students in the classroom can interact in solving a problem. There are several stages that exist for students to solve problems. In addition, there is a framework created by the author to improve student visualization in terms of problem-solving skills; the name is the VIMATE framework, where there is a link between the application of visualization in learning in the classroom. There are also visualization methods for improving visualization in problem-solving skills. In pedagogy, there are several forms of learning methods to be able to improve visualization in solving student problems.

Research on Pedagogical Design Visualization in Mathematical Problem-Solving

Researchers examined the positive impact of teaching methods and pedagogical design on learning in visualization in secondary school mathematical problem-solving. In well-planned visualization activities and programs, students are expected to apply knowledge, processes, and skills from different disciplines to solve problems. Correspondingly, problem-solving strategies, the VIMATE framework, Van Hiele, learning approaches and activities, modelling performance, and visualization methods.

The findings suggest the need for a more systematic approach to researching the development of visualization in secondary school mathematics to validate its role in improving pedagogical outcomes in problem-based, project-based, discovery, and inquiry learning. It is also noted that pedagogical principles related to online learning tools and platforms (e.g., flipped classrooms, mobile learning, and blended learning) are rarely adopted in this area.

Modeling performance is the result of students applying problem-solving skills using visualization. For example, students can present the results that have been done through activities in the classroom. the need for clear instructional design to be able to direct students in accordance with what is to be achieved through easy-to-use tools through visualization.

Research on the Subject Topic of Mathematics Visualization in Mathematical Problem-Solving

The results show that topic-integrated visualization in secondary school mathematical problem-solving. The most common topic of research is geometry (n = 5), including geometry transformation, basic geometry,

geometry measurement, and geometry problem-solving. The other topics are arithmetic, number sense, fractions, and algebra. Based on the curriculum topic of mathematics, it is noted that the topic of mathematics, such as operations, probability, and statistics, was rarely adopted in this area.

Mathematics topics related to visualization in problem-solving, especially for secondary school students, are mostly geometry topics. Geometry is widely used because geometry is an abstract topic to be understood by secondary school students. Align with Julius et al. (2021) that geometry, one of the subject's several topics, received the greatest attention. So that many tools or visualizations facilitate student understanding related to the topic of geometry. In addition to geometry, there is a topic that is arithmetic. In the topic of arithmetic, many problems are related to word problems, and students have difficulty in interpreting the problem into the final solution of mathematics, so this causes arithmetic to also be a serious topic for further study. There are some topics that are rarely discussed in the use of visualization of mathematical problem-solving skills, such as algebra and also probability and statistics. Algebra is unsurprising given the greater emphasis on function in secondary mathematics (Ellis & Özgür, 2024). Whereas statistics is a topic that needs a deeper interpretation in meaning a picture or diagram or table. So that future researchers can focus more on the topic of probability and statistics.

Research on the Types of Problem Visualization in Mathematical Problem-Solving

The result shows the types used in studies of word problems (n = 2); the other types are problem-equivalent problem, isomorphic problem, similar problem, and unrelated problem. This research did not address several types of problems, such as non-routine, open-ended, complex, and inquiry-based problems. This is consistent with the results in Verschaffel et al. (2020) of the review that there are three important elements of the learning/teaching environment for the development of students' problem-solving competencies, namely textbooks, software, and teachers.

There are several types of problems used in research; many studies discuss word problems. Word problems are the researcher's concern in improving problem-solving skills in word problems. Through visualization, word problems can be solved because it takes deep interpretation to understand the word in the problem. In this problem, solving ability is included in the problem-understanding category. This is why word problems are more often researched compared to other problems such as complex problems or non-routine problems. Complex problems are more complex than word problems. This means that if students are unable to solve or understand problems in word problems, then students cannot understand or solve more complex problems, such as complex problems. Meanwhile, non-routine problems are problems that are rarely done by students, and students are not used to solving them. Usually non-routine problems appear in international problems such as PISA/TIMSS. Thus, the variety of problem types should allow students to solve them, or at least students understand the problem well and then can be interpreted through images or prior knowledge they already have.

Research Methods Adopted in the Studies Reviewed

We classified the selected research methods into three groups that are based on

(1) target participants,

(2) data collection methods, and

(3) data analysis methods.

Target participants

After examining the data, it was determined that there were (n = 19) studies targeted at secondary school of education students, lower secondary school (68.42%), upper secondary school (26.31%), and both lower and upper secondary school (5.26%). The secondary school levels of the student participants are being looked into (**Figure 4**). 4) It was found that visualization mathematical problem-solving has been conducted more commonly in lower secondary schools than upper secondary schools. Participants in this study were not only secondary school students who had general abilities in general, but also two studies discussed the use of visualization for disabilities in secondary schools (10.5%). Not only for general students using visualization in secondary school mathematical problem-solving, but also students with disabilities could use visualization.

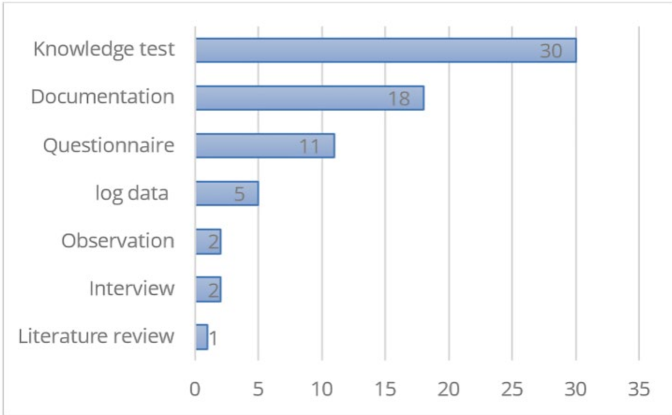

Figure 4. The studies employed a variety of instruments as data sources to collect data (Generated by Authors, 2025)

One study explains visualization for students with disabilities; one study focuses only on deaf and hearing disabilities (Blatto-Vallee et al., 2007).

Data collection method

Data collection cannot be determined precisely because each study will have different data collection and more than one. The author here tries to describe in detail each data collection used by the reviewed research. As **Figure 4** illustrates, the analysis of the data collection methods revealed that the most prevalent approach was the knowledge test (n = 30). Followed by documentation (n = 18), namely, the author categorizes documentation such as tasks, notes, videos, records, written work by students, text booklets, and worksheets. Questionnaires were also included in the third major data collection category (n = 11). Next is log data; there are several studies that collect data by taking into account timestamps and time on task, so they fall into the log data category (n = 5). Three kinds of data collection are observation, interview (n = 2), and literature review (n = 1).

Data analysis method

Further analysis of the data analysis methods used in the studies suggests that most of them (12 studies; 63%) used a purely quantitative method to investigate design. implementation of visualization in secondary school mathematical problem-solving. A considerable number of studies (4 studies, 21%) adopted a purely qualitative approach. The number of studies that employed mixed methods was very small, limited to three (16%).

Based on the findings, there are several classifications in the methodology found based on participants, data collection, and data analysis. For the previous participants, the type of school that was often discussed in the study was lower secondary school with an approximate grade (13–15 years old). Participants in these review studies have various criteria, ranging from a few participants to many. It depends on the purpose of each study. However, not only ordinary lower secondary students but also participants who have advantages such as disabilities. There are 3 studies that discuss disabilities for visualization of problem-solving skills, and there is 1 that discusses deaf and hearing disabilities and how students can still improve problem-solving skills through visualization. It turns out that visualization can also help students in the disability category.

More data collection is the ability test, where all studies measure students' abilities through tests because it is related to cognition. Although some used documentation and questionnaires to support the results of the tests. Knowledge tests are most predominant as measures of visualization in mathematical problem-solving in secondary school.

CONCLUSION

This study reviewed empirical studies on visualization in secondary school mathematics problem-solving published between 2006 and 2024. The findings reveal an increasing number of empirical investigations on this topic, particularly in 2024. Additionally, recognized research on the topic is becoming more diverse, with publications emerging from countries spanning a broad geographical scope. The two top countries' empirical studies are in the US, followed by Germany. We found that the characteristic visualization for mathematical problem-solving used in secondary school is divided into 5 categories, namely based on tools, based on learning methods, based on use, based on assessment, and also based on ability according to visualization in mathematical problem-solving. For tools that are usually dominantly used, drawing is followed by GeoGebra and advanced graphic calculators. The second characteristic is learning method, which is based on online mathematics games, the Van Hiele theory, technology-assisted instruction, augmented worked examples with animation, and visualization methods. The third characteristic is based on usage, such as generalization and visualization of mathematical content. The fourth characteristic based on assessment is computer-based multimedia testing, and the sixth characteristic based on ability is spatial visualization ability and visual-spatial representation. At the lower secondary school level, all 5 characteristics on visualization of mathematical problem-solving were used and for upper secondary school only 3 characteristics were used. Meanwhile, for the 1 study that measured both secondary school levels, only 1 characteristic was used.

In general, researchers prefer quantitative methods for data collection and analysis. More new findings were obtained by reviewing learning outcomes in terms of individual differences, learning objectives, and effects:

(1) Socioeconomic status, working memory, and intelligence were the neglected individual differences,

(2) critical thinking, representation skills, and problem-posing skills–or, particularly for 21st century skills–nothing has been researched yet, and

(3) motivation, interest, belief, self-resilience, self-esteem, and enjoyment studies were less attractive.

Future research on visualization in secondary school mathematical problem-solving is suggested to explore the development of individual differences in students in this context. And more attention can be paid to how to improve students' critical thinking, problem-posing skills, spatial abilities, or, particularly for 21st century skills and affective domain, such as interest, belief, self-esteem, and self-resilience when participating in visualization in secondary school mathematical problem-solving. Innovative pedagogical approaches should be developed in greater numbers for visualization in secondary school mathematical problem-solving to support activities such as problem-based learning, project-based learning, inquiry learning, discovery learning, or technology-based learning (i.e., blended learning and e-learning). The topics of mathematics that are often studied are geometry, arithmetic, number sense, algebra, and fractions, but no one has discussed probability and statistics. The most popular problem is word problems, and complex problems are never studied before. It was noted that fewer studies involved upper secondary schools. Finally, researchers were advised to collect more data through interviews, observations, and literature reviews.

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