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A Design Based Study: Characteristics of Differentiated Tasks for Mathematically Gifted Students

Duygu Ozdemir [1]*, Mine Isiksal Bostan [2]
[1] Department of Mathematics Education, Istanbul Aydin University, Istanbul, TURKEY
[2] Department of Mathematics Education, Middle East Technical University, Ankara, TURKEY
* Corresponding author: duyguaydemr@gmail.com

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Abstract:
Nowadays, the necessity of designing proper differentiated tasks for mathematically gifted students is a remarkable subject in international literature and practice. In this design-based study, it was aimed to analyze and construct characteristics for designing differentiated tasks of mathematically gifted students. Through this design process, preliminary and prototyping phases were used as a general framework to reveal these characteristics as design principles of the study. Literature reviews, experiences of practitioners and needs analysis studies about suggestions and key points reflecting the characteristics of differentiated tasks were conducted in preliminary research phase. After this phase, initial form of differentiated tasks with draft design principles were applied in classrooms in try-outs and field test with 12 mathematically gifted students and 4 mathematics teachers of these classrooms. The data obtained through qualitative methods lead to three categories; initial design principles as characteristics obtained through preliminary phase, modifications for tasks, and final design principles as characteristics obtained through prototyping phase.
Keywords: mathematically, gifted, students, differentiated, characteristics, design

INTRODUCTION

Gifted education studies gained much more importance and researchers seek to answer their questions about what the ways are to meet differentiated needs of gifted students (Mofield, 2020; VanTassel-Baska et al., 2021), who mostly attend regular classrooms and deprive of adjustments in classroom contexts or materials suitable to their differentiated needs, characteristics, and interests (Brigandi et al., 2019). However, based on the principle of educational equity (Van de Walle, Karp & Bay-Williams, 2013), it is the right of gifted students to benefit from differentiated educational opportunities (Wilkins, Wilkins, and Oliver, 2006). Therefore, inequity in educational opportunities for gifted students in most classrooms (Baykoç et al., 2014; Diezmann & Watters, 2001; Johnson, 2000; Sriraman, 2003) is a global problem that should be further explored and provided with solutions through well-structured research studies.

Most of the research shows the trend towards differentiation education including differentiating the materials that teachers should use for their gifted students in classrooms. As a result, most academic studies on mathematical giftedness have concentrated on the need and value of differentiating, recognizing, and directing mathematically gifted students (Deringöl & Davasligil, 2020; Diezmann & Watters, 2001; Dimitriadis, 2011; Fıçıcı & Siegle, 2008; Manuel & Freiman, 2017; Singer et al., 2016; Sriraman et al., 2013; Zedan & Bitar, 2017). Despite this significance, research that concentrate on

Table 1. Participants of the study

Participant teachers	TS	TN	TM	TR
Gender	Female	Female	Male	Female
Professional Qualification	Master	Bachelor	Bachelor	Master
Teaching Experience (years)	10	8	12	8
Number of Gifted students	3	2	2	8

suitable classroom tasks for mathematically gifted students are extremely rare, and as a result, schools in many countries lack the quality to include such instructional tasks for these students (Baykoç, 2014; Dimitriadis, 2011; Gavin et al., 2013; Mhlolo & Marumo, 2017; Özdemir & Işıksal-Bostan, 2021; Singer et al., 2016). Similarly, mathematically gifted students in Turkey need differentiated programs or materials in their classroom environments in which they spend most of their time (Özdemir & Işıksal Bostan, 2021). Even, in Science and Art Centers organized by the Ministry of National Education, where gifted students in Turkey can attend after school, differentiated materials are also necessity for these centers (Özdemir, 2018).

That is, research on curriculum and teaching models for gifted students and materials that could be used in classrooms are limited (Jolly & Jarvis, 2018). In line with this need both in theory and practice, the present study aimed to analyze and construct design principles needed for the differentiated tasks of the 5th and 6th grade mathematically gifted students based on the literature reviews, the views of the mathematics teachers and mathematically gifted students in Turkey. Dependently, construction and modification of the differentiated tasks based on these design principles as characteristics of the tasks were also conducted through this design-based study. In line with these, what are the initial and final design principles as characteristics for the differentiated tasks of 5th and 6th grade mathematically gifted students and what are the modifications conducted to the differentiated tasks were asked as the research questions of the study.

METHODOLOGY

Participants

Authors wanted to see the usage of the tasks and design principles with mathematically gifted students in different schools. Thus, the participants of the study were four purposively selected teachers and their fifteen 5th and 6th grade mathematically gifted students, who were identified as mathematically gifted by means of the Turkish Adaptation version of Test of Mathematical Abilities for Gifted Students (TOMAGS) (Özdemir, 2016; Ryser & Johnsen, 1998), in two different schools. Three of the teachers (TS, TN, TM) and their seven mathematically gifted students in four classes were from a public school in Ankara, the class size was about twenty-five and the students' families were generally of low socioeconomic status, while the other teacher (TR) and her eight mathematically gifted students were from a public school located in a middle social class environment in Ankara. The detailed information about these teachers were provided in **Table 1**.

Design Based Research

Collins (1992), as one of the pioneers of design-based research, explained it as a method that solves real problems in contexts using real users of those problems. Similarly, many researchers (Nieveen & Folmer, 2013; Plomp, 2013; Van den Akker et al., 2006) used this method to obtain a practical product that solves students' real educational problems. Furthermore, because design-based research allows the researcher to focus on the process during the research and produce a theory-based practical product, it has been presented as beneficial for gifted education (Jen et al., 2015). Since the goals of this study are focused on problems in practice and need theoretical support, design-based research was found to be the most useful for the study's requirements and goals. As a result, two stages of design-based study were adapted from studies: the preliminary phase and the prototyping phase (Kennedy-Clark, 2013;

Mafumiko, 2006; Masole, 2011; Plomp & Nieveen, 2013) and these phases (**Figure 1**) formed the framework of this study.

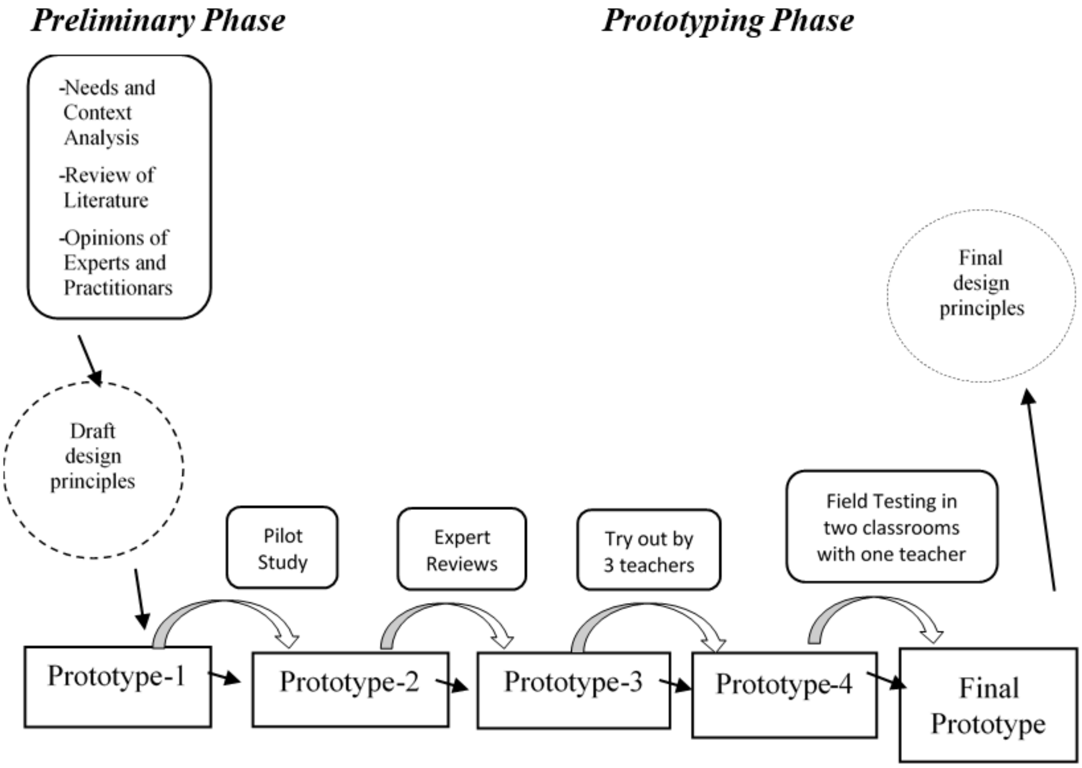

Figure 1. Phases of this design based study

Preliminary Research Phase

The preliminary research in this study provided direction and external confirmation for the study by contributing to both the understanding of the current situation and the creation of a conceptual theoretical framework (Kennedy-Clark, 2013; Masole, 2011; Van den Akker et al., 2006). In this phase, research both in the literature and application area on what characteristics should exist in differentiated tasks of mathematically gifted students was conducted.

First, the views and expectations of gifted students on their mathematics education in Turkey were analyzed and it was found that they complain about the lack of appropriate opportunities in their mathematics education and need appropriate tasks in the classroom that are more challenging and interesting for them (Özdemir & Işıksal Bostan, 2021). Furthermore, international literature was reviewed based on the needs of gifted students and teachers and as well as the suggestions for the characteristics of differentiated tasks that should be provided to them. In addition to the formal activities, informal activities such as discussions with critical people, researchers' own experiences, feedbacks from conferences, informal interviews with experts, school leaders and practitioners were also collected to show the problem in context and its suggestions.

Thus, all the data obtained from the preliminary research phase led to draft version of the design principles for the differentiated tasks of mathematically gifted students. That is, based on the characteristics mentioned in the literature, the views and concluding remarks from needs analysis studies, preliminary phase was completed, and characteristics obtained from this phase lead to the design principles of the differentiated tasks. By this way, forty differentiated tasks were created or modified based on the characteristics mentioned in these draft design principles. These initial form of the differentiated tasks and draft design principles were named as Prototype-1 and improved through the prototyping phases of the study.

Prototyping Phase

The prototyping phase eases to resolve real-life problems by systematically revising design products (McKenney et al., 2006). As mentioned earlier, Protype-1 in the way of draft design principles and initial differentiated tasks were developed in preliminary research phase. In the prototyping phase, modifications were made to these design principles and differentiated tasks through collaboration between researchers, experts, students, and teachers, and this collaboration helped to obtain the most structured form of the final products (Masole, 2011). To clearly explain this process, each prototype used in this study (**Figure 1**) was explained below.

Prototype-1 and Pilot Study

After regulating Prototype-1, the design principles and differentiated tasks were pilot tested with seven mathematically gifted students in two 5th and one 6th grade mathematics classrooms in a private school in Ankara, Turkey through one semester. The aim of this pilot study was to test the tasks and design principles by exploring the gifted students' reflections and intelligibility of tasks. At the end of this semester, based on the data obtained in semi-structured interviews and observation notes, some design principles were modified and tasks were revised in line with problems such as ambiguity of items, time management, language and grade level problems (Masole, 2011).

Prototype-2 and Expert Appraisal

Following to constructing the tasks using appropriate design principles in the form of Prototype-2, a peer review was conducted with four experts, three of whom were from the field of mathematics education and the fourth from gifted education. This phase ensured the validity and practicality of the differentiated tasks, as the experts evaluated the tasks individually in terms of content and practicality (Masole, 2011; Plomp & Nieveen, 2013), as well as critical issues relevant to gifted education. Therefore, Prototype-2 was evaluated in terms of mathematical background, suitability for gifted students, and usability in mathematics classrooms, and suggestions were considered to make appropriate revisions for Prototype-3.

Prototype-3: Try-outs

Because the main purpose of 'try-out' phase was to try the tasks and their design principles as in the form of Prototype-3, teachers (TS, TN, TM) tested the performance of the tasks in mathematics classes over six weeks and commented on design principles, content, corrections, usability, and suggestions for tasks. Through collaboration, the success of the tasks with design principles was measured against their practicality in real contexts (Gravemeijer, 2006). After the testing was completed, semi-structured interviews were conducted with teachers asking for their views and suggestions about characteristics and usability of the tasks with respect to mathematically gifted students. After each activity, teachers were also asked to complete a "teacher after-sheet" form in which their ideas about content related characteristics of the task, usability, and satisfaction with the students' needs were taken immediately after completing the task. Likewise, students' views of the task characteristics, how they felt during the activity and satisfaction with their differentiated needs in the classroom were addressed in the "student-after-sheet" forms. Additionally, the researcher made observations during the process and used a logbook to record notes about the suitability of the tasks for the characteristics, difficulties experienced, and students' nonverbal reflections. Based on the data obtained through this process, try-outs were conducted and the fourth prototype was formed.

Prototype-4 and Field Testing

Following try-outs, the fourth prototype was field tested for five weeks to learn about the perspectives of another mathematics teacher (TR). The teacher conducted classroom observations during this period to background the researchers. This was done to see whether the activities based on these design principles could be used by a typical teacher in a regular classroom without any direct feedback from the researchers. The teacher filled after sheet forms after each activity and collected after sheet forms from the students. During free times, the researcher and teacher continuously exchanged and revised tasks. After five weeks, both gifted students and the teacher were surveyed to get their views on the tasks and the design principles in terms of meeting the differentiated needs of mathematically gifted students in the classroom. In this way, the latest versions of the tasks and principles were evaluated and necessary changes were made to the tasks and design principles.

To sum up, through the prototyping phases, design principles and differentiated tasks with design characteristics were evaluated. Teachers as the real users of the tasks implemented these tasks independently with their own lesson plans. In each of their implementations, teachers and researchers made some changes to the tasks immediately or the points about design principles were noted to make the changes later. Based on these inferences, the tasks and principles were continually revised.

DATA ANALYSIS PROCEDURE

The main objective of this study was to analyze and construct design principles needed for the differentiated tasks of mathematically gifted 5th and 6th grade students. Continuous analysis was used in this study (Miles & Huberman, 1994) because the steps of data collection and data analysis were interrelated (Creswell, 2009; Glaser & Strauss, 1967). Constant comparative method, one of the most commonly used forms of analysis in qualitative studies (Merriam, 1998) and its steps defined by Glaser and Strauss (1967) were followed to describe the data in sufficient detail.

As the first step of analysis, the data obtained from the interviews were transcribed. Then, observation notes, student and teacher after sheet forms, assessment forms, activity sheets, and formal or informal interviews were combined as transcribed data. These various data sources provided us with more trustworthy and credible data through triangulation (Denzin, 1989). When needed, modification to the tasks and design principles was conducted based on the views of teachers and students within each phase. After the codes assigned to the data examined, it was seen that categories emerged from these similar codes showed parallelism with the prototyping structure of the design principles. That is, data obtained through these processes lead to the three categories as initial-draft design principles, modifications to the tasks, and final design principles, which were presented as the findings of this study below.

FINDINGS

Based on the aim of this study, design principles as in the characteristics of differentiated tasks for 5th and 6th grade mathematically gifted students were constructed as the final product of this study. Thus, in the following sub-headings, findings were presented as in three categories; initial principles as characteristics obtained through preliminary phase, modifications for tasks, and final principles as characteristics obtained through prototyping phase.

Initial Principles as Characteristics Obtained through Preliminary Phase

Preliminary research phase helped to compile the characteristics of the differentiated tasks for mathematically gifted students mentioned in the literature and by the practitioners. Review of the literature showed that although the importance of differentiating was highlighted in studies, few of them provide suggestions how to differentiate gifted students' tasks. However, it was seen that some characteristics that tasks of mathematically gifted students should have were mentioned by the researchers in separate studies. These characteristics were compiled in the preliminary phase and used as the framework for the initial design principles of the differentiated tasks.

Based on these data obtained in preliminary phase, it was seen that challenge was the dimension that mentioned mostly in the literature (Bochkareva et al., 2018; Chamberlin & Chamberlin, 2010; Diezmann & Watters, 2001, Kanevsky, 2011; Karaduman, 2010; Singer et al., 2016; Sriraman, 2003; Winebrenner, 2001; Zedan & Bitar, 2017) as should be included in the educational life of gifted students. Being

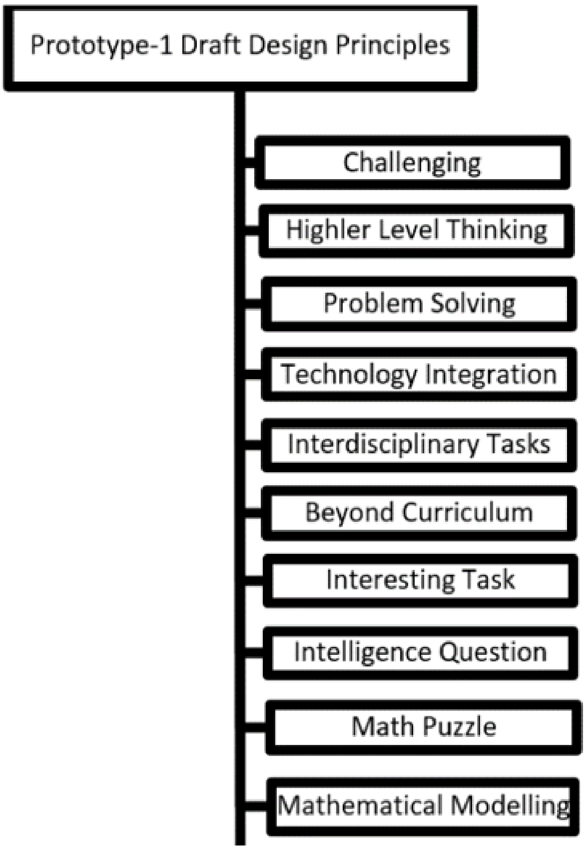

Figure 2. Initial Design Principles

interesting is the other mostly mentioned characteristics for the differentiated tasks because researchers (Johnson, 2000; Karaduman, 2010; Wilkins et al., 2006) suggest that gifted students need engaging tasks that catch their interests to get rid of boredom since they may not always have intrinsic motivation. Tasks requiring higher level thinking are the other characteristics because to differentiate instruction satisfying gifted students' needs, they should be provided with tasks promoting higher level thinking (Chamberlin & Chamberlin, 2010; Freiman, 2006; Karaduman, 2010; Sriraman, 2003; Winebrenner, 2001). In addition to these, problem solving (Freiman, 2006; Gavin et al., 2013; Karaduman, 2010; Renzulli, 1986; Tieso, 2002), mathematical modelling (Kim & Kim, 2010), interdisciplinary (Berger, 1991; Freiman, 2006; Karaduman, 2010; Renzulli, 1986) and technology integrated tasks (Johnson, 2000; Siegle, 2004) were also mentioned as opportunities that may be provided for the differentiated needs of gifted students. Additionally, providing intelligent question may be another opportunity that may meet gifted students' needs to think more and motivate them to worry and struggle about answer because they are abled and mostly motivated to think about complex or high-level questions (Baykoç, 2014; Freiman, 2006; Johnson, 2000). Besides, literature reviews and needs analysis studies in preliminary phase also reflected that math puzzles (Freiman, 2006; Gavin et al., 2013) and beyond curriculum tasks (Johnson, 2000; Karaduman, 2010; Rotigel & Fello, 2004) may be provided to gifted students as differentiated tasks.

To sum up, based on the data obtained from preliminary research phase, characteristics mentioned above compiled and provided as the initial design principles to construct differentiated tasks of mathematically gifted students as seen in **Figure 2**. These differentiated tasks with draft principles were tested in prototyping phases, and necessary modifications, which were presented as the other findings of this design-based study, were conducted.

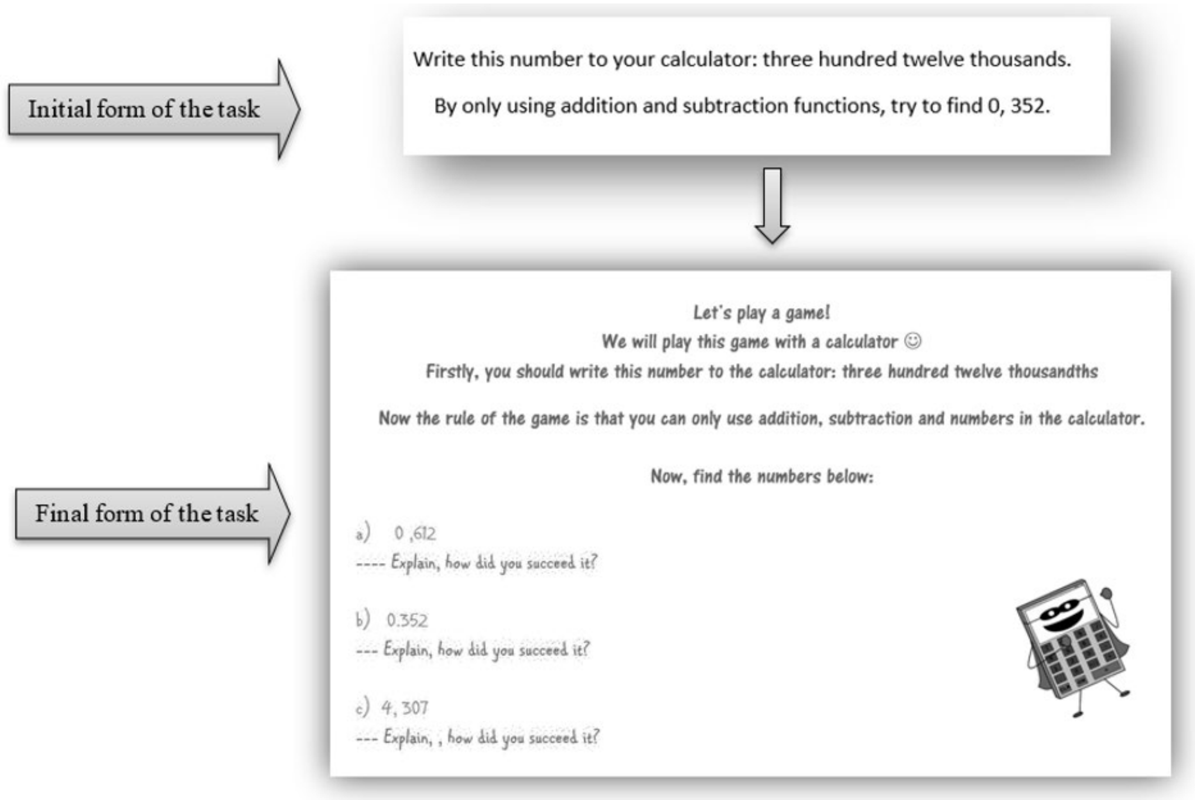

Figure 3. Initial and final form of the 25th differentiated task

Modifications for Tasks

As stated before, the tasks of the present study were designed based on the initial design principles and these tasks were evaluated by the experts, applied in real classrooms both in try-outs and field test. In this process, to better fit the tasks to the characteristics, some modifications to the tasks were conducted. Firstly, expert opinions helped to modify the content of the tasks and their suggestions were two-fold; modifications to avoid ambiguity in the tasks and modifications regarding the characteristics of the task. That is, some of the expressions in the tasks which may cause to lose in the mathematical meaning, typographical errors or spelling mistakes were modified based on the experts' suggestions. Additionally, some modifications to better fit the characteristics mentioned in the design principles were conducted.

The characteristics of the challenge were the most conspicuous on which the experts focused. Some of the activities (2nd, 10th, 25th, 26th, and 37th) were modified to better match the characteristics of challenge in line with the developmental characteristics of gifted students. To illustrate, most experts suggested changing the numbers in the 2nd and 25th tasks to make the task more challenging, requiring comprehensive operations with more complex reasoning. So, as can be seen in **Figure 3**, the decimal numbers in the options were changed to make them more challenging.

Moreover, 17th task was reorganized as more suitable for the puzzle form characteristic while the 6th (**Figure 4**) and 30th tasks formed as more interesting for the students at that grade to be more suitable for the interesting characteristic.

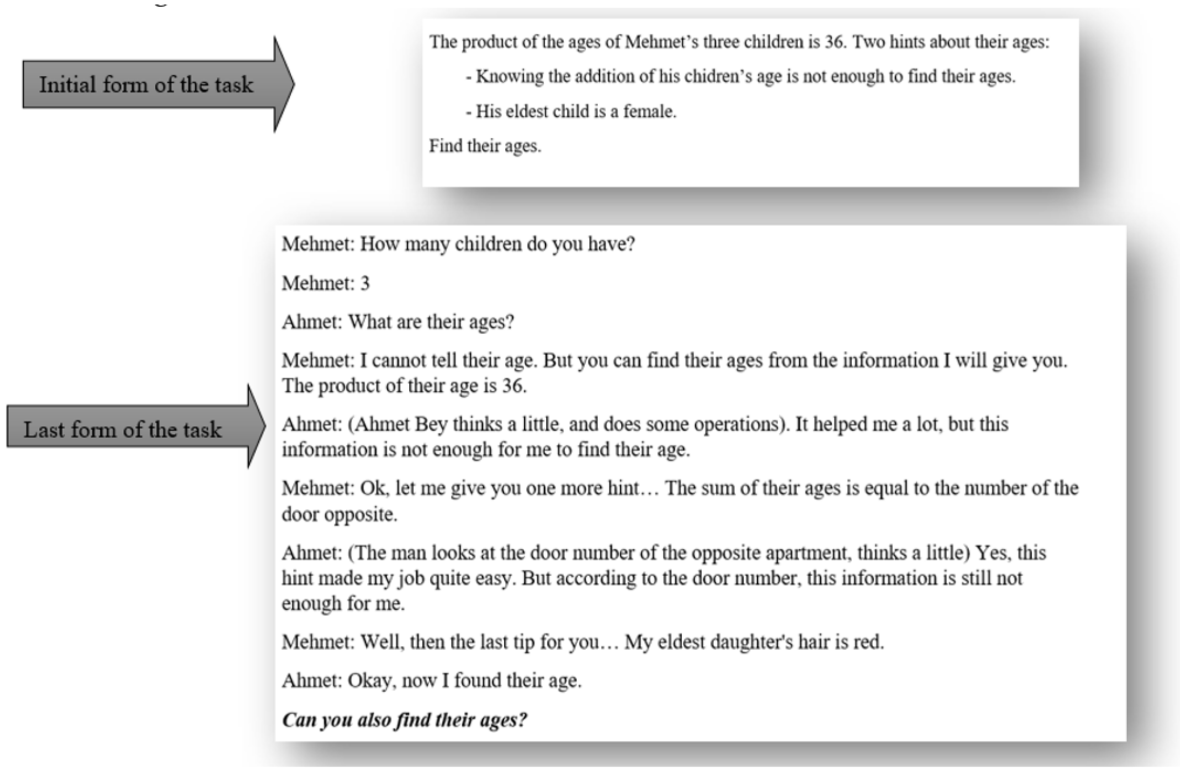

Figure 4. Initial and final form of the 25th differentiated task

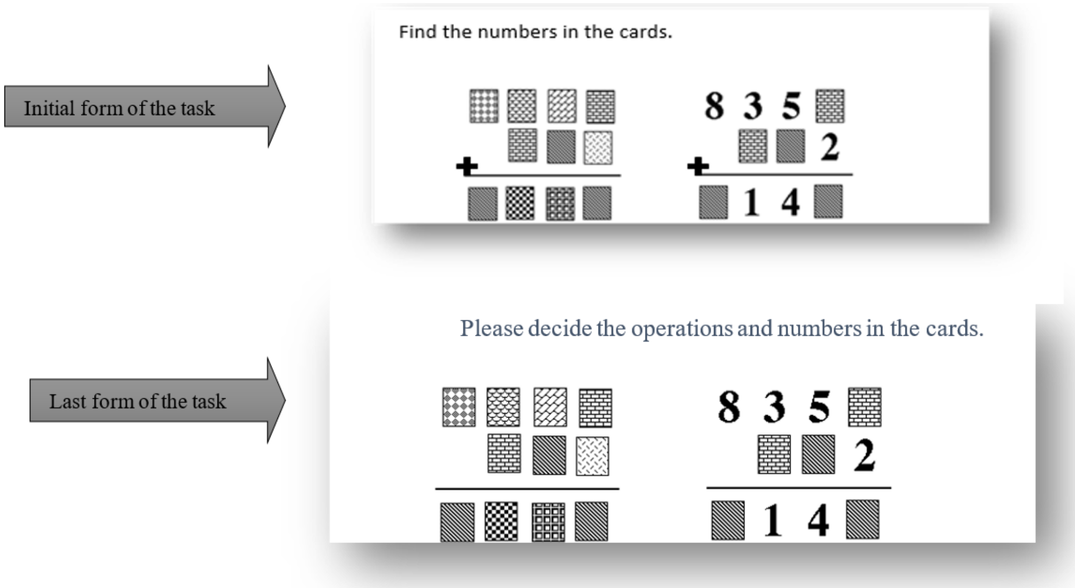

Figure 5. Initial and final form of the 3rd Differentiated Task

In addition, the modifications were also conducted based on the views of teachers and students in try-outs and field test to be more coherent to the design principles. For example, in the 3rd task (**Figure 5**), while two of the teachers (TS and TM) were using the task in the first week of the try-outs, they suggested to remove the addition symbol and force students to first guess what operation it might be. Both teachers and researcher discussed the activity and to make the task more challenging, it was decided that students should first decide which operation is suitable for the task. As a result, a note was added to the question stating that students should first decide the operation and then solve the problem. Following this update, the teacher (TN) used the new version of the task in her classroom in week-5 and expressed her observations about the challenge's success in promoting the cognitive skills of gifted students.

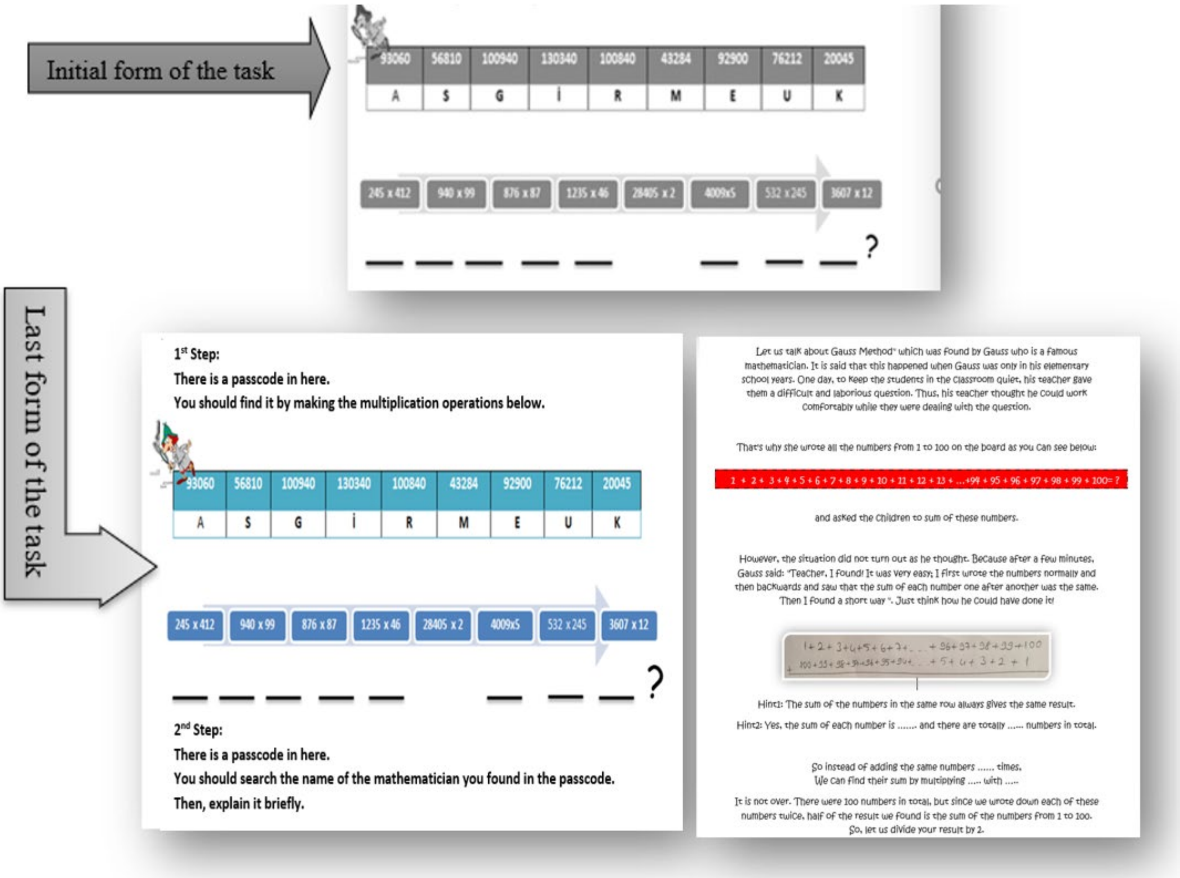

Figure 6. Initial and final form of the 1st differentiated task

As another modification, in the 1st task (**Figure 6**) about Gauss, the students found the name by multiplication but could not understand what they should do. When they could not understand the need for search about mathematician, the teachers and researcher observed that interdisciplinary characteristics of task became problem. Hence, to better reflect interdisciplinary characteristic of task, an additional question was added at the end, which provided a hint that the name they found belonged to a popular mathematician for whom they should look. Moreover, to make the task more challenging, an additional extension task that leads to discovery of Gauss's theorem was also added. By this way, the characteristics of the tasks were modified as not only interdisciplinary but also challenging and higher order thinking.

Furthermore, when performing the 12th task (**Figure 7**), all students initially answered "twenty"; thus, an additional cue was added to the query that "the answer is not twenty" to keep students focused on the challenge of the question. Prior to this addition, all gifted students expressed boredom and ease with the task because they believed the response was twenty and did not want to think about it. However, as indicated by the design principles, the task became more fascinating and challenging for them after the additional note. For example, the student indicated his pleasure with the challenge and interesting characteristics of this task;

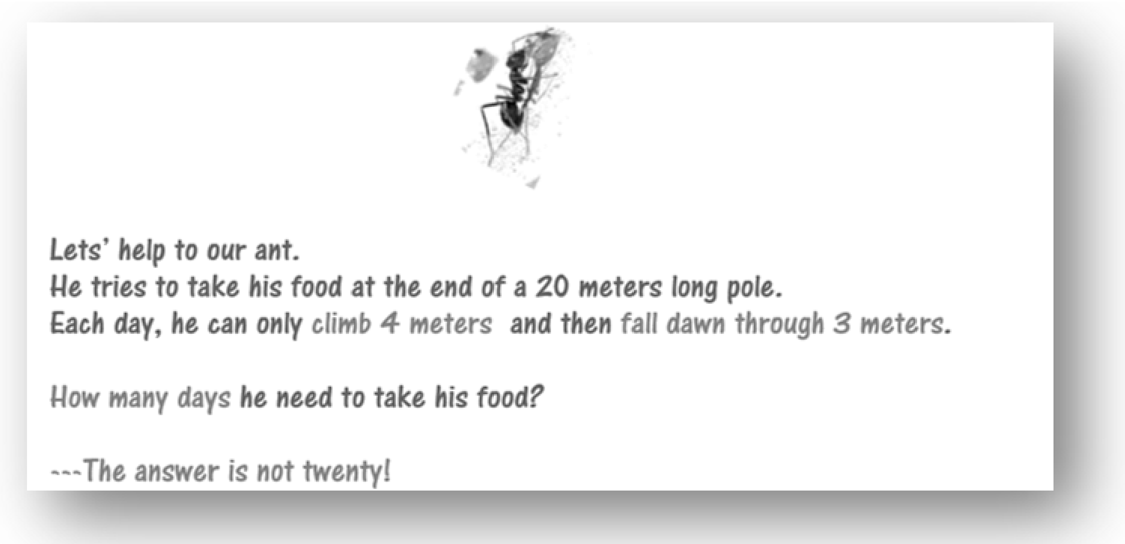

Figure 7. Last form of 12th differentiated task

> *"The activity seemed boring and I didn't want to try to solve it. But, when my teacher said that the answer was not 20, I was curious about it and tried to find. I did it! (S1, informal talks)."*

Additionally, for the 13th and 15th tasks having beyond curriculum characteristics, some hints for the students were suggested by the teachers so they could go beyond their level. For instance, a small-scale example for the 13th tasks and an explanative sentence about perimeter of the circle was added for the 15th tasks. By this way, students could comprehend these beyond curriculum ideas and focus on the challenge of the task. Besides, the tasks (21st and 22nd) and their structure were modified to make them more reasonable and interesting to attract gifted students' attention. After these modifications, it was observed that mathematically gifted students in classroom were more involved in the task and the teacher (TS) stated:

> *"After this modification, the activity became more effective for students' thinking way of difficult fraction problems because now, it is more interesting and non-routine for them. (TS, Week-4, try out)."*

In line with such kind of modifications obtained from the experts, students and teachers, the last forms of the tasks were obtained. By this way, not only differentiated tasks but also their characteristics took their final form and presented in the following heading as the other category of findings.

Final Design Principles: Characteristics of Differentiated Tasks

Through the prototyping phases, it was seen that initial, draft form of characteristics and differentiated tasks were insufficient to indicate the required design guidelines for differentiated tasks. These characteristics which were disorderly listed in one heading initially (**Figure 2**) are gathered in two main categories: characteristics in terms of content and characteristics in terms of type, and for each of these characteristics, some of the sub-characteristics were defined. That is, characteristics reflected in two main headings with their subheadings (**Figure 8**) were found most proper to clearly define and state the characteristics of differentiated tasks, as final design principles.

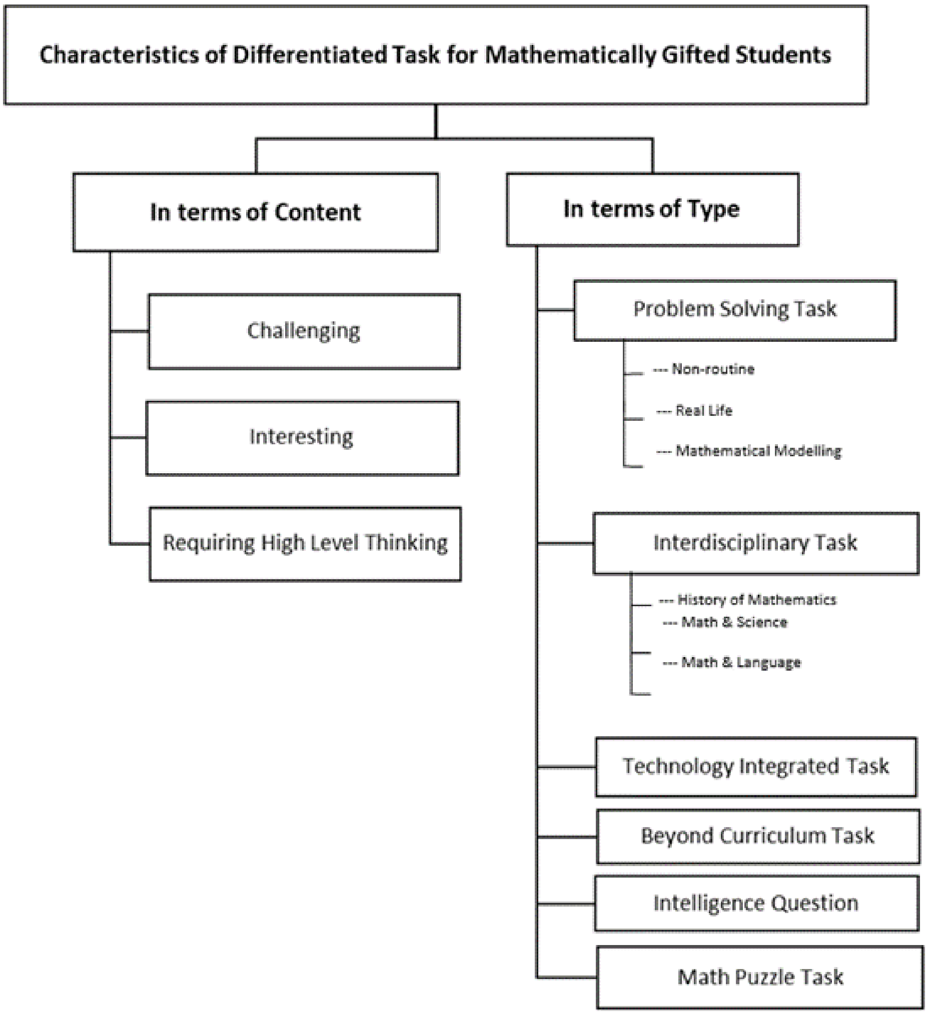

Figure 8. A Summary of Final Design Principles

For these final design principles, it was also concluded that any differentiated task for mathematically gifted students must have characteristics from each of these content and type characteristics. Besides, from each of these basic characteristics, at least one sub-characteristic should be selected. That is, any differentiated task for mathematically gifted students must have at least one content related characteristic and at least one type characteristic. For example, a challenging (content characteristic) and interesting (content characteristic) problem solving task (type) could be applied to the mathematically gifted student.

Characteristics in terms of Content. It was agreed that differentiated tasks should have at least one of the content characteristics that are challenging, interesting, or require higher level thinking. The ultimate version of the characteristics of the tasks was determined through various sources of data from students, teachers, and experts. The following presents the final version of these content characteristics with some underlying rationale for their use as differentiated tasks for mathematically gifted students.

Challenging: According to the data from preliminary research phase (Chamberlin, 2002; Diezmann & Watters, 2001; Gavin et al., 2013; Karaduman, 2010; Sriraman, 2003), the prototyping phase backed up the notion that tasks should be challenging for gifted students to maintain their interest during instruction by adding meaningful complexity to the tasks. In the context of this study, challenging means compelling the students with an appropriate level of difficulty, and the level of challenge may change with student level. Practitioner data also supported the idea that to be challenging, the student should not see the answer directly; it takes some effort and a thought process, as well as the integration of all knowledge and experience, to obtain the solution.

Data collected from teachers and students through tryouts and field tests revealed that some of the tasks adequately challenged and fulfilled the cognitive needs of mathematically talented students. During the implementation process, the students articulated themselves through different comments. For example, they stated that these tasks were coerced, or they posed a challenge.

> *"The activity (10th Activity) challenged my brain (S3, After sheet form, try out)."*

> *"This challenging activity (23rd Activity) compelled me to think more about fractions, the level of my intelligence increased (S7, After sheet form, try out)."*

As seen, the students were convinced from being challenged in mathematics lessons by means of the tasks labelled as having challenging characteristics, and it was observed in their reactions, too. Similarly, the teachers stated their satisfaction with the challenging tasks that their children could struggle, which they do not have in their regular tasks. They saw the tasks as challenging for the grade levels and developmental characteristics of the students. One of the teachers even stated that his perceptions of challenging tasks had changed,

> *"When I first see this activity (22nd task), I thought that my students cannot do and this excessive challenge makes my gifted ones be alienated from this type of tasks. However, interestingly, this challenge took my students' pleasure. They could manage this challenge and really enjoyed from difficulty. (TM, After Sheet Form)"*

Interesting: Within the scope of this study, the interesting side of activities means novelty that attracts students' attention, and tasks that are non-routine, attractive, novel, and oriented to students' interests were accepted as interesting. These interesting tasks were designed in the preliminary research phase and then developed through expert opinion, try outs and field testing. In these instructional practices, teacher and student data reflected that the activities captured the interest of mathematically gifted students and helped to capture their attention during instruction. Moreover, students paid more attention to these tasks and had more enthusiasm because they found them very interesting and different from their regular classroom tasks. It was also seen in the classroom observations and confirmed in the follow-up questionnaires that the tasks that were indicated as interesting tasks attracted the attention of the gifted students:

> *"This task (16th) was so interesting, I have never seen such a tortuous mathematics activity (S9, After sheet form, field test)."*

> *"Interesting side of the activity (30th) made me keep my shoulder to the wheel (S1, After sheet form, try-out)."*

Similar to these, teachers' after sheet forms and interviews went along with the students' ideas that they also specified students' engagement with interesting tasks. Exemplary:

> *"I can definitely say that the task (27th) was very interesting for my gifted students, which is one of the most important characteristics for them. If the task is not interesting, my gifted does not struggle (TS, After sheet form, try outs)."*

In brief, as supported by preliminary research phase (Johnson, 2000; Karaduman, 2010; Wilkins et al., 2006) and participants of the study, it was concluded that gifted students need engaging tasks that pique their interest to relieve boredom and lapses in concentration because they do not always have intrinsic motivation. Findings from the preparation and prototyping phases indicated that tasks should have interesting features for gifted students.

Requiring Higher Level Thinking: Tasks that require higher level thinking are the last characteristic in terms of content. As the reflections from the preliminary phase indicate, differentiated instruction that meets the needs of gifted students should provide them with tasks that promote higher-level thinking (Chamberlin & Chamberlin, 2010; Freiman, 2006; Karaduman, 2010; Sriraman, 2003). This higher level thinking helps to increase students' critical thinking. Although gifted students are born with critical thinking skills, they should be exposed to activities that teach them to think critically (Ktistis, 2014). Therefore, more emphasis should be placed on higher order skills for gifted students. Bloom's Taxonomy (Bloom, 1956) is particularly helpful in planning activities that require higher order thinking. Based on his revised taxonomy, tasks at the levels of analysis, evaluation, and creation serve this purpose and allow students to think at a higher level (Anderson et al., 2001).

Some of the tasks involving higher-order thought were structured to foster critical thinking skills in mathematically talented students based on these ideas. Students tended to be accustomed to thinking at the "Remember, Understand, or Apply" stage of Bloom's Taxonomy at the start of the prototyping and field test. However, when they participated in the exercises, their ability to think critically was enhanced by these tasks being at the analysis, evaluation or creation level. It was concluded that it is important to expose mathematically talented students to tasks that require higher level thinking in regular mathematics classes. Furthermore, classroom observations reflected that students even began to ask questions at these levels in their regular classes. Similarly, teachers shared their experiences that their students needed to think critically in the classrooms to complete these tasks. As an example,

> *"This task (25th) absolutely required thinking in higher order because it is not enough to know the decimals and their place value, they should analyze which numbers and operations to be used to obtain final numbers. It needs evaluation about place value in decimals. I think, higher level thinking is really necessary for these students' tasks (TN, After sheet form)."*

Characteristics in terms of Type of Task. At the end of the design-based process, in addition to content characteristics for differentiated tasks of mathematically gifted students, characteristics in terms of type were also determined. In the initial phase, the design principles consisted of a main heading and these type characteristics were included as content characteristics (**Figure 2**). However, problem-solving, interdisciplinary, beyond curriculum, technology-integrated, intelligence question, and math puzzle were found to indicate the type of tasks in prototyping phase and are presented below.

Problem Solving Task: Problem solving tasks were accepted as the characteristic for the differentiated tasks of gifted students based on the preliminary research phase (Freiman, 2006; Gavin et al., 2013; Greenes, 1997; Karaduman, 2010; Pierce et al., 2011; Renzulli, 1986; Tieso, 2002). Then, these tasks were used in classrooms and, mathematically gifted students in try outs and field test showed great interest in problem solving activities that are non-routine, related with real life or include mathematical modelling, which are stated as the sub-characteristic of problems solving characteristics. That is, non-routine problems which cannot be solved by using knowledge directly and unordinary problems that could not be solved with routine methods (Arslan & Altun, 2007), real life problems that are related with daily life and use realistic contexts or mathematical modelling problems were taken as proper for the sub-dimension of mathematically gifted students' problem-solving tasks.

It was obvious that the students were used to solving repetitive problems or drills, and gifted students could solve these types of problems easily, although sometimes with boredom at the start of the tryouts. It was noticed that they did not spend much time thinking deeply about the issues. And when asked about issues in their pre-interviews, all but two students mumbled. Nevertheless, when confronted with the differentiated tasks, they realized that mathematical problems are not limited to those they can easily solve. The students spent a lot of time on the problems without getting bored and indicated that they were happy and satisfied with these different problem-solving activities. Therefore, at the end of the trial, teachers and researchers decided to allow problem solving tasks as a good option for differentiated tasks and examples from the students' statements showing their satisfaction with non-routine and real-life problems are given below:

> *"The activity (9th task) was very different from the ones we usually do in our classroom. It was enjoyable and struggled me to find the correct numbers. In fact, addition is a simple job but this was an unusual problem which I prefer to do (S3, informal talk, try out)."*

> *"The activity (20th task) reminded me my marbles and I visualized the scenario. I had a good time because it was like a problem in my life (S9, After sheet form, field testing)."*

During the discussions with teachers in try outs and field testing, mathematical modelling tasks were also determined as a type of the problem-solving task because it is a process that a problem in real life is formed as a mathematical problem whose solutions are again adapted to that real life problem (Berry, 2002; Blum, 2002). Solving mathematical modelling problems is complicated process that needs mathematical thinking to reach a goal by using conceptual tools and multiple interpretations (English; 2003; English and Watters, 2005). More clearly, they require explaining, making sense and interpreting the case from the mathematical ways (Lesh & Doerr, 2003) and show the application of math in real life (Heymann, 2003). In classroom practices, the teachers were content with these problems due to their role in making their students think various dimensions of the problem. The students were also satisfied with the differentiated structure of mathematical modelling problems. For instance,

> *"I liked this different and difficult employee problem. I feel like a real determinant of a company. It was not boring and similar as in our regular lesson. I had a good time while using my brain. I want much more from these problems. Do you have any more? (S7, Informal talks, try out)."*

All the data from the teachers, students and observations reflected the idea that these problem-solving tasks seem different, unusual, more realistic, and interesting to solve for the mathematically gifted students, which makes it as a good option for differentiated tasks of mathematically gifted students.

Interdisciplinary Task: Interdisciplinary task which allows integrating more than one discipline into the teaching of any concepts (Beane, 1997) was accepted as another characteristic because learning the application of math in other fields helps students to get the answers for their questions (Berger, 1991; Freiman, 2006; Greenes, 1997; Karaduman, 2010; Renzulli, 1986; Sriraman & Sondergaard, 2009). In line with this data obtained from preliminary phase, since talented students could see mathematics more holistically by comparing it to other disciplines or lessons, their passion for applying mathematics to other disciplines was observed in classroom applications. When students were able to link their knowledge through disciplines, they realized that mathematics is everywhere and were content to learn.

Among those interrelationships with other lessons, some interdisciplinary tasks were specified in the final design principles. For instance, mathematical history tasks, tasks related with mathematics and environmental issues, integration of mathematics and language lessons were determined as the subdimensions of these interrelations. Because gifted students have great curiosity about historical background of concepts and wonder about issues that happened in the past (Yevdokimov, 2007), students' engagement and curiosity about historical background of the mathematical concepts could be observed through the classroom applications in mathematical history tasks, which were also suggested in Aydemir and Çakıroğlu' s (2013) study. Additionally, as suggested in some studies (Aydemir & Teksöz; 2014; Jianguo, 2004), the fusion of mathematics and environmental issues provided a practical way to raise environmental awareness and present real-life reflections of mathematics. It was found that students were curious about the real-life effects of their own behavior and were satisfied when they could see them using mathematical operations, which helped them draw conclusions about both the mathematical and scientific sides of the environmental problem. Lastly, since the students have Turkish, English and mathematics classes in almost every class, linking the tasks to these subjects made the students feel confident in solving mathematical problems and enjoyed using these disciplines together.

Beyond Curriculum Task: Consistent with the data from preliminary research phase (Johnson, 2000; Karaduman, 2010; Rotigel & Fello, 2004), the prototyping phase also supported the idea that tasks beyond the curriculum should be included in the tasks for gifted students because they seek to identify, discover, teach, or use any mathematical concepts that are ahead of the standard sub-learning domain in the curriculum. Parallel with this, it was seen in classroom applications that these tasks helped to fulfil gifted students' need in learning more. While doing beyond curriculum tasks (16^{th}, 23^{rd} and 28^{th} tasks), discovering or using new concepts, students did not realize that they were learning a new mathematical concept. In this way, they became open to discovering the logic of the new concept by relating it to their existing knowledge. For example, in the 23rd task, they had to share half or part of the pizza with other people; they needed to discover how to divide fractions by a number. Although they did not know how to divide fractions, they were able to reason and understand the real meaning of division by fractions instead of memorizing the rules, which made the mathematically gifted students task-oriented and excited during the activity. In conclusion, mathematically gifted students enjoyed discovering and applying new concepts and this helped to increase in their cognitive potential. In this way, they were not limited to grade-level curriculum objectives; on the contrary, they were able to engage in rich mathematics and showed great enthusiasm in discovering new concepts.

Technology Integrated Task: As indicated in the preliminary research phase, most of the gifted students are very talented, motivated and successful in using technology and the role of technology in their motivation could be used as another option to meet their needs (Johnson, 2000; Siegle, 2004). Since technology allows students to proceed at their own pace (Kaput, 1992; Özçakır, 2013), in this study, some tasks (2^{nd}, 24^{th}, 25^{th}, and 26^{th}) provided gifted students with individualized pathways and it was inferred from student and teacher data that the use of technology helped to engage gifted students in effective learning. In these activities, students understood the mathematical concepts while enjoying with the technology-integrated tasks because it was unfamiliar and interesting for them to use technology in the classroom. Therefore, during the long discussions in the prototyping phase, technology-integrated tasks were considered as proper for differentiated tasks.

Intelligence Question: With the help of data obtained from preliminary phase (Baykoç, 2014; Freiman, 2006; Johnson, 2000) and prototyping phases, tasks including intelligent question was determined another type of characteristic. It was observed that tasks (the 3^{rd}, 4^{th}, 5^{th}, 31^{st}, 32^{nd}, 33^{rd} and 34^{th}) helped to satisfy gifted students' needs to think more and motivated them to struggle about the answer. Furthermore, the teachers in the study said that the intelligence questions helped gifted students improve their ability to look at situations from different perspectives because, in order to get a meaningful response to an intelligence question, students had to think in depth and were forced to look at cases from different perspectives. As a result, students had a good time contemplating these activities and stretching their minds to find the solution. As an example,

> *"This mind question annoyed me. I went crazy to find the answer. I had to think all the points and details to find the answer. I liked all these feelings and efforts. I would like more questions like this (S4, informal talk)."*

Math Puzzle: A math puzzle is described in this study as a problem that requires mathematics to solve using some specific rules, and the solver must find a solution that satisfies the given condition. As stated in the design principles (Freiman, 2006; Gavin et al., 2013), a math puzzle is a distinct category that can be used as a differentiated task for mathematically gifted students. As a result, some tasks (1st, 3^{rd}, 7^{th}, 8^{th}, 10^{th}, 14^{th}, 17^{th}, 30th tasks) were formed as math puzzle tasks, allowing gifted students' mathematical ideas to be evaluated and synthesized with the rules of the puzzle while inspiring them. The teacher explained how to use math puzzles to learn new or old math concepts while keeping students engaged. As a result, during tryouts and field tests, students had to recall and apply their mathematical skills when actively participating in these activities. Furthermore, some students perceived these activities as scrambling and found fun in looking for clues or keys, and they were content with participating in these types of tasks in the classroom. Similarly, some of them saw these activities as a mathematical game to be played in class. As an example,

> *"In this challenging math puzzle, I felt like playing a game in mathematics lesson. I can meet my game need by this way (S9, informal talks).*

DISCUSSION

Characteristics of differentiated tasks, whose validity and practicability were satisfied by means of key points in preliminary and prototyping phases were obtained as the main output of the present study. Thus, four quality criteria; relevance, consistency, practicality, and effectiveness (Nieveen, 2013) were satisfied by means of this design-based study and all these helped to fine-tune design principles to better address the need of differentiated tasks in classrooms (Herrington et al., 2007; Plomp & Nieveen, 2013).

According to the findings, tasks designed for mathematically gifted students should be challenging, interesting, and require higher level thinking. In addition to content-related characteristics, certain task styles have also been found to be more successful and advantageous for mathematically gifted students in both the literature and implementations. Therefore, how the content was presented to these students is another significant dimension; challenge, interest and high-level thinking should be integrated to the problem-solving task, interdisciplinary task, intelligence question, math puzzle, technology-integrated task. It was seen in the findings of the study that existence of these design principles reflected the effectiveness of differentiated tasks for mathematically gifted students. For instance, when the tasks lacked novelty and challenge, gifted students did not want to struggle with them. As a result, it is reasonable to conclude that these differentiated tasks should be included in mathematics classrooms to assist gifted students in reaching their full potential.

During this design-based study, the teachers, the guidance service and school administration collaborated and worked in systematic discipline. Thus, such collaborations in schools are highly recommended to obtain the whole picture for gifted education. In fact, in Turkey, guidance services mostly and sometimes only carry out studies for the inclusive or problematic students, but the gifted ones also need special education and guidance service is the crucial point in gifted education in schools.

Additionally, teacher education should be prioritized in order to address the root cause of the issue. In Turkey, teacher education systems do not place enough emphasis on gifted students (Baykoç, 2104) since pre-service teachers graduate from universities with little awareness of giftedness. At best, some of them attend a special education course, but the emphasis is mostly on children with disabilities. Therefore, it is important that teacher education programs include courses related to gifted students and that pre-service teachers are exposed to gifted students and observe their characteristics. In addition, the general competence of teachers is another point to be emphasized because teachers of gifted students should be well equipped with the subject knowledge, knowledge of gifted students, pedagogical and pedagogical content knowledge. Nonetheless, most teachers lack expertise and skills in areas such as specialized subject knowledge and alternative pedagogical methods for talented students (Rakow, 2012; Martin & Pickett, 2013). Teachers should broaden their pedagogical material awareness at this stage, as gifted students can become more troublesome in terms of both classroom management and cognitive demands. As a result, teacher plans should include student knowledge, knowledge of instructional strategies and subject matter knowledge to discover gifted students' ability and sustain their interests (Park, 2005).

Since mathematically gifted students are not given differentiated tasks in the regular curriculum and textbooks, curriculum developers should give these types of tasks a place in the mathematics curriculum from the earliest grades. This will allow them to learn and improve how to think mathematically, critically, and analytically, and provide earlier experience with these types of questions. They should also have the opportunity to discover the connections of mathematics with other real-life disciplines. For example, the objectives of mathematics should be integrated with those of other subjects such as history of mathematics or science. In this way, students could see the holistic picture that all sciences are nested and interconnected. Moreover, the modern age requires technological competence and expertise, and these technological advances arouse the interest of gifted students (Periathiruvadi & Rinn, 2012). Thus, this role should be included in the mathematical tasks for their motivation and, the integration of technology, including through the use of computer programs, calculators, Geogebra, etc., may be used to motivate and improve the intellectual understanding of mathematically talented students. Last but not least, the content of selective mathematics lessons or math club studies should be enriched, and these lessons could be used as an opportunity to encourage and afford the full potential of mathematically gifted students. In this manner, teachers' worries about time management and the overload of curriculum demands may be alleviated.

Since the sample of this study consists of mathematically gifted students, an uncommon subset of the population (Baykoç, 2014), the number of mathematically gifted students in six classrooms was a few. As a result, future research should consider this restriction, and the same study may be repeated with a larger sample and different teachers involved in the design, development and evaluation process. Following that, the preschool and early elementary school years are critical in recognizing and influencing gifted students' values and behaviors (Baykoç, 2014). If students are cognitively, emotionally, and socially fulfilled in their early years, they will be more accessible and able to think objectively, mathematically, and analytically because they will be used to it. Given the value of recognizing and fostering giftedness at a young age (Karnes & Johnson, 1991; Pfeiffer & Petscher, 2008; Sankar-DeLeeuw, 1999), undertaking a similar study with different grade levels and designing tasks specifically for the preschool and elementary grades is strongly advised. A longitudinal study that looks at students' needs in their early years or over a longer period may also be performed. Another potential research subject may be the degree to which teachers, families, peers, and curriculum designers are aware of these essential characteristics of tasks for mathematically gifted students. Since the purpose of design principles is to encourage people to benefit from their own environments by choosing the most suitable principles in terms of content and procedure (McKenney et al., 2006), the design principles of this study may be evaluated in other settings with other participating teachers and students. In this manner, their efficacy could be checked, or the principles could be changed to better suit the needs of gifted students in mathematics classrooms.

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